On the Possible Existence of a Second Form of Electrical Current in the ECE Equations of Electromagnetism

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ABSTRACT:
In the non-traditional areas of physics it has been postulated and perhaps demonstrated that a second form of electrical current, called by some a "cold current", coexists with the standard electrical current that is traditionally explained by the Maxwell-Heaviside theory of electromagnetism. Further, the ECE equations of electromagnetism have demonstrated unusual electrical behaviour that is not explained by the traditional Maxwell-Heaviside theory whereby a transient zero value for one of the field potentials generates secondary components to the fields associated with the spin connection.

It is postulated that a new form of current exists within the framework of the ECE equations of electromagnetism as one of the possible solutions of ECE theory. The properties of such a current are briefly speculated upon.

A basis is laid for creating a well-posed problem in the ECE theory of electromagnetism. For time dependent problems, nine inter-dependent wave-like equations are proposed. The equations whose original formulation was expressed in pairs of Curl and Divergence equations were recast into wave equations with a divergent equation representing one possible boundary condition. A similar formulation based on coupled Poisson equations is proposed for the static situation. Limiting equations for magnetic only, quasi-magnetostatic, and electric only formulations as well as linearized versions of all equation sets, are also presented.

I. INTRODUCTION

The solutions to the ECE electromagnetic equations, other than for simple analytic and numerical models [1],[2],[3] has remained elusive. This has been in part due to the under-posed nature of the equation system. What this means typically is that there is not enough information in the form of independent equations to define the dependent variables completely. This has required that some limiting simplifications be imposed in order to get a balanced equation system. Usually these simplifications amounted to a global specification for one of the dependent variables.

Besides being under-posed, the equations tend to be unstable from a solution standpoint, and are subject to singularities in the solution. This paper addresses the issue of the under-posed nature of the equations, and proposes a modified set of equations that is better suited to numerical solution.

II: THE PROBLEM

In the non-traditional areas of physics it has been postulated [4], [5], taught [6], [7], and perhaps demonstrated [8] that a second form of electrical current, called by some a "cold current", coexists with the standard electrical current that is traditionally explained by the Maxwell-Heaviside theory of electromagnetism. This second form of current seems to conduct electrical energy in circuits for example, but does so with a resistive loss that is significantly less than the traditional Ohmic loss explained by Maxwell.

Further, the ECE equations of electromagnetism have demonstrated both analytically and numerically, unusual electrical behaviour that is not explained by the traditional Maxwell-Heaviside theory [1], [2], [3]. This behaviour has been observed mathematically in time dependent situations where two or more spatial dimensions are involved.

It is postulated that a new form of current exists within the framework of the ECE equations of electromagnetism. Its existence is one of the possible solutions of ECE theory resulting from the separation of one of the key equations into two distinct equations acting in a coupled fashion.

To demonstrate this, we start with the ECE electromagnetic equations expressed in engineering form in terms of potentials and spin connections [9] in a Euclidean vector space.

(1) $$\nabla \bullet (\boldsymbol{\omega} \times \boldsymbol{A}) = 0$$

(2) $$\nabla \times (\boldsymbol{\omega}\phi - \omega_o \boldsymbol{A}) - \frac{\partial(\boldsymbol{\omega} \times \boldsymbol{A})}{\partial t} = 0$$

(3) $$\nabla \bullet \left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla\phi - \boldsymbol{\omega}\phi + \omega_o \boldsymbol{A} \right) = -\frac{\rho}{\varepsilon_o}$$

(4) $$\nabla \times \nabla \times \boldsymbol{A} - \nabla \times (\boldsymbol{\omega} \times \boldsymbol{A}) + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \nabla\left(\frac{\partial \phi}{\partial t}\right) + \frac{\partial \omega_o \boldsymbol{A}}{\partial t} - \frac{\partial \boldsymbol{\omega}\phi}{\partial t} \right) = \mu_o \boldsymbol{J}$$

where ϕ and $\boldsymbol{A}$ are the scalar and vector potentials respectively, ω_o and $\boldsymbol{\omega}$ are the scalar and vector spin connections respectively, and ρ and $\boldsymbol{J}$ are the charge density and traditional current density respectively.

According to Appendix I, equations (1) and (2) represent three (not four) independent equations in a three-dimensional Euclidean vector space. In fact, equation (2) can be treated as the field equation, and equation (1) as the flux boundary condition [3]. A similar situation can be seen for equations (3) and (4), but is more easily seen if (3) and (4) are expressed in terms of electric field $\boldsymbol{E}$ and magnetic induction field $\boldsymbol{B}$ [9].

(5) $$\nabla \bullet \boldsymbol{E} = \frac{\rho}{\varepsilon_o}$$

(6) $$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

By Appendix I, equation (6) has the following equation as one component.

(7) $$\nabla \bullet \left(\mu_o \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \right) = 0$$

This we can recognize with a little algebra, as equation (5) by virtue of the charge conservation equation.

Thus equations (1) through (4) represent six equations in unknowns ϕ, $\boldsymbol{A}$, ω_o and $\boldsymbol{\omega}$ indicating that the equation system the way it is presented is lacking two equations for complete specification. That is, it is under-posed or under-specified.

A well-posed problem has the number of independent equations equal to the number of dependent variables. In addition, it requires that some form of boundary condition specification exist somewhere on the boundaries, and that initial conditions be specified for each of the dependent variables. The number of equations can exceed the number of dependent variables. This offers a confirmation to the solution that is found, but care must be taken to choose the equations that provide the most information for generating a solution.

