3. PLANCK CONSTANT, PLANCK-EINSTEIN AND DE BROGLIE EQUATIONS, AND THE SCHRÖDINGER EQUATION.

In the rest frame the ECE lagrangian density is the rest energy divided by the rest volume. Therefore the action in the rest frame is:

$$S_0 = \int \frac{mc}{V_0} d^4x . \qquad - (37)$$

The four volume in the rest frame is:

$$d^4x = V_0 \, c \, dt_0 . \qquad - (38)$$

Now identify the rest action with the Planck constant:

$$S_0 = \hbar \qquad - (39)$$

and the integral over the time interval with the inverse rest frequency:

$$\int dt_0 = \frac{1}{\omega_0} \qquad - (40)$$

to obtain the Planck Einstein equation in the rest frame:

$$E_0 = \hbar \omega_0 . \qquad - (41)$$

If applied to the photon rest mass this is also known {1-38} as the de Broglie equation for photon rest mass. It is known experimentally that Eq. (41) also holds for the photon when it travels for all practical purposes infinitesimally close to the speed of light with respect to an observer in the rest frame. In this case rest frequency is changed to ω . In special relativity this would be a Lorentz transformation of angular frequency but in general relativity a general coordinate transformation. However, the rest mass of the photon cannot be identically zero in ECE theory, because the action would be identically zero. The rest mass of the photon is very small but not zero. In ECE theory the Planck constant is the rest frame limit of the action:

$$S = c \int T \, d^4x . \quad -(42)$$

In the rest frame:

$$\hbar = mc \int \frac{d^4x_0}{V_0} \quad -(43)$$

and if:

$$\int \frac{d^4x_0}{V_0} = \frac{c}{\omega_0} \quad -(44)$$

we obtain:

$$E_0 = \hbar\omega_0 = mc^2 . \quad -(45)$$

More generally, and for any particle, the action that gives the ECE wave equation in any frame of reference is given by:

$$S = \frac{1}{c} \int \mathcal{L} \, d^4x \quad -(46)$$

and is the generalization of the Planck constant to any frame of reference. The Planck constant in the rest frame is:

$$S_0 = \hbar = \frac{1}{c} \int \mathcal{L}_0 \, d^4x_0 \quad -(47)$$

where V_0 is the rest volume defined by Eq. (7). Therefore ECE theory shows that the Planck constant in the rest frame must have an internal structure defined by the four volume d^4x_0 and rest volume V_0 . The same is true in any other frame, the rest volume being replaced by the volume in that frame of reference. If the four volume is:

$$d^4x_0 = \bar{V}_0 c\, dt_0 \quad -(48)$$

it is found that

$$\hbar = mc^2 \int \frac{\bar{V}_0}{\bar{V}_0}\, dt_0 = mc^2 t_0 . \quad -(49)$$

The time interval t_0 must be a constant for a given mass m. The existence of the Planck constant means that a particle is never quite at rest, it must have a rest frequency defined by Eq. (40) So there must be zero point energy defined by Eq. (45). Classically the particle in the rest frame does not move relative to the observer in the same rest frame, and there is no rest energy in a classical theory. The rest energy mc^2 is the result of special relativity theory as is well known. ECE theory gives both the rest energy and the Planck energy $\hbar\omega_0$, showing that it is a unified field theory . The fact that the Planck constant has an internal structure that depends on volume is of key importance in modifying the Heisenberg commutator equation in accordance with the experimental findings {42} of Croca et al. This modification has been initiated in volume 2 of ref. {1}.

The Fermat principle of least time {48} is the classical principle that governs the propagation of light in optics. The path taken by the light through a medium is such that the time of passage is a minimum. The amplitude of a light wave at point P_1 is related to the amplitude at point P_2 by:

$$\phi(P_2) = e^{i\phi}\, \phi(P_1) \quad -(50)$$

where the phase ϕ is defined by:

$$\phi = 2\pi \frac{x}{\lambda} . \quad -(51)$$

Here x is the coordinate and λ the wavelength {48}. Eq. (50) is the fundamental origin of the Schrödinger equation. Light takes paths such that the phase is minimized. This is the precise statement of the Fermat principle. In the limit of geometrical optics ϕ is infinite, the light appears to travel in straight lines. There is no curvature, and this is a "weak field limit" of ECE theory in which the interval t_0 is minimized.

