1)

Notes for Paper 54, Part 5

The basic intent of paper 54 is to develop causal and generally covariant wave mechanics. The starting point is to find the origin of the minimum action, $\hbar$, in general relativity. This was covered in volume one but due to rapidly increasing understanding it is developed more systematically in this paper. So far in these notes we have derived the Planck Einstein and de Broglie equations: $E = \hbar\omega$ and $p = \hbar\kappa$, and have also found the fundamental lagrangian density of ECE theory:

$$\mathcal{L} = c^2 T + D_\mu q^a{}_\nu D^\mu q_a^\nu. \quad -(1)$$

It is important to note that $\mathcal{L}$ is a density, with units of energy divided by volume. So at the outset it is seen that the action is going to involve volume. This lead to my papers on the modification of the Heisenberg commutator eqn. in volume 2, explaining the data by Croca. In one of those papers $\hbar$ was replaced by $\hbar/V$, an action density.

The action is defined by:

$$S = \frac{1}{c}\int \mathcal{L}\, d^4x \quad -(2)$$

and has the units of energy multiplied by time. These

2) are also the units of angular momentum. The phase of a wave is defined as:

$$\Phi = \exp(iS/\hbar) \quad -(3)$$

In the rest frame the lagrangian density (1) is

$$\mathcal{L}_0 = c^2 T_0 = \frac{mc^2}{V_0}, \quad -(4)$$

which is the rest energy:

$$En_0 = mc^2 \quad -(5)$$

divided by the rest volume V_0. The rest action is then:

$$\hbar = S_0 = \frac{1}{c}\int \mathcal{L}_0 \, d^4x. \quad -(6)$$

Using eq. (4):

$$\boxed{\hbar = mc \int \frac{d^4x}{V_0}}. \quad -(7)$$

Therefore:

$$\frac{1}{c}\int \frac{d^4x}{V_0} = \frac{\hbar}{mc^2} \quad -(8)$$

for given m is a universal constant, the Planck constant divided by the rest energy, two famous quantities in physics.

3)

It follows that the rest four volume d^4x must be:

$$d^4x = V_0 \, c dt_0 \quad - (9)$$

where dt_0 is a time interval in the rest frame.

Therefore:

$$\int \frac{V_0 \, dt_0}{V_0} = \frac{\hbar}{mc^2} \quad - (10)$$

$$= \int dt_0 = t_0 .$$

For a given m, this time interval must be a universal constant.

The volume V_0 cancels out on the left hand side of eqn. (10), but more generally in general relativity $\hbar$ is defined by eqn (7).

The universal time interval $\int dt_0$ in eq. (10) has been obtained by minimizing the action S, i.e by finding the action S in the rest frame. This is the meaning of the Planck constant $\hbar$ in ECE theory. The minimized time interval $\int dt_0$ is in accordance with both the Hamilton Principle of Least Action and the Fermat

4) Principle of Least Time. Thus $\int d\tau_0$ is the least time. These two principle of causal physics form the basis of wave mechanics. This conclusion is well described in P. W. Atkins, Molecular Quantum Mechanics (OUP, 2nd. ed., 1983). We base the following description on Atkins.

As described by Atkins in his pp. 24 ff. the Fermat principle of least time is the basic rule governing the propagation of light in optics. The path taken by a light ray through a medium is such that <u>the</u> <u>time of passage</u> <u>is a</u> <u>minimum</u>. From this principle, Snell's law for example can be obtained; the angle of incidence is equal to the angle of reflection. The wave nature of light accounts for this through the Fermat principle of least time (Atkins pp. 25 ff.). The essence of the argument (p. 27) is that light consists of waves:

$$\phi(x) = a \exp(2\pi i x / \lambda). \quad -(11)$$

The amplitude at one point can be related to that at another by:

$$\boxed{\phi(P_2) = e^{i\phi}\phi(P_1).} \quad -(12)$$

5) Eq. (12) is the fundamental origin of the Schrödinger equation, (Atkins pp. 27 and 28).

Light takes paths such that the phase length is minimized. This is the precise form of the Fermat principle. In the limit of geometrical optics the phase length ϕ is infinite, the light appears to travel in straight lines. There is no curvature and this is a kind of weak field limit of general relativity or ECE theory. This argument indicates the fundamental meaning of the interval / etc.

