96(7): True Schwarzschild Solution, Analytical Charge and Current Densities

In this case:

$$J^0 = -\phi\left(\frac{\ddot{C}}{4ABC^2}\right) \quad -(1)$$

$$J_r = -\frac{A^{(0)}}{\mu_0}\left(\frac{5C\ddot{C} - 4\dot{C}^2}{4B^2C^3}\right) \quad -(2)$$

where:

$$C = (r^3 + \alpha^3)^{2/3} \quad -(3)$$

$$\dot{C} = 2r^2(r^3 + \alpha^3)^{-1/3} \quad -(4)$$

$$\ddot{C} = 4r(r^3 + \alpha^3)^{-1/3} - 2r^4(r^3 + \alpha^3)^{-4/3} \quad -(5)$$

So:

$$\boxed{J^0 = -\phi\left(\frac{4r}{(r^3+\alpha^3)^{5/3}} - \frac{2r^4}{(r^3+\alpha^3)^{8/3}}\right)}$$

for $A = B = 1$. $\quad -(6)$

As $r \to \infty$, $J^0 \to 0$ for all α.

So the static electric field is described by:

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(7)$$

because the only constant value of J^0 occurs at $r \to \infty$.

In calculating the radial part of the current density we use:

2)

$$\frac{\ddot{c}}{c^2} = \frac{4r}{(r^3+d^3)^{5/3}} - \frac{2r^4}{(r^3+d^3)^{8/3}} \quad -(8)$$

$$\frac{\dot{c}^2}{c^3} = \frac{4r^4}{(r^3+d^3)^{8/3}} \quad -(9)$$

$$J_r = -\frac{A^{(0)}}{\mu_0}\left(5\frac{\ddot{c}}{c^2} - \frac{\dot{c}^2}{c^3}\right) \quad -(10)$$

$$= -\frac{A^{(0)}}{\mu_0}\left(\frac{20r}{(r^3+d^3)^{5/3}} - \frac{10r^4}{(r^3+d^3)^{8/3}} - \frac{4r^4}{(r^3+d^3)^{8/3}}\right)$$

$$\boxed{J_r = -\frac{A^{(0)}}{\mu_0}\left(\frac{20r}{(r^3+d^3)^{5/3}} - \frac{14r^4}{(r^3+d^3)^{8/3}}\right)} \quad -(11)$$

If we consider a vacuum Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(12)$$

then this is possible if and only if:

$$r \to \infty. \quad -(13)$$

Otherwise in the original Schwarzschild line element the $\underline{B}$ and $\underline{E}$ fields cannot be static and are described by:

$$\underline{\nabla} \cdot \underline{E} = \rho(\mathrm{vac})/\epsilon_0 \quad -(14)$$

$$\underline{\nabla} \times \underline{B} + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}(\mathrm{vac}) \quad -(15)$$