Solution of the ECE Vacuum Equations

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Abstract

The vacuum of Einstein Cartan Evans (ECE) theory is a curved and twisted space-time filled by electromagnetic potentials. The vacuum potential can be computed by the condition that electromagnetic force fields vanish. In classical electrodynamics, this condition is not sufficient to compute a distribution of potential from given boundary conditions. ECE theory has a richer structure with spin connections of Cartan geometry and additional constraints due to the antisymmetry of the connections in general relativity. Expressions for describing the energy and momentum density are given. It can be shown for the first time that the high vacuum potentials known from quantum effects are consequences of the structure of space-time itself.

Keywords: Einstein Cartan Evans (ECE) field theory; antisymmetry; vacuum field.
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1 Introduction

The space free of masses and charges has been a subject of physical interpretation for hundreds of years. Sometimes it was assumed to be empty, sometimes to be a medium called Ether, for example to explain electromagnetic wave propagation. The notion of vacuum often describes empty space, but quantum physicists speak of a "quantum vacuum" that is not empty at all. In this paper we use "vacuum" synonymously with space-time itself in the sense of general relativity. We will see that space is filled with a potential, called "background potential".

In Einstein's theory the vacuum is empty, this result is in sharp contrast to quantum physics, where the vacuum is a sea of virtual particles with a huge energy density. This quantum sea gives rise to the radiative corrections of physics such as the Lamb shift or Casimir effect. In standard physics this is a huge and irreconcilable discrepancy. In Einstein-Cartan-Evans (ECE) theory [1]- [2] both worlds are reconciled, the background in ECE is filled with a potential energy defined directly by the tetrad of Cartan. The potential energy is physical and cannot be arbitrarily changed as in gauge theory. The radiative corrections come from fluctuations in this potential of the generally covariant unified field.

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Energy transfer in ECE can take place through resonant solutions of the field equations. This energy from spacetime in ECE comes from the background potential energy. There is a transfer of energy but the total energy is constant. The conventional continuity equation can be generalized in such a way that curvature and torsion of spacetime can even produce charge carriers. There are experimental hints that such effects can actually happen.

In the past there have been some attempts to explain the background fields by solving Maxwell-Heaviside equations for electromagnetic fields. However, the field equations naturally result to a state where the energy is contained in electric and magnetic fields which are not observed in vacuo (with exception of a 2.7 K background radiation which may exist all over the universe or not). In the current paper we avoid this difficulty by using the experimental fact that such fields are not existent. Therefore the background energy observed by quantum processes must be contained in the potentials. We will show how these can be calculated from the basic geometrical properties of space. These are defined by Cartan geometry which is used exclusively in ECE theory. Since the force fields are assumed to be zero, the field equations identically vanish. Instead we use the basic antisymmetry conditions of the Cartan spin connections and the Maurer-Cartan structure equations. We neglect polarization effects and use the vector form of the equations. In this way we are able to present a general solution of the background potential. We calculate the energy and momentum density by extending a suggestion found in the literature. A discussion section concludes the paper. As a result, we find a plausible explanation why there are huge energy densities in the vacuum. This theory is based on first principles, leading to a new view on this subject without need for obscure quantum effects.

2 Antisymmetry conditions and equations of state

2.1 Direct setup of an equation set

The electric and magnetic field of ECE theory under omission of polarization effects are

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} + \boldsymbol{\omega}\Phi, \tag{1}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}. \tag{2}$$

The antisymmetry conditions for the potentials are the electric vector-valued relation

$$\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} - \boldsymbol{\omega}\Phi = 0, \tag{3}$$

and the three magnetic (scalar-valued) relations, writte in vector form:

$$\mathbf{C} := \begin{pmatrix} \frac{\partial A_2}{\partial x_3} + \frac{\partial A_3}{\partial x_2} + \omega_2 A_3 + \omega_3 A_2 \\ \frac{\partial A_1}{\partial x_3} + \frac{\partial A_3}{\partial x_1} + \omega_1 A_3 + \omega_3 A_1 \\ \frac{\partial A_1}{\partial x_2} + \frac{\partial A_2}{\partial x_1} + \omega_1 A_2 + \omega_2 A_1 \end{pmatrix} = 0. \tag{4}$$