A word at this point should be said about boundary conditions. Some numerical algorithms for partial differential equation solving require that boundary conditions be expressed in terms of boundary condition types known as Dirichlet, Neumann and a combination of the two called Robin. Other algorithms prefer a Dirichlet condition and what is called a "natural" boundary condition that represents the flux of a dependent variable or dependent variable combination across a boundary. All dependent variables need some form of specification somewhere on the boundaries to the problem, although they don't need to be specified everywhere. Explicit specification is not required for each dependent variable, but each variable should be included in an algebraic condition representing its boundary value if nothing else. Care must be taken however, because sometimes an algorithm assumes certain boundary conditions if they are not otherwise explicitly expressed. For example, sometimes a zero flux will be assumed at a boundary if nothing else is specified.

III. A SOLUTION

Equations (1) through (4) represent two pairs of Curl and Divergence equations, each with three independent equations in a three-dimensional vector space. It would be very convenient, to form a better posed problem, if there was another set of Curl and Divergence equations to add the remaining equations. This can be obtained if one accepts the following postulate.

Assume that there exists a second form of current density $\boldsymbol{J_1}$ *that is different from the traditional current density* $\boldsymbol{J_o}$ *where the traditional current density is governed by Maxwell-Heaviside equations (8) and that the second form of current density is governed by equation (9).*

(8) $$\nabla \times \nabla \times \boldsymbol{A} + \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \nabla\left(\frac{\partial \phi}{\partial t}\right)\right) = \mu_o \boldsymbol{J_o}$$

(9) $$-\nabla \times (\boldsymbol{\omega} \times \boldsymbol{A}) + \frac{1}{c^2}\left(\frac{\partial \omega_o \boldsymbol{A}}{\partial t} - \frac{\partial \boldsymbol{\omega} \phi}{\partial t}\right) = \mu_o \boldsymbol{J_1}$$

We note that this is just the separation of equation (4) into two parts, with a new current $\boldsymbol{J_1}$, such that

(10) $$\boldsymbol{J} = \boldsymbol{J_o} + \boldsymbol{J_1}$$

We can perform a similar separation for equation (3) to get

(11) $$\nabla \bullet \left(-\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi\right) = \frac{\rho_o}{\varepsilon_o}$$

(12) $$\nabla \bullet (-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}) = -\frac{\rho_1}{\varepsilon_o}$$ where

(13) $$\rho = \rho_o + \rho_1$$

In equation (13), the total charge density ρ is made up of two components, the traditional charge density ρ_o and a new component ρ_1, corresponding to the second form of current $\boldsymbol{J_1}$.

A. Master Equation Set

The above nine independent equations are re-written here for easy reference, in pairs of Curl, and Divergence equations. (Appendix II contains proof of equation independence.)

(14) $\nabla\times\nabla\times\boldsymbol{A}+\frac{1}{c^2}\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\nabla\left(\frac{\partial\phi}{\partial t}\right)\right)=\mu_o\boldsymbol{J_o}$

(15) $\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\phi\right)=-\frac{\rho_o}{\varepsilon_o}$

(16) $-\nabla\times(\boldsymbol{\omega}\times\boldsymbol{A})+\frac{1}{c^2}\left(\frac{\partial\omega_o\boldsymbol{A}}{\partial t}-\frac{\partial\boldsymbol{\omega}\phi}{\partial t}\right)=\mu_o\boldsymbol{J_1}$

(17) $\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$

(18) $\nabla\times(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})+\frac{\partial(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t}=0$

(19) $\nabla\bullet(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=-\frac{\rho_1}{\varepsilon_o}$

The equation set (14),(16) and (18) consists of nine equations. These equations can be derived from the twelve equations above by using the technique given in Appendix I. The dependent variables are ϕ, $\boldsymbol{A}$, ω_o, and $\boldsymbol{\omega}$, with some of ρ_o, $\boldsymbol{J_o}$, ρ_1, *and* $\boldsymbol{J_1}$, being driving terms.

This equation system, as stated above consists of nine independent equations in eight unknowns and so is over specified. It is highly non-linear so could make its use in numerical calculations difficult.

B: Time-Dependent Equations

The master equation set is easily shown be wave-like in form, for time dependent situations. In Appendix II it is shown that the master equation set can be expressed as nine wave equations with coupling at the boundaries and through specification of initial conditions. The following equations, easily derived from the master equation set in the same manner as the homogeneous forms of Appendix II, expressed in non-homogeneous form are

(20) $\nabla^2\left(-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi\right)-\frac{1}{c^2}\frac{\partial^2\left(-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi\right)}{\partial t^2}=\mu_o\frac{\partial\boldsymbol{J_o}}{\partial t}$

(21) $\nabla^2(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})-\frac{1}{c^2}\frac{\partial^2(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})}{\partial t^2}=\mu_o\frac{\partial\boldsymbol{J_1}}{\partial \boldsymbol{t}}$

(22) $\nabla^2(\boldsymbol{\omega}\times\boldsymbol{A})-\frac{1}{c^2}\left(\frac{\partial^2(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t^2}\right)=\mu_o\nabla\times\boldsymbol{J_1}$

which together with the divergent equations, often used to specify flux type boundary conditions [5],

(23) $\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\phi\right)=-\frac{\rho_o}{\varepsilon_o}$

(24) $\nabla \bullet (\boldsymbol{\omega} \times \boldsymbol{A}) = 0$

(25) $\nabla \bullet (-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}) = -\frac{\rho_1}{\varepsilon_o}$

other boundary conditions appropriate to the problem, and initial conditions, completely define the problem. This system is over specified with one more equation than dependent variables.

