1)

99(5): Pure Rotational Limit

In previous work the pure rotational case has been considered to be that where $R^a{}_b$ is dual to T^c in the Minkowski tangent spacetime:

$$R^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bd}T^d. \quad -(1)$$

In this note this idea is developed with the round trip with covariant derivatives:

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma - T^\lambda{}_{\mu\nu}D_\lambda V^\rho \quad -(2)$$

The pure rotational limit is considered to be

$$R^\rho{}_{\sigma\mu\nu}V^\sigma = -T^\lambda{}_{\mu\nu}D_\lambda V^\rho, \quad -(3).$$

which is similar to eq. (1) but written completely in the base manifold. With eq. (3) eq. (2) becomes a rotation generator type equation:

$$[D_\mu, D_\nu]V^\rho = -2T^\lambda{}_{\mu\nu}D_\lambda V^\rho, \quad -(4)$$

which is an operator type equation:

$$\boxed{[D_\mu, D_\nu] = -2T^\lambda{}_{\mu\nu}D_\lambda.} \quad -(5)$$

The operators or group generators always obey the Jacobi identity:

$$[D_\rho, [D_\mu, D_\nu]] + [D_\nu, [D_\rho, D_\mu]] + [D_\mu, [D_\nu, D_\rho]]$$

2)

As given for example in Ryder, p. 113, the generators of any group obey the Jacobi identity, so D_μ, D_ν and D_λ can be considered to be group generators. For example the generators of $SO(3)$ (or $SU(2)$) obey:

$$[I_i, I_j] = i\epsilon_{ijk} I_k$$
$$= C_{ijk} I_k \quad -(7)$$

where the group structure constant is:

$$C_{ijk} = i\epsilon_{ijk} \quad -(8)$$

which obeys the condition:

$$C_{lin} C_{njk} + C_{ljn} C_{nki} + C_{lkn} C_{nij} = 0 \quad -(9)$$

and can be defined by the adjoint representation:

$$C_{imn} = (I_i)_{mn} \quad -(10)$$

where:

$$I_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$-(11)$$

There is a clear similarity between eqs. (5) and (7), except that eq. (5) is written in the general manifold and eq. (7) in flat spacetime.

Therefore it is possible to think of the torsion tensor in eq. (4) as a group

3) structure constant of the group whose generators are D_μ, D_ν and D_λ. Taking this analogy further it is well known that the SU(3) group is defined by:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = i f_{abc} \frac{\lambda_c}{2} \quad -(12)$$

where λ_a are the Gell-Mann matrices and $i f_{abc}$ are the structure constants of SU(3).

It therefore becomes clear that if eq. (5) is considered to be rotational in nature, analogous to (7) & (12), the possible values of $T^\lambda{}_{\mu\nu}$ are $T^1{}_{23}$, $T^2{}_{31}$, $T^3{}_{12}$, the these are spacelike totally anti-symmetric values. These are analogous to ϵ_{123}, ϵ_{231}, ϵ_{312} and play a role analogous or f_{abc}. Their Hodge duals are $T^1{}_{01}$, $T^2{}_{02}$ and $T^3{}_{03}$. In ECE theory they define the electric and magnetic field components

$$E_X = cA^{(0)} T^1{}_{01}, \quad B_X = A^{(0)} T^1{}_{23},$$
$$E_Y = cA^{(0)} T^2{}_{02}, \quad B_Y = A^{(0)} T^2{}_{31},$$
$$E_Z = cA^{(0)} T^3{}_{03}, \quad B_Z = A^{(0)} T^3{}_{12} \quad -(13)$$

which are used in:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 ; \quad \underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(14)$$