63(22): The $r \to \infty$ Limit of Eq. (5) of Notes 63(21)

In this limit the spin connection vanishes and:

$$\frac{d^2\phi}{dr^2} = -\frac{\rho}{\epsilon_0} \quad -(1)$$

Eq. (5) of Notes 63(21) is:

$$\phi = \frac{\rho(0)}{\epsilon_0}\left(\frac{\cos^2(\kappa r)}{\frac{1}{\kappa^2 r^4} + \kappa^2\cos(\kappa r) + 5\frac{\kappa}{r}\sin(\kappa r) - \frac{4}{r^2}\cos(\kappa r)}\right)$$

$$\xrightarrow[r\to\infty]{} \frac{\rho(0)}{\epsilon_0}\left(\frac{\cos(\kappa r)}{\kappa^2}\right) \quad -(2)$$

So:

$$\frac{d^2\phi}{dr^2} = -\frac{\rho(0)}{\epsilon_0}\cos(\kappa r) = -\frac{\rho}{\epsilon_0} \quad -(3)$$

Q.E.D.

In this limit, we may choose ρ from eq (3) to represent the Poisson equation (1), whose solution is well known to be:

$$\phi = \frac{e}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(r')}{|r-r'|}\,d^3x' \quad -(4)$$

So from eq. (2) and (4):

$$\frac{\rho(0)}{\epsilon_0}\frac{\cos(\kappa r)}{\kappa^2} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(r')}{|r-r'|}\,d^3x',$$

$r \to \infty$,

$$\boxed{\frac{\cos(\kappa r)}{\kappa^2} \to \frac{1}{4\pi\rho(0)}\int\frac{\rho(r')}{|\underline{r}-\underline{r}'|}\,d^3x'} \quad -(5)$$