1)

Notes 81(1): Inverse Faraday Effect and Faraday Effect from the $B^{(3)}$ Spin Field.

These notes define a new method of directly solving the Einstein equation by expressing the relativistic kinetic energy as:

$$T = E - E_0 = \frac{p^2 c^2}{E + E_0} \quad -(1)$$

where the relativistic energy is:

$$E = \gamma m c^2, \quad -(2)$$

the rest energy is:

$$E_0 = m c^2, \quad -(3)$$

the relativistic momentum is:

$$\underline{p} = \gamma m \underline{v}, \quad -(4)$$

and:

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2} \quad -(5)$$

One electron is considered to interact with an electromagnetic field through the minimal prescription:

$$p^\mu \rightarrow p^\mu + e A^\mu. \quad -(6)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \quad -(7)$$

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right). \quad -(8)$$

Thus:

$$E = \gamma m c^2 \rightarrow \gamma m c^2 + e\phi, \quad -(9)$$

$$\underline{p} = \gamma m \underline{v} \rightarrow \gamma m \underline{v} + e\underline{A}. \quad -(10)$$

So the relativistic kinetic energy describing the interaction is:

$$\boxed{T = \frac{\gamma^2 m^2 v^2 c^2 + e^2 A^2 c^2}{\gamma m c^2 + e\phi + m c^2}} \quad -(11)$$

2) The interaction part is:

$$T_{int} = \frac{e^2 A^2 c^2}{\gamma m c^2 + m c^2 + e\phi} \quad - (12)$$

where $$|A| = A^{(0)} = \frac{cB^{(0)}}{\omega}. \quad - (13)$$

Thus: $$T_{int} = \frac{e^2 B^{(0)2} c^4}{\omega^2 (mc^2(1+\gamma) + e\phi)}. \quad - (14)$$

<u>Non-Relativistic Limit</u>

This is the limit:

$$mc^2(1+\gamma) \gg e\phi, \quad - (15)$$

$$\gamma \to 1, \quad - (16)$$

so: $$T_{int} \to \left(\frac{e^2 c^2}{2m\omega^2}\right) B^{(0)2}. \quad - (17)$$

In this limit:

$$T_{int} = \frac{1}{2}\omega J \quad - (18)$$

where J is the magnitude of the orbital angular momentum of the electron in the e/m field. So

$$\boxed{J = \frac{e^2 c^2}{m\omega^3} B^{(0)2}} \quad - (19)$$

as from the <u>Hamilton - Jacobi method</u>.

3) The non-relativistic limit is the condition for the Prague experiments, to detect the Larmor radius and the RFR effect. The induced magnetic dipole moment from eq. (19) is:

$$\underline{m}^{(3)} = \left(\frac{e^3 c^2}{2 m^2 \omega^3} \right) B^{(0)} \underline{B}^{(3)} . \quad -(20)$$

The RFR effect is given directly from eq. (17) as:

$$\hbar \omega_{res} = \frac{e^2 c^2 B^{(0)2}}{2 m \omega^2} \left(1 - (-1) \right) \quad -(21)$$

using the SU(2) basis. This gives:

$$\boxed{f_{res} = \left(\frac{e^2 \mu_0 c}{2\pi \hbar m} \right) \frac{I}{\omega^2}} . \quad -(22)$$

<u>So the RFR effect is very fundamental, it can be derived directly from the minimal prescription.</u> Relativistic corrections to RFR are given by eq. (14).

In this non-relativistic limit the Larmor radius may be worked out from:

$$J = pr = \frac{e^2 c^2 B^{(0)2}}{m \omega^3} , \quad -(23)$$

$$p = eA = \frac{ec}{\omega} B^{(0)} \quad -(24)$$

so:

$$\boxed{r_L = \frac{p}{m\omega} = \frac{ecB^{(0)}}{m\omega^2} = \frac{ec}{m\omega^2} \left(\frac{\mu_0 I}{c} \right)^{1/2}} \quad -(25)$$

using

$$I = \frac{c B^{(0)2}}{\mu_0} . \quad -(26)$$

CROSS-CHECKED