93(15): Calculation of the ϕ component of $\underline{J}$

This is:

$$J_\phi = J^3_3 = -\frac{A^{(0)}}{\mu_0}\left(R^3{}_0{}^{30} + R^3{}_1{}^{31} + R^3{}_2{}^{32}\right) -$$

Here: $R^3{}_1{}^{31} = g^{33} g^{11} R^3{}_{131}$

where $R^3{}_{131} = g^{33} R_{3131} = -g^{33} R_{1331}$

$$= -g^{33} g_{11} R^1{}_{331} = g^{33} g_{11} R^1{}_{313},$$

so $R^3{}_1{}^{31} = g^{33} g^{11} g^{33} g_{11} R^1{}_{313}$

$$= \frac{1}{r^4 \sin^4\theta} R^1{}_{313}$$

$$\boxed{R^3{}_1{}^{31} = \frac{1}{2r^4 \sin^2\theta}(x-2)} \quad -(2)$$

Secondly:

$$R^3{}_2{}^{32} = g^{22} g^{33} R^3{}_{232} = -g^{22} g^{33} R^3{}_{223}$$

where $R^3{}_{223} = g^{33} R_{3223} = -g^{33} R_{2323}$

$$= -g^{33} g_{22} R^2{}_{323}.$$

So $R^3{}_2{}^{32} = g^{22} g^{33} g^{33} g_{22} R^2{}_{323}$

$$= \frac{1}{r^4 \sin^4\theta} R^2{}_{323}$$

$$\boxed{R^3{}_2{}^{32} = \frac{(2-x)}{r^4 \sin^2\theta}} \quad -(3)$$

2)

Finally:

$$\boxed{R^{3}{}_{0}{}^{30} = -\frac{2(1+x)}{r^4 \sin^2\theta}} \quad -(4)$$

Therefore:

$$R^{3}{}_{0}{}^{30} + R^{3}{}_{1}{}^{31} + R^{3}{}_{2}{}^{32}$$

$$= \frac{1}{r^4 \sin^2\theta}\left(\frac{x-2}{2} + 2 - x - 2 - 2x\right)$$

$$= \frac{1}{2r^2} \cdot \frac{1}{r^2 \sin^2\theta}\left(x - 2 + 4 - 2x - 4 - 4x\right)$$

$$= -\frac{1}{2r^2} \cdot \frac{1}{r^2 \sin^2\theta}\left(2 + 5x\right)$$

So:

$$\boxed{J_\phi = J^3_3 = \frac{A^{(0)}}{\mu_0} \cdot \frac{1}{2r^2} \cdot \frac{1}{r^2 \sin^2\theta}\left(2 + 5x\right)} \quad -(5)$$

where $x = \frac{2mG}{rc^2}$.

As $x \to 0$

$$\boxed{r^2 \sin^2\theta \, J_\phi \to \frac{A^{(0)}}{\mu_0} \cdot \frac{1}{r^2}} \quad -(6)$$