The propagation of particles is given classically by the Hamilton principle of least action. Particles select paths between two points such that the action associated with that path is a minimum. This classical statement is equivalent to Newtonian dynamics in the weak field limit of ECE field theory. Particles adopt a least path and waves a least time. The reason is the same, the phase ϕ is minimized. So particles and waves become indistinguishable if the phase is made proportional to action and this is the fundamental idea of wave mechanics. Thus ϕ is proportional to S and so the constant of proportionality must have the units of inverse action because ϕ is unitless. In the classical limit ϕ is infinite so the constant of proportionality approaches zero. Schrödinger's equation is recovered from this argument if:

$$\phi = \frac{S}{\hbar} . \qquad -(52)$$

Eq. (50) describes a path from $P_1\,(x_1, t_1)$ to $P_2\,(x_2, t_2)$ {48}. Thus:

$$\phi(x_2, t_2) = e^{iS/\hbar}\,\phi(x_1, t_1) . \qquad -(53)$$

Differentiate {48} Eq. (53) with respect to t_2 :

$$\frac{\partial}{\partial t_2}\phi(x_2, t_2) = \frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\,\phi(x_1, t_1)\right) . \qquad -(53)$$

Now use the Leibnitz Theorem:

$$\frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\phi(x_1,t_1)\right) = \phi(x_1,t_1)\frac{\partial}{\partial t_2}e^{iS/\hbar} + e^{iS/\hbar}\frac{\partial\phi}{\partial t_2}(x_1,t_1). \quad -(54)$$

Since $\phi(x_1,t_1)$ is not a function of t_2:

$$\frac{\partial\phi}{\partial t_2}(x_1,t_1) = 0 \quad -(55)$$

and

$$\frac{\partial}{\partial t_2}e^{iS/\hbar} = \frac{i}{\hbar}\frac{\partial S}{\partial t_2}.e^{iS/\hbar}. \quad -(56)$$

Thus:

$$\frac{\partial}{\partial t_2}\phi(x_2,t_2) = \frac{i}{\hbar}\frac{\partial S}{\partial t_2}e^{iS/\hbar}\phi(x_1,t_1). \quad -(57)$$

Finally use Eq. (53) to obtain the Schrödinger equation in time dependent form { 48 }:

$$\frac{\partial}{\partial t}\phi = \frac{1}{\hbar}\frac{\partial S}{\partial t}\phi. \quad -(58)$$

This is not strictly a wave equation because a wave equation in mathematics contains second derivatives, but it is the famous equation of non-relativistic quantum mechanics. The more familiar form of the Schrödinger equation is obtained by using { 48 }:

$$E = -\frac{\partial S}{\partial t} \quad -(59)$$

where E is the total energy, the sum of kinetic and potential energy. So Eq. (58) becomes:

$$i\hbar\frac{\partial\phi}{\partial t} = E\phi. \quad -(60)$$

Finally define the operator:

$$H = i\hbar \frac{\partial}{\partial t} \quad -(61)$$

to obtain the familiar:

$$H\phi = E\phi. \quad -(62)$$

It is seen that the Schrödinger equation is a causal differential equation and as such cannot be interpreted as an expression of something that is acausal or unknowable. Using the operator (61) for energy it is seen that the Schrödinger equation is mathematically the same as:

$$[H, t]\phi = i\hbar\phi \quad -(63)$$

where the time t multiplies the function ϕ. Eq. (63) is an example of a Heisenberg commutator equation in the non-relativistic quantum limit. There is no more meaning to Eq. (63) than Eq. (62) because Eq. (63) is a restatement of Eq. (62) and thus contains the same mathematical information. This is the causal deterministic view of Einstein, de Broglie, Schrödinger, Bohm, Vigier and followers. The Copenhagen interpretation of Eq. (63) is that if t is known exactly, E is unknowable, and vice versa. This is the view of Bohr, Heisenberg and followers.