C: Static Equations

For situations where there is no time dependence, the master set of equations reduce to the following

(26) $\nabla \times \nabla \times \boldsymbol{A} = \mu_o \boldsymbol{J_o}$

(27) $\nabla^2 \phi = -\frac{\rho_o}{\varepsilon_o}$

(28) $-\nabla \times (\boldsymbol{\omega} \times \boldsymbol{A}) = \mu_o \boldsymbol{J_1}$

(29) $\nabla \bullet (\boldsymbol{\omega} \times \boldsymbol{A}) = 0$

(30) $\nabla \times (-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}) = 0$

(31) $\nabla \bullet (-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}) = -\frac{\rho_1}{\varepsilon_o}$

Equations (26) and (27) are decoupled from the rest of the equation set. However, once again, these traditional equations of electromagnetism do not act independently from the equations that define the spin connection since they are coupled to (28) through (31) and cannot be solved independently.

By virtue of equation (30), $(-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A})$ can be written as the gradient of a scalar η, ie.

(32) $\nabla \eta = -\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}$

Equation (31) then becomes

(33) $\nabla^2 \eta = -\frac{\rho_1}{\varepsilon_o}$

Equation (28) can be put into similar form by taking its Curl and noting equation (29) to get

(34) $\nabla^2 (\boldsymbol{\omega} \times \boldsymbol{A}) = \mu_o \nabla \times \boldsymbol{J_1}$

The equation system consisting of equations (26), (27), (33), and (34) consists of eight Poisson-like equations in eight unknowns $\boldsymbol{A}$, ϕ, $\boldsymbol{\omega}$, and η so is completely specified.

Flux boundary conditions (29) and (31) together with Dirichlet and/or other conditions on ϕ make the problem well-posed.

D: Linearization of the Time-Dependent Equations

We note that we have nine non-homogeneous wave equations in eight unknowns, $\boldsymbol{A}$, ϕ, $\boldsymbol{\omega}$, and ω_o. This system can be forced into a well-posed formulation by transforming the non-linear partial differential equations to linear differential equations. Linearization of non-linear partial differential equations is not new [10], [11] and has already been applied to the original form of the ECE engineering equations [12].

If we define new variables

(35) $$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi$$

(36) $$\boldsymbol{F} = \omega_o \boldsymbol{A} - \boldsymbol{\omega}\phi$$

(37) $$\boldsymbol{G} = -\boldsymbol{\omega} \times \boldsymbol{A}$$

then the wave equations (20), (21), and (22) become

(38) $$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu_o \frac{\partial \boldsymbol{J_o}}{\partial t}$$

(39) $$\nabla^2 \boldsymbol{F} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{F}}{\partial t^2} = \mu_o \frac{\partial \boldsymbol{J_1}}{\partial \boldsymbol{t}}$$

(40) $$\nabla^2 \boldsymbol{G} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{G}}{\partial t^2} = -\mu_o \nabla \times \boldsymbol{J_1}$$

The boundary conditions are given by

(41) $$\nabla \bullet \boldsymbol{E} = \frac{\rho_o}{\varepsilon_o}$$

(42) $$\nabla \bullet \boldsymbol{F} = -\frac{\rho_1}{\varepsilon_o}$$

(43) $$\nabla \bullet \boldsymbol{G} = 0$$

Equations (38) through (40) constitute a well-posed problem with nine independent equations in nine unknowns, ***E, F***, and ***G*** and compatible boundary conditions (41) through (43) for flux through the boundaries, should these be appropriate. Non-linearity enters the system through Dirichlet or Neumann type boundary conditions and initial conditions through the transformation equations (35) through (37).

E: Linearization of the Static Equations

One can linearize the static equations in a manner analogous to that done for the time dependent equations. Redefining

(44) $\boldsymbol{G} = -\boldsymbol{\omega} \times \boldsymbol{A}$

the static equations become rewritten here for easy reference

(45) $\nabla \times \nabla \times \boldsymbol{A} = \mu_o \boldsymbol{J_o}$

(46) $\nabla^2 \phi = -\dfrac{\rho_o}{\varepsilon_o}$

(47) $\nabla^2 \eta = -\dfrac{\rho_1}{\varepsilon_o}$

(48) $\nabla^2 \boldsymbol{G} = -\mu_o \nabla \times \boldsymbol{J_1}$

along with boundary flux conditions,

(49) $\nabla \bullet \boldsymbol{G} = 0$

and the second scalar potential

(50) $\nabla \eta = -\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}$

completely define the static problem.

Equations (45) through (48) are eight equations in eight unknowns $\boldsymbol{A}$, ϕ, $\boldsymbol{G}$, and η so is completely specified. Flux boundary condition (49) together with Dirichelt and/or Neumann conditions on ϕ, make the problem linear and well-posed. Non-linearity enters the system through Dirichelt or Neumann type boundary conditions and initial conditions through the transformation equations (44) and (50).

F: Constitutive Relations

To complement a set of field equations one often specifies constitutive relations that form functional relationships between various field variables. For example, relationships between the electric displacement field and the electric field give a relative permittivity for materials that behave in a linear fashion. Similarly, relations exist between the magnetic inductive and magnetic fields defining a relative permeability for linearly behaving materials. For materials that conduct a current, a linear relationship between current density and electric field strength known as Ohm's Law in traditional electromagnetic theory, is often assumed ie.

(51) $\boldsymbol{J_0} = \sigma\left(-\dfrac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi \right)$

At this point, it is not known if we can assume a similar form for $\boldsymbol{J_1}$ to Ohm's Law. We will however assume this, ie.,

(52) $$\boldsymbol{J_1} = a\left(-\boldsymbol{\omega}\phi + \omega_o \boldsymbol{A}\right)$$ for the secondary current,

where a is a material property somewhat akin to conductivity σ in traditional Maxwell-Heaviside theory. We have assumed homogeneity and isotropy in specifying a constant value for a and σ. From a purely speculative standpoint, if $\boldsymbol{J_1}$ dissipates power in a manner analogous to $\boldsymbol{Jo}$ then a must be larger than σ for the secondary current to have less "Ohmic" loss, as postulated in the literature [4], [8].

