Notes 63(5) : Check a the Hartree-type spin constraint

The electric field is:

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} q_i \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^3} \quad -(1)$$

Now use:

$$-\underline{\nabla}\left(\frac{1}{|\underline{r} - \underline{r}_i|}\right) = \frac{\underline{r} - \underline{r}_i}{|\underline{r} - \underline{r}_i|^3} \quad -(2)$$

If

$$\phi = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\underline{r} - \underline{r}_i|} \quad -(3)$$

then

$$\underline{E} = -\underline{\nabla}\phi = \underline{\omega}\phi \quad -(4)$$

Thus:

$$-\sum_i \underline{\nabla}\phi_i = \sum_i \underline{\omega}_i \phi_i \quad -(5)$$

i.e.

$$\sum_i \frac{\underline{r} - \underline{r}_i}{|\underline{r} - \underline{r}_i|^3} q_i = \sum_i \underline{\omega}_i \frac{q_i}{|\underline{r} - \underline{r}_i|} \quad -(6)$$

and

$$\boxed{\underline{\omega}_i = \frac{\underline{r} - \underline{r}_i}{|\underline{r} - \underline{r}_i|^2}} \quad -(7)$$

Each electron or proton feels its own spin constraint.