Paper 61: Summary of the Coulomb Law in ECE Theory

The law is given from the first Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(1)$$

and the first Bianchi identity:

$$d \wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b, \quad -(2)$$

with the ECE Ansatz:

$$\left.\begin{aligned} A^a &= A^{(0)} q^a, \\ F^a &= A^{(0)} T^a. \end{aligned}\right\} \quad -(3)$$

Eqs (1) to (3) lead to:

$$\underline{\nabla} \cdot \underline{E}^a = c\mu_0 \tilde{J}^{0a} \quad -(4)$$

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - \underline{\nabla}\phi^a - c\omega^{0a}{}_b \underline{A}^b + \underline{\omega}^a{}_b \phi^b. \quad -(5)$$

Eq. (4) may be written for each index a as:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0. \quad -(6)$$

If we restrict attention to the scalar potential then for each index a, eq. (5) is:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}_b \phi^b. \quad -(7)$$

In eq (7) ϕ^b is interpreted as a scalar quantity indexed by a label b. For a particular b index therefore, eq. (7) is:

$$\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(8)$$

2) Eqs (6) and (8) give the equation:

$$\underline{\nabla} \cdot \left(- \underline{\nabla} \phi + \underline{\omega} \phi \right) = \rho / \epsilon_0 \quad -(9)$$

i.e

$$\boxed{\nabla^2 \phi - \underline{\nabla} \cdot \left(\phi \underline{\omega} \right) = - \rho / \epsilon_0} \quad -(10)$$

The spin connection vector is:

$$\left.\begin{aligned} \underline{\omega} &= \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k} \\ &= \omega_r \underline{e}_r + \omega_\phi \underline{e}_\phi + \omega_\theta \underline{e}_\theta \end{aligned}\right\} -(11)$$

where ω_r is the radial component of $\underline{\omega}$. If the latter is purely radial, then:

$$\underline{\omega} = \omega_r \underline{e}_r \quad -(12)$$

and:

$$\underline{\omega} \cdot \underline{\nabla} \phi = \omega_r \frac{\partial \phi}{\partial r} \quad -(13)$$

$$\phi \underline{\nabla} \cdot \underline{\omega} = \frac{\phi}{r^2} \frac{\partial}{\partial r} \left(r^2 \omega_r \right) -(14)$$

$$\nabla^2 \phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} \quad -(15)$$

It is reasonable to assume that:

$$\omega_r = \frac{A}{r} \quad -(16)$$

where A is dimensionless. Then eqns. (10) to (16)

give the result:

3)

$$\boxed{\frac{\partial^2 \phi}{\partial r^2} + \frac{(2-A)}{r}\frac{\partial \phi}{\partial r} - \frac{A\phi}{r^2} = -\frac{\rho}{\epsilon_0}} \quad -(17)$$

This equation has a similar structure to the well known one-dimensional Schrödinger equation:

$$\frac{d^2 P}{dr^2} + \frac{2\mu}{\hbar^2}\frac{e^2}{4\pi\epsilon_0}\frac{P}{r} - \frac{2\mu}{\hbar^2}\frac{l(l+1)\hbar^2}{2\mu r^2}P$$
$$= -\frac{2\mu E}{\hbar^2}P \quad -(18)$$

for motion in the effective potential:

$$V_{eff} = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2\mu r^2} \quad -(19)$$

However, eq. (17) contains second and first r partial derivatives in ϕ, and a term in ϕ. The comparison of structure of eqns. (17) and (18) shows that the term in $-A\phi/r^2$ is repulsive in nature.

If $\quad A = 2 \quad -(20)$

eq. (17) becomes:

$$\boxed{\frac{\partial^2 \phi}{\partial r^2} - \frac{2\phi}{r^2} = -\frac{\rho}{\epsilon_0}} \quad -(21)$$

4) The ~~Laplace~~ Poisson equation is:

$$\nabla^2 \phi = -\rho/\epsilon_0 \quad - (22)$$

and this describes the attraction part of the Coulomb law. If ρ is very small, eq. (21) becomes:

$$\frac{\partial^2 \phi}{\partial r^2} \sim \frac{2\phi}{r^2} \quad -(23)$$

so:

$$\boxed{\phi = \alpha r^2 + \frac{\beta}{r}} \quad - (24)$$

The Coulomb potential energy is:

$$V_c = -e\phi_c = -\frac{e^2}{4\pi\epsilon_0 r}. \quad - (25)$$

The effect of the spin connection is to add a repulsion of approximate form (24) to the attractive Coulomb potential (25). It is seen from eq. (21) that:

$$\frac{\partial^2 \phi}{\partial r^2} = \frac{2\phi}{r^2} - \rho/\epsilon_0 \quad - (26)$$

so as $r \rightarrow 0$, $\phi \rightarrow \infty$, and the repulsion overwhelms the attraction.

For one electron a <u>positive</u> energy

5) is inputted of the form:

$$\boxed{V = e\left(\alpha r^2 + \frac{\beta}{r}\right)} \quad -(27)$$

and if this exceeds 13.6 eV the H atom is ionized.

Eq (27) is a general law and shows that it is possible to input energy from spacetime into a material to release free electrons. The mechanism to do this must be found experimentally.

At macroscopic distances in laboratory experiments the term $2\phi / r^2$ is very small compared with $-\rho / \epsilon_0$, and the Coulomb law applies.