IV: DISCUSSION

A summary table of all of the non-linear equations presented in this paper is included in Appendix V. Linearized versions are presented in Appendix VI.

A: Divergence of $\boldsymbol{J_1}$

Equations (16) through (19) result in a second charge conservation equation

(53) $$\nabla \bullet \boldsymbol{J_1} + \frac{\partial \rho_1}{\partial t} = 0$$

B: Spike-Like Nature Of Dependent Variables

It has been repeatedly demonstrated [1], [2], and [3], that some of the dependent field variables exhibit a spike-like behaviour which occurs whenever one of the potentials crosses zero.

As is shown earlier, the master equation set for time dependent problems can be expressed as nine independent wave equations. The interconnectivity of the wave equations is not overly apparent until a boundary is reached. At that time, the infinite number of possible solutions is reduced to a single valued function for each variable, given a well-posed problem.

If one considers a one dimensional case for example, equation (24) is trivial, and equation (25) reduces to

(54) $$\frac{\partial}{\partial x}\left(-\omega_x \phi + \omega_o A_x\right) = -\frac{\rho_1}{\varepsilon_o}$$

This illustrates the origin of "spike like" behaviour of both $\boldsymbol{\omega}$ and ω_o when either ϕ or $\boldsymbol{A}$ is zero and also shows where the calculation difficulties associated with singularities arise.

C: Significance of the $\boldsymbol{\omega}\times \boldsymbol{A}$ Field

Consider equation (22). In [3], it was shown that for a thin flat conductive plate, the $\boldsymbol{\omega}\times \boldsymbol{A}$ field is perpendicular to the surface of the plate. If one imagines this plate wound into a tube, in essence one has a wire radiating the $\boldsymbol{\omega}\times \boldsymbol{A}$ field in a radial fashion, perpendicular to the wire. This perpendicular field has been discussed at length in [4], and has ascribed healing properties to it. It also has been described as being "electrostatic" in nature, most likely because it radiates perpendicular to the source. In the near field limit, in the absence of current flow, which one would expect for a radiating source, the field is Laplacian and in this particular situation has only one component, directed in the direction of propagation. Scalar waves have been described in the literature [13] and it may be that this perpendicular component is such a wave.

V: CONCLUSIONS

A basis is laid for creating well-posed problems in the ECE theory of electromagnetism. The equations whose original formulation were expressed in pairs of Curl and Divergence equations were recast into inter-dependent wave equations with divergent equations representing the boundary conditions. This allowed the separation of the equivalent of the Maxwell-Ampere equation in ECE theory into two separate equations, the first defining the traditional current density of Maxwell-Heaviside theory, and the second introducing a new secondary current that is a result of spin connection and heretofore only speculated upon in the non-traditional areas of physics. One of the results of this separation is the introduction of a wave field that could explain the phenomenon of "scalar" waves proposed in the literature.

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APPENDIX I

Theorem: *In three spatial dimensions, the equations*

(1) $\nabla \times \boldsymbol{F} = \boldsymbol{G}$
(2) $\nabla \bullet \boldsymbol{G} = 0$

where ***F*** *and* **G** *are two independent vectors in three dimensions, represents at most three independent equations.*

Proof:
Assume that the vectors ***F*** and ***G*** are expressed on a three basis vector Euclidean coordinate system which we will assume to be Cartesian, without loss of generality.

We will express equation (1) in matrix format for convenience,

(3) $$\begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix}$$

If we perform the following operations; $\frac{\partial}{\partial x}$ on the first row, $\frac{\partial}{\partial y}$ on the second row, and $\frac{\partial}{\partial z}$ on the third row, we get the following equation set.

(4) $$\begin{pmatrix} 0 & -\frac{\partial^2}{\partial x \partial z} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial y \partial z} & 0 & -\frac{\partial^2}{\partial x \partial y} \\ -\frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial x \partial z} & 0 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial G_x}{\partial x} \\ \frac{\partial G_y}{\partial y} \\ \frac{\partial G_z}{\partial z} \end{pmatrix}$$

If we replace the top row with the sum of rows 1, 2 and 3, we get

(5) $$\begin{pmatrix} 0 & 0 & 0 \\ \frac{\partial^2}{\partial y \partial z} & 0 & -\frac{\partial^2}{\partial x \partial y} \\ -\frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial x \partial z} & 0 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} \\ \frac{\partial G_y}{\partial y} \\ \frac{\partial G_z}{\partial z} \end{pmatrix}$$

We immediately recognize the first equation as equation (2).

Thus we have at most three independent equations in the four equations of equations (1) and (2).

This can be immediately reduced to show that in two dimensions we get only two independent equations and in one dimension we have one independent equation.

APPENDIX II

To illustrate the degree of independence of the equations in the master equation set, we consider their homogenous forms for simplicity.

