1)

Notes 62(2): Second Example of the ECE Lemma, Electromagnetic Waves

In this case:

$$\Box A^a_\lambda = R A^a_\lambda \quad -(1)$$

$$R := q^\nu_a \partial^\mu (\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}) = q^\nu_a R^a_\nu \quad -(2).$$

The ECE Ansatz has been used:

$$A^a_\lambda = A^{(0)} q^a_\lambda . \quad -(3)$$

If

$$R = 0 \quad -(4)$$

we obtain the d'Alembert equation for each a

$$\Box A^a_\mu = 0 . \quad -(5)$$

Eq (5) implies:

$$\Gamma^a_{\mu\nu} = \omega^a_{\mu\nu} . \quad -(6)$$

Therefore in ECE theory the d'Alembert eqn. is given by condition (6). This is equivalent to the use of the Lorenz condition:

$$\partial_\mu A^\mu = 0 \quad -(7)$$

in the standard model.

If the Lorenz condition is not used

2) Here:

$$R A^a_\mu = \mu_0 J^a_\mu \quad -(8)$$

where J^a_μ is a four current. Thus:

$$J^a_\mu = \frac{A^{(0)}}{\mu_0} R q^a_\mu . \quad -(9)$$

Using: $$R = q^\mu_a \partial^\nu (\Gamma^a_{\nu\mu} - \omega^a_{\nu\mu}) \quad -(10)$$

gives:

$$\boxed{ J^a_\mu = \frac{A^{(0)}}{\mu_0} \partial^\nu (\Gamma^a_{\nu\mu} - \omega^a_{\nu\mu}) = \frac{A^{(0)}}{\mu_0} R^a_\mu } \quad -(11)$$

The photon mass is given by:

$$R = -\frac{m^2 c^2}{\hbar^2} . \quad -(12)$$

Eq. (11) is the generally covariant form of the Lehnert / Proca current, in which:

$$m \neq 0 \quad -(13)$$

$$\Gamma^a_{\nu\mu} \neq \omega^a_{\nu\mu} . \quad -(14)$$

S.I. Units

$A = J s C^{-1} m^{-1}$, $\mu_0 = J s^2 C^{-2} m^{-1}$, $R = m^{-2}$,
$\rho = C m^{-3}$, $\underline{J} = C s^{-1} m^{-2} = A m^{-2}$, $J^a_\mu = (\rho, \underline{J})^a$