1)

64(1) (TBS): Resonance Equation from the Generally Covariant Gauss Law of Magnetism.

The generally covariant Gauss Law of magnetism is

$$\underline{\nabla} \cdot \underline{B}^a = \mu_0 \tilde{j}^{0a} \quad -(1)$$

where:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b. \quad -(2)$$

The standard model is:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(3)$$

$$\underline{B} = \underline{\nabla} \times \underline{A}. \quad -(4)$$

Substituting eq. (3) in eq. (4) gives the vector identity:

$$\underline{\nabla} \cdot (\underline{\nabla} \times \underline{A}) := 0. \quad -(5)$$

and <u>no resonances</u>.

However, resonances occur from eqs (1) and (2) as follows. From eq (2) in eq. (1):

$$\underline{\nabla} \cdot (\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) = \mu_0 \tilde{j}^{0a} \quad -(6)$$

i. e.

$$\underline{\nabla} \cdot (\underline{\omega}^a{}_b \times \underline{A}^b) = -\mu_0 \tilde{j}^{0a} \quad -(7)$$

Using a vector identity of vector analysis ("Vector Analysis Problem Solver", p. 316):

$$\boxed{\underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b = -\mu_0 \tilde{j}^{0a}} \quad -(8)$$

2)

Denote:

$$\underline{\Omega}^a{}_b := \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(9)$$

to obtain:

$$\underline{A}^b \cdot \underline{\Omega}^a{}_b = -\mu_0 \tilde{j}^{0a} \quad -(10)$$

In one dimension:

$$A_Z^b \Omega_{Z}{}^a{}_b = -\mu_0 \tilde{j}^{0a}. \quad -(11)$$

This simplifies to:

$$A_Z \Omega_Z = -\mu_0 \tilde{j}^0. \quad -(12)$$

Now let: $$\tilde{j}^0 = \tilde{j}^0(0) \cos(\kappa Z) \quad -(13)$$

meaning an oscillatory charge in a circuit or magnetic material.

Thus: $$A_Z \Omega_Z = -\mu_0 \tilde{j}^0(0) \cos(\kappa Z) - (14)$$

$$\frac{d}{dZ}(A_Z \Omega_Z) = \Omega_Z \frac{dA_Z}{dZ} + \frac{d\Omega_Z}{dZ} A_Z = \mu_0 \kappa \tilde{j}^0(0) \sin(\kappa Z)$$

$$\boxed{\frac{d^2}{dZ^2}(A_Z \Omega_Z) = \Omega_Z \frac{d^2 A_Z}{dZ^2} + 2\left(\frac{d\Omega_Z}{dZ}\right)\frac{dA_Z}{dZ} + \left(\frac{d^2\Omega_Z}{dZ^2}\right) A_Z = \mu_0 \kappa^2 \tilde{j}^0(0) \cos(\kappa Z)}$$

$$-(15)$$

This is a resonance equation

3) The spectrum of resonances from eq. (15) is a graph of A_Z against Z. The graph depends on $\Omega_Z(Z)$, i.e. on the way that Ω_Z depends on Z. A numerical solution of eq. (15) gives the resonance spectrum and depends on the function $\Omega_Z(Z)$. This has to have non-zero first and second derivatives. Also, A_Z must have non-zero first and second derivatives.

Therefore we could model $\Omega_Z(Z)$ with a Fourier series, the simplest model is:

$$\Omega_Z(Z) = \Omega_Z(0) \exp(\kappa Z) \quad -(16)$$

$$\frac{d\Omega_Z(Z)}{dZ} = \kappa \Omega_Z(0) e^{\kappa Z} \quad -(17)$$

$$\frac{d^2\Omega_Z(Z)}{dZ^2} = \kappa^2 \Omega_Z(0) e^{\kappa Z} \quad -(18)$$

In this model, eq. (15) is: $-(19)$

$$\boxed{\frac{d^2 A_Z}{dZ^2} + 2\kappa \frac{dA_Z}{dZ} + \kappa^2 A_Z = \mu_0 \kappa^2 \frac{\tilde{j}^{\,0}(0)}{\Omega_Z(0)} e^{-\kappa Z} \cos(\kappa Z)}$$

4) Various other models may be tried for Ω_z, and each resonance equation worked out by computer.

Exponentially Decreasing Ω_z

In this case:

$$\Omega_z(z) = \Omega_z(0)\, e^{-\kappa z} \quad -(20)$$

$$\frac{d\Omega_z}{dz} = -\kappa\,\Omega_z(0)\,e^{-\kappa z} \quad -(21)$$

$$\frac{d^2\Omega_z}{dz^2} = \kappa^2\,\Omega_z(0)\,e^{-\kappa z} \quad -(22)$$

and:

$$\boxed{\frac{d^2 A_z}{dz^2} - 2\kappa\frac{dA_z}{dz} + \kappa^2 A_z = \mu_0\kappa^2\left(\frac{\tilde{J}^{\,0}(0)}{\Omega_z(0)}\right)e^{\kappa z}\cos(\kappa z)}$$

$$-(23)$$

Complex Circular Ω_z

In this case:

$$\Omega_z(z) = \Omega_z(0)\, e^{i\kappa z} \quad -(24)$$

$$\frac{d\Omega_z}{dz} = i\,\Omega_z(0)\,\kappa\, e^{i\kappa z} \quad -(25)$$

$$\frac{d^2\Omega_z}{dz^2} = -\Omega_z(0)\,\kappa^2\, e^{i\kappa z} \quad -(26)$$

giving the resonance equation:

5)

$$\frac{d^2A_z}{dz^2} + i\kappa\frac{dA_z}{dz} - \kappa^2 A_z = \mu_0\kappa^2 \frac{\tilde{j}^{\,0}(0)}{\Omega_z(0)} e^{-i\kappa z}\cos(\kappa z) \quad -(27)$$