(1) $$\nabla\times\nabla\times\boldsymbol{A}+\frac{1}{c^2}\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\nabla\left(\frac{\partial\phi}{\partial t}\right)\right)=0$$

(2) $$\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\phi\right)=0$$

(3) $$\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$$

(4) $$\nabla\times(\boldsymbol{\omega}\phi-\omega_o\boldsymbol{A})-\frac{\partial(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t}=0$$

(5) $$-\nabla\times(\boldsymbol{\omega}\times\boldsymbol{A})+\frac{1}{c^2}\left(\frac{\partial\omega_o\boldsymbol{A}}{\partial t}-\frac{\partial\boldsymbol{\omega}\phi}{\partial t}\right)=0$$

(6) $$\nabla\bullet(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=0$$

The equation set (1) through (6) offers a convenient linearization [12]. If one defines new field variables by

(7) $$\boldsymbol{E}=-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi$$

(8) $$\boldsymbol{F}=-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A}$$

(9) $$\boldsymbol{G}=-\omega\times\boldsymbol{A}$$

the equation set can be cast in a simpler format, namely

(10) $$\nabla\times\nabla\times\boldsymbol{E}+\frac{1}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2}=0$$ (see [3])

(11) $$\nabla\bullet E=0$$

(12) $$\nabla\bullet\boldsymbol{G}=0$$

(13) $$-\nabla\times\boldsymbol{F}+\frac{\partial\boldsymbol{G}}{\partial t}=0$$

(14) $$\nabla\times\boldsymbol{G}+\frac{1}{c^2}\frac{\partial\boldsymbol{F}}{\partial t}=0$$

(15) $$\nabla\bullet\boldsymbol{F}=0$$

Equations (12), (13), (14), (15) can be written in wave-like form by taking the Curl of (13) and (14) and eliminating the “cross” terms. The resulting equations are

(16) $$\nabla\times\nabla\times\boldsymbol{G}+\frac{1}{c^2}\frac{\partial^2\boldsymbol{G}}{\partial t^2}=0$$

(17) $\nabla \times \nabla \times \boldsymbol{F} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{F}}{\partial t^2} = 0$

Using the vector identity

(18) $\nabla \times \nabla \times \boldsymbol{F} = \nabla(\nabla \bullet \boldsymbol{F}) - \nabla^2 \boldsymbol{F}$

and noting (12) and (15), these reduce to

(19) $\nabla^2 \boldsymbol{G} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{G}}{\partial t^2} = 0$

(20) $\nabla^2 \boldsymbol{F} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{F}}{\partial t^2} = 0$

and

(21) $\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$

These represent nine independent wave equations. The taking of time and spatial derivatives in their derivation removes some of their generality however, limiting solutions to those of an oscillatory nature, and in a visual sense, looses the interdependence of the equations.

The interdependence of the equations occurs in the specification of the initial conditions, and at the boundaries. The boundary conditions require a specification of the field variables where these are known, and a specification of gradients or flux for the field variables ***E,F,*** and ***G,*** where known.

APPENDIX III MAGNETIC ONLY EQUATIONS

For conductive materials it can be assumed that there are no free charges and that the scalar potential is zero. Given linear constitutive relations,

(1) $\boldsymbol{J_0} = -\sigma \frac{\partial \boldsymbol{A}}{\partial t}$ and

(2) $\boldsymbol{J_1} = a\,\omega_o \boldsymbol{A}$ for the secondary current,

the wave-like equation set reduces to

(3) $$\nabla^2\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right) - \frac{1}{c^2}\frac{\partial^2\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right)}{\partial t^2} - \mu_o \sigma \frac{\partial\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right)}{\partial t} = 0$$

(4) $$\nabla^2(\omega_o \boldsymbol{A}) - \frac{1}{c^2}\frac{\partial^2(\omega_o \boldsymbol{A})}{\partial t^2} - \mu_o a \frac{\partial(\omega_o \boldsymbol{A})}{\partial t} = 0$$

(5) $$\nabla^2(\boldsymbol{\omega} \times \boldsymbol{A}) - \frac{1}{c^2}\left(\frac{\partial^2(\boldsymbol{\omega} \times \boldsymbol{A})}{\partial t^2}\right) - \mu_o a \nabla \times (\omega_o \boldsymbol{A}) = 0$$

which together with boundary conditions [3]

(6) $$\nabla \bullet \left(\frac{\partial \boldsymbol{A}}{\partial t}\right) = 0$$

(7) $$\nabla \bullet (\boldsymbol{\omega} \times \boldsymbol{A}) = 0$$

(8) $$\nabla \bullet (\omega_o \boldsymbol{A}) = 0$$

forms the equation system.

Equation (3) becomes upon integration through time,

(9) $$\nabla^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \mu_o \sigma \frac{\partial \boldsymbol{A}}{\partial t} = 0$$

In two dimensions equations (9), (4), (6) and (8) apply since $\boldsymbol{\omega} \times \boldsymbol{A}$ is trivial.

This model supports damped wave propagation of $\boldsymbol{A}$, $\omega_o \boldsymbol{A}$ and $\boldsymbol{\omega x A}$ all with propagation speed c. Given the spike-like nature expected for $\boldsymbol{\omega}$ and ω_o, one would expect a spike-like waveform for $\omega_o \boldsymbol{A}$ and $\boldsymbol{\omega}$**x**$\boldsymbol{A}$**.**

As discussed in [3], for a harmonic field in frequencies in the lower rf range (hundreds of kilohertz), the wave-like component in equation (9) is insignificant in comparison to the dispersive component. This occurs when

$$\left|\frac{i\beta}{\mu_o \sigma c^2}\right| \prec\prec 1 \qquad \text{where } \beta \text{ is a typical frequency of oscillation.}$$

It it unknown whether this applies to the other two wave components, because at this time, we have no knowledge of the value of the secondary conductivity a.

However, if we can make a similar assumption for equation (4), a similar ratio of terms is

$$\left|\frac{i\beta}{\mu_o a c^2}\right| \prec\prec 1$$

The magnitude of a is unknown, but if it is on par, or greater than σ, then the above inequality certainly is valid.

We can postulate a similar relationship for the terms in equation (5).