The propagation of particles is given by Hamilton's principle of least action. Particles select paths between two points such that the action associated with the path is minimized. This statement is equivalent to Newtonian dynamics in the weak field limit of ECE theory. Particles adopt a least path and light waves adopt a least ~~to~~ time. The reason is the same, the phase length ϕ is minimized. This is the famous wave / particle dualism. In ECE theory the wave function is the tetrad.

6) The particles are described by eqn (12). Waves are propagated along a path that makes ϕ a minimum, particles are propagated along a path that makes S a minimum. Thus ϕ is proportional to S. The constant of proportionality must ~~be~~ have units of inverse action. In the limit of classical mechanics, ϕ goes to infinity, so the constant of proportionality must be very small.

- Schrodinger's equation is recovered if:

$$\phi = S/\hbar, \quad -(13)$$

as in eqn. (3).

Eq. (12) describes a path from P_1 (a point at x_1, t_1) to P_2 (a point at x_2, t_2).

Thus:

$$\psi(x_2, t_2) = e^{iS/\hbar}\psi(x_1, t_1) \quad -(14)$$

- Differentiate eq. (14) with respect to t_2:

$$\frac{\partial}{\partial t_2}\psi(x_2, t_2) = \frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\psi(x_1, t_1)\right). \quad -(15)$$

Now use the Leibnitz theorem:

$$\frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\psi(x_1, t_1)\right) = \psi(x_1, t_1)\frac{\partial}{\partial t_2}e^{iS/\hbar} + e^{iS/\hbar}\frac{\partial}{\partial t_2}\psi(x_1, t_1) \quad -(16)$$

7) Since $\phi(x_1, t_1)$ is not a function of t_2 we obtain:

$$\frac{\partial}{\partial t_2}\phi(x_1, t_1) = 0. \quad -(17)$$

Also:

$$\frac{\partial}{\partial t_2} e^{iS/\hbar} = \frac{i}{\hbar}\frac{\partial S}{\partial t_2} e^{iS/\hbar}. \quad -(18)$$

Thus:

$$\frac{\partial}{\partial t_2}\phi(x_2, t_2) = \frac{i}{\hbar}\frac{\partial S}{\partial t_2} e^{iS/\hbar}\phi(x_1, t_1). \quad -(19)$$

Finally use eq. (14) in eq. (19) to obtain the Schrödinger equation

$$\boxed{\frac{\partial}{\partial t_2}\phi(x_2, t_2) = \frac{1}{\hbar}\frac{\partial S}{\partial t_2}\phi(x_2, t_2)} \quad -(20)$$

This is the famous non-relativistic wave equation of Schrödinger equation of wave mechanics. In the

$$\frac{\partial S}{\partial t} = -E \quad -(21)$$

where

$$E = T_k + V_p. \quad -(22)$$

Here E is the total energy, T_k is the kinetic energy and V_p the potential energy.

8) So:

$$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar}\psi \quad -(23)$$

or:

$$E\psi = i\hbar\frac{\partial}{\partial t}\psi \quad -(24)$$

i.e

$$\boxed{E = i\hbar\frac{\partial}{\partial t}} \quad -(25)$$

Thus E is identified with the hamiltonian operator:

$$H\psi = E\psi, \quad -(26)$$

and:

$$H\psi = i\hbar\frac{\partial}{\partial t}\psi \quad -(27)$$

i.e. the time dependent Schrödinger equation.

It is seen that this is a causal differential equation, and as such cannot be interpreted as an expression of something acausal, or indeterminate. This was always Schrödinger's view, and also that of both Einstein and de Broglie.

In volume one of M. W. Evans,

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Generally Covariant Unified Field Theory (Abramis, 2005) this analysis was extended to the tetrad as follows:

$$\boxed{q_\mu^a(x^\mu) = e^{iS(x^\mu)/\hbar} q_\mu^a(0)} \quad -(28)$$

where $S(x^\mu)$ is now known (paper 54) to be described by eqs (1) and (2) of these notes.