Comparing real parts of eq. (27):

$$\boxed{\frac{d^2A_z}{dz^2} - \kappa^2 A_z = \mu_0\kappa^2\left(\frac{\tilde{j}^{\,0}(0)}{\Omega_z(0)}\right)\cos^2(\kappa z)} \quad -(28)$$

If there is dielectric absorption and dispersion then κ may be complex valued:

$$\kappa = \kappa' - i\kappa'' \quad -(29)$$

and

$$\kappa^2 = (\kappa' - i\kappa'')(\kappa' - i\kappa'')$$
$$= \kappa'^2 - 2i\kappa'\kappa'' - \kappa''^2 \quad -(30)$$

If $\kappa'' \gg \kappa'$ then:

$$\kappa^2 \sim -\kappa''^2 \quad -(31)$$

and:

$$\frac{d^2A_z}{dz^2} + \kappa''^2 A_z \quad -(32)$$
$$= -\mu_0\kappa''^2\left(\frac{\tilde{j}^{\,0}(0)}{\ldots}\right) e^{-\kappa'' z}\cos(\kappa z)$$

6) where:

$$\cos(\kappa z) \sim \cos(-i\kappa'' z) \quad -(33)$$

Now use:

$$\cos(ix) = \cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right) \quad -(34)$$

and

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right) \quad -(35)$$

$$= \cos(-x) \quad -(36)$$

So:

$$\cos(-ix) = \cosh(-x) = \cosh(x)$$

and

$$\cos(\kappa z) = \cosh(\kappa'' z).$$

Thus:

$$\boxed{\begin{aligned}\frac{d^2 A_z}{dz^2} + \kappa''^2 A_z &= -\mu_0 \kappa''^2 \left(\frac{\tilde{j}^{\circ}(0)}{\Omega_z(0)}\right) \cosh(\kappa'' z)\, e^{-\kappa'' z} \\ &= -\frac{\mu_0 \kappa''^2}{2}\left(\frac{\tilde{j}^{\circ}(0)}{\Omega_z(0)}\right)\left(1 + e^{-2\kappa'' z}\right)\end{aligned}}$$

$$-(37)$$

This is an <u>undamped oscillator</u>.

7)

Comments

Depending on the modelling of Ω, a variety of resonance equations have been obtained from the ECE Gauss law of magnetism, eq. (1). The latter has been reduced to its simplest form. A rich spectrum of resonance is obtained if no simplifying assumptions are made. Even with simplifying assumptions, several resonance peaks are expected.

Therefore magnetism is a far richer subject in general relativity ~~than~~ in special relativity.

The next stage in these notes is the resonance Ampère Law of ECE theory.

8)

PS : Reciprocal Distance Model

If : $\Omega_Z(Z) = \Omega_Z(0) / Z$ — (38)

then
$$\frac{d\Omega_Z}{dZ} = -\frac{\Omega_Z(0)}{Z^2} \quad \text{— (39)}$$
$$\frac{d^2\Omega_Z}{dZ^2} = \frac{\Omega_Z(0)}{Z^3} \quad \text{— (40)}$$

and from eq. (15) :

$$\boxed{\frac{d^2A_Z}{dZ^2} - \frac{1}{Z}\frac{dA_Z}{dZ} + \frac{1}{Z^2}A_Z = \mu_0 Z \kappa^2 \left(\frac{\tilde{j}^0(0)}{\Omega_Z(0)}\right)\cos(\kappa Z) \quad \text{— (41)}}$$

This is another possible resonance equation. The operative quantity in this analysis is :

$$\underline{\Omega}^a{}_b = \underline{\nabla} \times \underline{\omega}^a{}_b . \quad \text{— (9)}$$

This may be interpreted as a kind of magnetic field, because $\underline{\omega}^a{}_b$ is approximately dual to the potential for initially small $\tilde{j}^{0a}$.

9)

It is clear that Ω_2 may be modelled in many different ways, depending on the material being considered and circuit design. There are ferromagnetics paramagnetics and diamagnetics, and each will have its characteristic $\underline{\Omega}^a{}_b := \underline{\nabla} \times \underline{\omega}^a{}_b$. It is clear that the spin connection itself, $\underline{\omega}^a{}_b$, must have a non-zero curl, and $\underline{\Omega}^a{}_b$ must have non-zero first and second derivatives. Under these conditions, resonances occur even from the Gauss law.

We have not yet considered the Ampère Law or Ampère - Maxwell law. We have considered the Coulomb law with a reciprocal distance connection, but have not yet considered the Faraday law of induction in any detail.

<u>Reduction to the Standard Model</u>

This has been considered in earlier papers. The reduction occurs by:

$$\underline{\nabla} \times \underline{A}^a \rightarrow -\underline{\omega}^a{}_b \times \underline{A}^b \quad -(10)$$

$$\tilde{j}^{oa} \rightarrow 0 \quad -(11)$$

The reduction in eq. (11) means that the driving force disappears, so resonance disappears. The reduction (10) means that the form of the standard model (eq. (4)) is regained but the magnetic field doubled, or for the same magnetic field, the vector potential is halved.

The modelling of $\underline{\Omega}^a{}_b$ suggested in these notes reduces to the standard model using:

$$\underline{\nabla} \times \underline{A}^a \longrightarrow - \underline{\omega}^a{}_b \times \underline{A}^b$$

$$\underline{\Omega}^a{}_b := \underline{\nabla} \times \underline{\omega}^a{}_b$$

so any model used for $\underline{\Omega}^a{}_b$ translates into a model of $\underline{\omega}^a{}_b$ and thus of $\underline{A}^a$. With this technique, the standard model Gauss law is always regained off-resonance. This can be checked by computer.