The result is that equations (4), (5) and (9) can be simplified from the hyperbolic wave equations to the much more manageable parabolic equations, ie.

(10) $$\nabla^2 \boldsymbol{A} - \mu_o \sigma \frac{\partial \boldsymbol{A}}{\partial t} = 0$$

(11) $$\nabla^2 \omega_{\boldsymbol{o}} \boldsymbol{A} - \mu_o a \frac{\partial (\omega_{\boldsymbol{o}} \boldsymbol{A})}{\partial \boldsymbol{t}} = 0$$

(12) $$\nabla^2 (\boldsymbol{\omega} \times \boldsymbol{A}) - \mu_o a \nabla \times (\omega_{\boldsymbol{o}} \boldsymbol{A}) = 0$$

The boundary conditions remain equations (7), and (8) and (6) with the removal of the time derivative should a flux type boundary condition be required. Note that equation (12) now supports instant propagation of its dependent variable $\boldsymbol{\omega} \times \boldsymbol{A}$, whereas equations (10) and (11) offer a diffusive mode of propagation in a coupled manner.

The equation system is over specified but through linearization results in a well-posed problem.

Writing

(13) $\boldsymbol{F} = \omega_o \boldsymbol{A}$ and

(14) $\boldsymbol{G} = -\omega \times \boldsymbol{A}$ equations (11) and (12) become

(15) $$\nabla^2 \boldsymbol{F} - \mu_o a \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{t}} = 0$$

(16) $$\nabla^2 \boldsymbol{G} - \mu_o a \nabla \times \boldsymbol{G} = 0$$

which together with equation (10) and

(17) $\nabla \bullet \boldsymbol{A} = 0$

(18) $\nabla \bullet \boldsymbol{F} = 0$

(19) $\nabla \bullet \boldsymbol{G} = 0$

or other appropriate boundary conditions constitutes a well posed problem. Non-linearities enter through equations (13) and (14).

APPENDIX IV ELECTRIC ONLY EQUATIONS

For situations where there is no vector potential, the time dependent equations reduce to

(1) $$\nabla^2(\nabla\phi)-\frac{1}{c^2}\frac{\partial^2(\nabla\phi)}{\partial t^2}=-\mu_o\frac{\partial \boldsymbol{J_o}}{\partial t}$$

(2) $$\nabla^2(\boldsymbol{\omega}\phi)-\frac{1}{c^2}\frac{\partial^2(\boldsymbol{\omega}\phi)}{\partial t^2}=-\mu_o\frac{\partial \boldsymbol{J_1}}{\partial \boldsymbol{t}}$$

which together with

(3) $$\nabla^2\phi=-\frac{\rho_o}{\varepsilon_o}$$

(4) $$\nabla\bullet(-\boldsymbol{\omega}\phi)=-\frac{\rho_1}{\varepsilon_o}$$

and initial conditions, completely define the problem. Equations (3) is the divergence boundary condition on $\nabla\phi$.

If we write, which we can do by virtue of equation (18) in the body of the paper

(5) $$\nabla\eta=-\boldsymbol{\omega}\phi$$

where η is a second scalar potential, then equations (2) and (4) take a form analogous to (1) and (3), namely

(6) $$\nabla^2(\nabla\eta)-\frac{1}{c^2}\frac{\partial^2(\nabla\eta)}{\partial t^2}=\mu_o\frac{\partial \boldsymbol{J_1}}{\partial \boldsymbol{t}}$$

(7) $$\nabla^2\eta=\frac{\rho_1}{\varepsilon_o}$$

For the electric only equations, we are left with solving the equation pair (1), (6) and the pair of (3) and (7) as the flux boundary conditions.

This system is greatly over specified and only becomes well posed if there is no current present, in which case two wave equations result, ie.

(8) $$\nabla^2\eta-\frac{1}{c^2}\frac{\partial^2\eta}{\partial t^2}=0$$

(9) $$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$$

to form a well-posed system given boundary and initial conditions.

APPENDIX V SUMMARY TABLE OF NON-LINEAR EQUATIONS FOR TWO CURRENT MODEL OF ECE EM EQUATIONS

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Master Equation Set $\nabla\times\nabla\times\boldsymbol{A}+\frac{1}{c^2}\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\nabla\left(\frac{\partial\phi}{\partial t}\right)\right)=\mu_o\boldsymbol{J_o}$ $-\nabla\times(\boldsymbol{\omega}\times\boldsymbol{A})+\frac{1}{c^2}\left(\frac{\partial\omega_o\boldsymbol{A}}{\partial t}-\frac{\partial\boldsymbol{\omega}\phi}{\partial t}\right)=\mu_o\boldsymbol{J_1}$ $\nabla\times(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})+\frac{\partial(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t}=0$	$\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\phi\right)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$ $\nabla\bullet(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=-\frac{\rho_1}{\varepsilon_o}$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\boldsymbol{J}=\boldsymbol{J_o}+\boldsymbol{J_1}$ $\rho=\rho_o+\rho_1$
Time Dependent Equations $\nabla^2\left(-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi\right)-\frac{1}{c^2}\frac{\partial^2\left(-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\phi\right)}{\partial t^2}=\mu_o\frac{\partial\boldsymbol{J_o}}{\partial t}$ $\nabla^2(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})-\frac{1}{c^2}\frac{\partial^2(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})}{\partial t^2}=\mu_o\frac{\partial\boldsymbol{J_1}}{\partial\boldsymbol{t}}$ $\nabla^2(\boldsymbol{\omega}\times\boldsymbol{A})-\frac{1}{c^2}\left(\frac{\partial^2(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t^2}\right)=\mu_o\nabla\times\boldsymbol{J_1}$	$\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\phi\right)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$ $\nabla\bullet(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=-\frac{\rho_1}{\varepsilon_o}$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\boldsymbol{J}=\boldsymbol{J_o}+\boldsymbol{J_1}$ $\rho=\rho_o+\rho_1$