This non-relativistic analysis can be extended to ECE theory {1- 48 } by using the equation for the propagation of the tetrad wave function:

$$q^a_\mu(x^\mu) = e^{iS(x^\mu)/\hbar}\, q^a_\mu(0) \quad -(64)$$

where $S(x^\mu)$ is defined by Eqs. (8) and (9). Apply the d'Alembertian operator to both sides of Eq. (64):

$$\Box q^a_\mu(x^\mu) = \Box\left(e^{iS(x^\mu)/\hbar} q^a_\mu(0)\right). \quad -(65)$$

It is known from the ECE Lemma that:

$$\Box q^a_\mu(x^\mu) = R q^a_\mu(x^\mu) \quad -(66)$$

so:

$$\Box\left(e^{iS(x^\mu)/\hbar} q^a_\mu(0)\right) = R q^a_\mu(x^\mu). \quad -(67)$$

In analogy with the derivation of Eqs. (53) to (58) it is found that:

$$\Box\left(e^{iS(x^\mu)/\hbar} q^a_\mu(0)\right) = q^a_\mu(0)\,\Box\, e^{iS(x^\mu)/\hbar} \quad -(68)$$

where:

$$\Box q^a_\mu(0) = 0. \quad -(69)$$

Eq. (69) follows from:

$$\Box := \frac{1}{c^2}\frac{\partial^2}{\partial t_2^2} - \frac{\partial^2}{\partial X_2^2} - \frac{\partial^2}{\partial Y_2^2} - \frac{\partial^2}{\partial Z_2^2} \quad -(70)$$

and

$$q^a_\mu(0) := q^a_\mu(ct_1, X_1, Y_1, Z_1). \quad -(71)$$

Now use:

$$\Box e^{iS(x^\mu)/\hbar} = \frac{i}{\hbar} \Box S(x^\mu)\, e^{iS(x^\mu)/\hbar} \quad -(72)$$

and Eq. (64) to find:

$$\Box q^a_\mu(x^\mu) = R q^a_\mu(x^\mu) = \frac{i}{\hbar} \Box S(x^\mu)\, q^a_\mu(x^\mu). \quad -(73)$$

Therefore:

$$R = \frac{i}{\hbar} \Box S(x^\mu), \quad -(74)$$

and use Eq. (23) to obtain an expression for the ECE action in terms of Cartan geometry:

$$\Box S = -i\hbar\, q^\lambda_a\, \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda \right). \quad -(75)$$

In the rest frame:

$$R = -kT = -\frac{km}{V_0} \quad -(76)$$

so

$$\Box S = i\hbar \frac{km}{V_0} = \frac{i\hbar}{\lambda_c^2} \quad -(77)$$

where:

$$\lambda_c = \frac{\hbar}{mc} \quad -(78)$$

is the Compton wavelength { 48 }. So in the rest frame the action of ECE theory is:

$$\square S = \frac{i\hbar}{\lambda_c^2} \qquad -(79)$$

and this is the action of the Dirac equation of special relativistic quantum mechanics. The Dirac equation is a limit of the ECE wave equation (1). As is well known, the Schrödinger equation is the non-relativistic limit of the Dirac equation. If we are to interpret the Planck constant as a least action, then according to Eq. (47) S in Eq. (79) must approach $\hbar$, so the latter is an expectation value of the wave equation of action in the rest frame:

$$\square S = \frac{i}{\lambda_c^2} S. \qquad -(80)$$

More generally, for any reference frame, Eq. (80) is:

$$\square S = iRS \qquad -(81)$$

and so the Planck constant is the expectation value:

$$\hbar = S_0 = \langle SS^* \rangle^{1/2}. \qquad -(82)$$

This is a causal deterministic derivation which shows that the Planck constant is an expectation value of the wave equation for action. In the standard model the Planck constant is introduced empirically to explain phenomena such as the photoelectric effect and black body radiation { 48 }. The standard model does not give the internal structure of the Planck constant, Eq. (47). In consequence the Heisenberg commutator equation (63) does not contain volume, and this is the reason { 1 } why the Copenhagen interpretation of Eq. (63) fails drastically when sufficiently discriminating experiments { 42 – 45 } are used to test it.