Now apply the d'Alembertian $\Box$ to both sides of eq. (28):

$$\Box q_\mu^a(x^\mu) = \Box\left(e^{iS(x^\mu)/\hbar} q_\mu^a(0)\right). \quad -(29)$$

We know from the ECE Lemma that:

$$\Box q_\mu^a(x^\mu) = R q_\mu^a(x^\mu) \quad -(30)$$

so:

$$\Box\left(e^{iS(x^\mu)/\hbar} q_\mu^a(0)\right) = R q_\mu^a(x^\mu). \quad -(31)$$

In analogy with the derivation (14) to (19)

10) it is found that:

$$\Box\left(e^{iS(x^\mu)/\hbar}\, q_\mu^a(0)\right)$$
$$= q_\mu^a(0)\,\Box\, e^{iS(x^\mu)/\hbar} \quad -(31)$$

where:
$$\Box\, q_\mu^a(0) = 0 \quad -(32)$$

i.e.:
$$\Box := \frac{1}{c^2}\frac{\partial^2}{\partial t_2^2} - \frac{\partial^2}{\partial X_2^2} - \frac{\partial^2}{\partial Y_2^2} - \frac{\partial^2}{\partial Z_2^2} \quad -(33)$$

and:
$$q_\mu^a(0) := q_\mu^a(X_1, Y_1, Z_1, ct_1). \quad -(34)$$

Now use:

$$\Box \exp\left(iS(x^\mu)/\hbar\right)$$
$$= \frac{i}{\hbar}\,\Box S(x^\mu)\exp\left(iS(x^\mu)/\hbar\right) \quad -(35)$$

and eqn. (28) to find:

$$\boxed{\begin{aligned}\Box\, q_\mu^a(x^\mu) &= \frac{i}{\hbar}\,\Box S(x^\mu)\, q_\mu^a(x^\mu)\\ &= R\, q_\mu^a(x^\mu)\end{aligned}} \quad -(36)$$

Therefore:

$$R = \frac{i}{\hbar} \Box S(x^\mu) . \quad -(37)$$

However we know that:

$$R = q^\lambda_a \left(\partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu \right) - \partial^\mu \left(\omega^a_{\mu b} q^b_\lambda \right) \right)$$

$$-(38)$$

so we obtain an expression for the ECE action in terms of Cartan geometry:

$$\boxed{\Box S = -i \hbar q^\lambda_a \left(\partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu \right) - \partial^\mu \left(\omega^a_{\mu b} q^b_\lambda \right) \right)}$$

$$-(39)$$

In the rest mass limit it is known that:

$$R = -kT \rightarrow -\frac{km}{V_0} \quad -(40)$$

so:

$$\Box S \rightarrow \frac{i\hbar km}{V_0} . \quad -(41)$$

$$= \frac{i\hbar}{\lambda^2} \quad -(42)$$

where:

$$\lambda_c = \frac{\hbar}{mc} \quad -(43)$$

is the Compton wavelength.

So in the rest frame limit, the action of ECE theory is:

$$\boxed{\square S \rightarrow \frac{i\hbar}{\lambda_c^2}} \quad -(44)$$

Therefore this is the fundamental action of the Dirac equation of relativistic wave mechanics

$$\left(\square + \frac{m^2c^2}{\hbar^2} \right) q_\mu^a = 0. \quad -(45)$$

Eq. (45) is the rest frame limit of the ECE wave equation of generally covariant wave mechanics:

$$\left(\square + kT \right) q_\mu^a = 0 \quad -(46)$$

where:

$$R = -kT, \quad -(47)$$

and where R is given by eqns. (37) or (38). This is a causal derivation of the Dirac equation, from which we can obtain the Schrödinger equation in the non-relativistic limit.

If we are to interpret the Planck constant as a least action then according to eq. (6) S in eq. (44) must approach $\hbar$, so the latter is an expectation value of the wave equation for action in the rest frame limit:

$$\Box S = \frac{i}{\lambda_c^2} S, \quad -(48)$$

which more generally is:

$$\boxed{\Box S = i R S} \quad -(49)$$

and

$$S_0 = \hbar = \langle S S^* \rangle^{1/2}. \quad -(50)$$

This is again a purely causal derivation, showing that $\hbar$ is an expectation value of a wave equation of motion for action.