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Static Equations $\nabla\times\nabla\times\boldsymbol{A}=\mu_o\boldsymbol{J_o}$ $\nabla\times(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=0$ $-\nabla\times(\boldsymbol{\omega}\times\boldsymbol{A})=\mu_o\boldsymbol{J_1}$	$\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$ $\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A})=-\frac{\rho_1}{\varepsilon_o}$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\nabla\eta=-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A}$
Magnetic Only Equations $\nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}-\mu_o\sigma\frac{\partial\boldsymbol{A}}{\partial t}=0$ $\nabla^2(\omega_o\boldsymbol{A})-\frac{1}{c^2}\frac{\partial^2(\omega_o\boldsymbol{A})}{\partial t^2}-\mu_o a\frac{\partial(\omega_o\boldsymbol{A})}{\partial t}=0$ $\nabla^2(\boldsymbol{\omega}\times\boldsymbol{A})-\frac{1}{c^2}\frac{\partial^2(\boldsymbol{\omega}\times\boldsymbol{A})}{\partial t^2}-\mu_o a\nabla\times(\omega_o\boldsymbol{A})=0$	$\nabla\bullet A=0$ $\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$ $\nabla\bullet(\omega_o\boldsymbol{A})=0$	$\boldsymbol{A}$	$\boldsymbol{A},\ \boldsymbol{\omega},\omega_o$	$\boldsymbol{J_0}=-\sigma\frac{\partial A}{\partial t}$ $\boldsymbol{J_1}=a\omega_o\boldsymbol{A}$ $\phi=0$ $\rho_0=0$ $\rho_1=0$
Quasi-Magnetostatic Equations $\nabla^2\boldsymbol{A}-\mu_o\sigma\frac{\partial\boldsymbol{A}}{\partial t}=0$ $\nabla^2\omega_{\boldsymbol{o}}\boldsymbol{A}-\mu_o a\frac{\partial(\omega_{\boldsymbol{o}}\boldsymbol{A})}{\partial\boldsymbol{t}}=0$ $\nabla^2(\boldsymbol{\omega}\times\boldsymbol{A})-\mu_o a\nabla\times(\omega_{\boldsymbol{o}}\boldsymbol{A})=0$	$\nabla\bullet\left(\frac{\partial\boldsymbol{A}}{\partial t}\right)=0$ $\nabla\bullet(\boldsymbol{\omega}\times\boldsymbol{A})=0$ $\nabla\bullet(\omega_o\boldsymbol{A})=0$	$\boldsymbol{A}$	$\boldsymbol{A},\ \boldsymbol{\omega},\omega_o$	$\boldsymbol{J_0}=-\sigma\frac{\partial A}{\partial t}$ $\boldsymbol{J_1}=a\omega_o\boldsymbol{A}$ $\phi=0$ $\rho_0=0$ $\rho_1=0$

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Electric Only Equations $\nabla^2(\nabla\phi)-\frac{1}{c^2}\frac{\partial^2(\nabla\phi)}{\partial t^2}=-\mu_o\frac{\partial \boldsymbol{J_o}}{\partial t}$ $\nabla^2(\boldsymbol{\omega}\phi)-\frac{1}{c^2}\frac{\partial^2(\boldsymbol{\omega}\phi)}{\partial t^2}=-\mu_o\frac{\partial \boldsymbol{J_1}}{\partial \boldsymbol{t}}$	$\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\boldsymbol{\omega}\phi)=-\frac{\rho_1}{\varepsilon_o}$	ϕ	$\phi, \boldsymbol{\omega}$	$\boldsymbol{A}=0$
Electric Only Zero Current Equations $\nabla^2\eta-\frac{1}{c^2}\frac{\partial^2\eta}{\partial t^2}=0$ $\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$	$\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\nabla\eta)=-\frac{\rho_1}{\varepsilon_o}$	ϕ	$\phi, \boldsymbol{\omega}$	$\boldsymbol{A}=0$ $\nabla\eta=-\boldsymbol{\omega}\phi$

