1)

Notes 58(4) a : Oscillatory or Wave Cosmologies

For the general metric in spherically symmetric spacetime:

$$ds^2 = -e^{2\alpha(t,r)} dt^2 + e^{2\beta(t,r)} dr^2 + r^2 d\Omega^2 \quad -(1)$$

and:

$$q^0_0 = e^{\alpha} ; \; q^1_1 = e^{\beta} ; \; q^2_2 = q^3_3 = 1. \quad -(2)$$

The ECE Lemma's are:

$$\Box q^0_0 = R_0 q^0_0 \quad -(3)$$

$$\Box q^1_1 = R_1 q^1_1 , \quad -(4)$$

i.e

$$\Box e^{\alpha} = R_0 e^{\alpha} \quad -(5)$$

$$\Box e^{\beta} = R_1 e^{\beta} . \quad -(6)$$

The differentiation in eqs (5) and (6) are defined by:

$$\Box e^{\alpha} = \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) e^{\alpha} \quad -(7)$$

where:

$$\frac{\partial}{\partial t} e^{\alpha} = \frac{\partial \alpha}{\partial t} e^{\alpha}$$

$$\frac{\partial^2}{\partial t^2} e^{\alpha} = \frac{\partial}{\partial t} \left(\frac{\partial \alpha}{\partial t} e^{\alpha} \right)$$

$$= \left(\frac{\partial^2 \alpha}{\partial t^2} + \left(\frac{\partial \alpha}{\partial t} \right)^2 \right) e^{\alpha} \quad -(8)$$

using the Leibnitz Theorem.

2)

Thus:

$$\Box e^{\alpha} = \left(\Box \alpha + \frac{1}{c^2}\left(\frac{\partial \alpha}{\partial t}\right)^2 - \left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2 - \left(\frac{\partial \alpha}{\partial z}\right)^2 \right) e^{\alpha} \quad -(9)$$

Therefore:

$$R_0 = \Box \alpha + \frac{1}{c^2}\left(\frac{\partial \alpha}{\partial t}\right)^2 - \left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2 - \left(\frac{\partial \alpha}{\partial z}\right)^2 \quad -(10)$$

$$R_1 = \Box \beta + \frac{1}{c^2}\left(\frac{\partial \beta}{\partial t}\right)^2 - \left(\frac{\partial \beta}{\partial x}\right)^2 - \left(\frac{\partial \beta}{\partial y}\right)^2 - \left(\frac{\partial \beta}{\partial z}\right)^2 . \quad -(11)$$

For the Schwarzschild Metric:

$$\alpha = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2}, \quad \beta = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \quad -(12)$$

$$r = \left(x^2 + y^2 + z^2\right)^{1/2}. \quad -(13)$$

For other metrics obeying the EH field equation, α and β are different, but all ~~metrics~~ ~~and~~ tetrads of the EH field equation obey equations (5) and (6).

3)

Equations (5) and (6) are classical. If it were possible to find complex valued solutions:

$$\alpha = \alpha' + i\alpha'' \quad -(14)$$

$$\beta = \beta' + i\beta'' \quad -(15)$$

then eqns (5) and (6) would be eigenequations via the imaginary parts. For real valued α and β however there is only one R_0 and only one R_1. These questions are very important for cosmology, because in the standard model the existence of the ECE Lemma is not realized. To develop wave solutions, complex-valued tetrads are needed. The equations defining the tetrads are:

$$q^0_0 q^0_0 = e^{2\alpha} \quad -(16)$$

$$q^1_1 q^1_1 = e^{2\beta} \quad -(17)$$

$$q^2_2 q^2_2 = 1 \quad -(18)$$

$$q^3_3 q^3_3 = 1 . \quad -(19)$$

So far we have considered the real solution:

$$q^0_0 = e^{\alpha}, \; q^1_1 = e^{\beta} \quad -(20)$$

Now consider q^0_0 and q^1_1 to be complex valued:

$$q^0_0 = e^{\alpha' + i\alpha''}, \; q^{0*}_0 = e^{\alpha' - i\alpha''} \quad -(21)$$

$$q^1_1 = e^{\beta' + i\beta''}, \; q^{1*}_1 = e^{\beta' - i\beta''} \quad -(22)$$

and

$$q^0_0 q^{0*}_0 = e^{2\alpha} \quad -(23)$$

$$q^1_1 q^{1*}_1 = e^{2\beta} \quad -(24)$$

4)

Eqs. (21) to (24) are satisfied for all α'' by:

$$\alpha = \alpha', \quad \beta = \beta'. \quad -(25)$$

Therefore:

$$q^0_0 = e^{\alpha} e^{i\alpha''}, \quad q^1_1 = e^{\beta} e^{i\beta''} \quad -(26)$$

and the transformations:

$$q^0_0 \rightarrow q^0_0 e^{i\alpha''}, \quad q^1_1 \rightarrow q^1_1 e^{i\beta''} \quad -(27)$$

leave the metric elements unchanged:

$$g_{00} \rightarrow e^{-i\alpha''} g_{00} e^{i\alpha''} \quad -(28)$$

$$g_{11} \rightarrow e^{-i\beta''} g_{11} e^{i\beta''}. \quad -(29)$$

<u>Oscillatory</u> <u>Universe</u>

The oscillatory universe is defined by the <u>eigen-equations</u>:

$$\Box e^{i\alpha''} = R''_0 e^{i\alpha''} \quad -(30)$$

$$\Box e^{i\beta''} = R''_1 e^{i\beta''} \quad -(31)$$

These are <u>wave</u> - <u>equations</u>, because of the wave-functions $e^{i\alpha''}$ and $e^{i\beta''}$. Therefore in addition to the uniformly expanding cosmology represented by eqs. (3) and (4), there can be <u>oscillatory</u> <u>cosmologies</u> or <u>wave cosmologies</u>.