APPENDIX VI SUMMARY TABLE OF LINEAR EQUATIONS FOR TWO CURRENT MODEL OF ECE EM EQUATIONS

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Master Equation Set $\nabla\times\nabla\times \boldsymbol{A}-\frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}=\mu_o \boldsymbol{J_o}$ $\nabla\times \boldsymbol{G}+\frac{1}{c^2}\frac{\partial \boldsymbol{F}}{\partial t}=\mu_o \boldsymbol{J_1}$ $\nabla\times \boldsymbol{F}-\frac{\partial \boldsymbol{G}}{\partial t}=0$	$\nabla\bullet \boldsymbol{E}=\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet \boldsymbol{F}=-\frac{\rho_1}{\varepsilon_o}$ $\nabla\bullet \boldsymbol{G}=0$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\boldsymbol{J}=\boldsymbol{J_o}+\boldsymbol{J_1}$ $\rho=\rho_o+\rho_1$ $\boldsymbol{E}=-\frac{\partial \boldsymbol{A}}{\partial t}-\nabla\phi$ $\boldsymbol{F}=\omega_o\boldsymbol{A}-\boldsymbol{\omega}\phi$ $\boldsymbol{G}=-\boldsymbol{\omega}\times\boldsymbol{A}$ $\phi=0$ $\rho_0=0$ $\rho_1=0$
Time Dependent Equations $\nabla^2\boldsymbol{E}-\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}=\mu_o\frac{\partial \boldsymbol{J_o}}{\partial t}$ $\nabla^2\boldsymbol{F}-\frac{1}{c^2}\frac{\partial^2 \boldsymbol{F}}{\partial t^2}=\mu_o\frac{\partial \boldsymbol{J_1}}{\partial \boldsymbol{t}}$ $\nabla^2\boldsymbol{G}-\frac{1}{c^2}\frac{\partial^2 \boldsymbol{G}}{\partial t^2}=-\mu_o\nabla\times\boldsymbol{J_1}$	$\nabla\bullet \boldsymbol{E}=\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet \boldsymbol{F}=-\frac{\rho_1}{\varepsilon_o}$ $\nabla\bullet \boldsymbol{G}=0$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\boldsymbol{J}=\boldsymbol{J_o}+\boldsymbol{J_1}$ $\rho=\rho_o+\rho_1$ $\boldsymbol{E}=-\frac{\partial \boldsymbol{A}}{\partial t}-\nabla\phi$ $\boldsymbol{F}=\omega_o\boldsymbol{A}-\boldsymbol{\omega}\phi$ $\boldsymbol{G}=-\boldsymbol{\omega}\times\boldsymbol{A}$ $\phi=0$ $\rho_0=0$ $\rho_1=0$

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Static Equations $\nabla\times\nabla\times\boldsymbol{A}=\mu_o\boldsymbol{J_o}$ $\nabla^2\boldsymbol{G}=-\mu_o\nabla\times\boldsymbol{J_1}$	$\nabla\bullet\boldsymbol{G}=0$ $\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\nabla\eta)=-\frac{\rho_1}{\varepsilon_o}$	$\boldsymbol{A},\phi$	$\boldsymbol{A},\phi$ $\boldsymbol{\omega},\omega_o$	$\nabla\eta=-\boldsymbol{\omega}\phi+\omega_o\boldsymbol{A}$ $\boldsymbol{G}=-\boldsymbol{\omega}\times\boldsymbol{A}$
Magnetic Only Equations $\nabla^2\boldsymbol{A}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}-\mu_o\sigma\frac{\partial\boldsymbol{A}}{\partial t}=0$ $\nabla^2\boldsymbol{F}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{F}}{\partial t^2}-\mu_o a\frac{\partial\boldsymbol{F}}{\partial t}=0$ $\nabla^2\boldsymbol{G}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{G}}{\partial t^2}-\mu_o a\nabla\times G=0$	$\nabla\bullet\boldsymbol{A}=0$ $\nabla\bullet\boldsymbol{F}=0$ $\nabla\bullet\boldsymbol{G}=0$	$\boldsymbol{A}$	$\boldsymbol{A},\ \boldsymbol{\omega},\omega_o$	$\boldsymbol{J_0}=-\sigma\frac{\partial\boldsymbol{A}}{\partial t}$ $\boldsymbol{J_1}=a\,\omega_o\boldsymbol{A}$ $\boldsymbol{F}=\omega_o\boldsymbol{A}$ $\boldsymbol{G}=-\omega\times\boldsymbol{A}$
Quasi-Magnetostatic Equations $\nabla^2\boldsymbol{A}-\mu_o\sigma\frac{\partial\boldsymbol{A}}{\partial t}=0$ $\nabla^2\boldsymbol{F}-\mu_o a\frac{\partial\boldsymbol{F}}{\partial t}=0$ $\nabla^2\boldsymbol{G}-\mu_o a\nabla\times\boldsymbol{G}=0$	$\nabla\bullet\boldsymbol{A}=0$ $\nabla\bullet\boldsymbol{F}=0$ $\nabla\bullet\boldsymbol{G}=0$	$\boldsymbol{A}$	$\boldsymbol{A},\ \boldsymbol{\omega},\omega_o$	$\boldsymbol{J_0}=-\sigma\frac{\partial\boldsymbol{A}}{\partial t}$ $\boldsymbol{J_1}=a\,\omega_o\boldsymbol{A}$ $\boldsymbol{F}=\omega_o\boldsymbol{A}$ $\boldsymbol{G}=-\omega\times\boldsymbol{A}$

EQUATION SET	FLUX BOUNDARY CONDITIONS	DIRICHLET OR NEUMANN BOUNDARY CONDITIONS	INITIAL CONDITIONS REQUIRED	DEFINITIONS & ASSUMPTIONS
Electric Only Equations $\nabla^2(\nabla\phi)-\frac{1}{c^2}\frac{\partial^2(\nabla\phi)}{\partial t^2}=-\mu_o\frac{\partial \boldsymbol{J_o}}{\partial t}$ $\nabla^2(\nabla\eta)-\frac{1}{c^2}\frac{\partial^2(\nabla\eta)}{\partial t^2}=\mu_o\boldsymbol{J_1}$	$\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\boldsymbol{\omega}\phi)=-\frac{\rho_1}{\varepsilon_o}$	ϕ	$\phi, \boldsymbol{\omega}$	$\boldsymbol{A}=0$ $\nabla\eta=-\boldsymbol{\omega}\phi$
Electric Only Zero Current Equations $\nabla^2\eta-\frac{1}{c^2}\frac{\partial^2\eta}{\partial t^2}=0$ $\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$	$\nabla\bullet(\nabla\phi)=-\frac{\rho_o}{\varepsilon_o}$ $\nabla\bullet(\nabla\eta)=-\frac{\rho_1}{\varepsilon_o}$	ϕ	$\phi, \boldsymbol{\omega}$	$\boldsymbol{A}=0$ $\nabla\eta=-\boldsymbol{\omega}\phi$