The vacuum conditions are obtained by setting

$$\mathbf{E} = 0, \tag{5}$$

$$\mathbf{B} = 0. \tag{6}$$

These are twelve equations for eight unknowns $\Phi, \mathbf{A}, \omega_0, \boldsymbol{\omega}$. Taking into account the antisymmetry constraints (3)-(4), this would lead to an over-determined system. Therefore we smplify the constraints by taking the divergence of Eqs. (3) and (4):

$$\Delta\Phi - \boldsymbol{\nabla}\cdot\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\cdot(\omega_0\mathbf{A}) - \boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\Phi) = 0, \tag{7}$$

$$\boldsymbol{\nabla}\cdot\mathbf{C} = 0. \tag{8}$$

The last equation leads to mixed second derivatives and can be tried to simplify by using the Lindstrom constraint which is a simplification of Eq. (4):

$$\boldsymbol{\nabla}\times\mathbf{A} = -\boldsymbol{\omega}\times\mathbf{A}. \tag{9}$$

On the other hand, from Eqs. (2) and (6) we have

$$\boldsymbol{\nabla}\times\mathbf{A} = \boldsymbol{\omega}\times\mathbf{A}. \tag{10}$$

which is not compatible with the Lindstrom constraint (9) (see section 5.2). Therefore we have to use the full condition (4). In total we have to solve the equation set

$$-\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0\mathbf{A} + \boldsymbol{\omega}\Phi = 0, \tag{11}$$

$$\boldsymbol{\nabla}\times\mathbf{A} - \boldsymbol{\omega}\times\mathbf{A} = 0, \tag{12}$$

$$\Delta\Phi - \boldsymbol{\nabla}\cdot\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\cdot(\omega_0\mathbf{A}) - \boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\Phi) = 0, \tag{13}$$

$$\begin{aligned}&\frac{\partial}{\partial x_1}\left(\frac{\partial A_2}{\partial x_3} + \frac{\partial A_3}{\partial x_2} + \omega_2 A_3 + \omega_3 A_2\right)\\ &+\frac{\partial}{\partial x_2}\left(\frac{\partial A_1}{\partial x_3} + \frac{\partial A_3}{\partial x_1} + \omega_1 A_3 + \omega_3 A_1\right)\\ &+\frac{\partial}{\partial x_3}\left(\frac{\partial A_1}{\partial x_2} + \frac{\partial A_2}{\partial x_1} + \omega_1 A_2 + \omega_2 A_1\right) = 0.\end{aligned} \tag{14}$$

2.2 Pre-evaluation of magnetic constraints

The direct equation set derived in the previous section is not very handable to numerics and the structure of the equations and possible solutions is not obvious. Therefore we follow a different line of development. By adding Eqs. (4) and (10) we obtain

$$\frac{\partial A_2}{\partial x_3} + \omega_2 A_3 = 0, \tag{15}$$

$$\frac{\partial A_1}{\partial x_3} + \omega_1 A_3 = 0, \tag{16}$$

$$\frac{\partial A_1}{\partial x_2} + \omega_1 A_2 = 0. \tag{17}$$

Subtracting (4) from (10) gives

$$\frac{\partial A_3}{\partial x_2} + \omega_3 A_2 = 0, \tag{18}$$

$$\frac{\partial A_3}{\partial x_1} + \omega_3 A_1 = 0, \tag{19}$$

$$\frac{\partial A_2}{\partial x_1} + \omega_2 A_1 = 0. \tag{20}$$

Comparing (16) with (17) etc. leads to

$$\omega_1 = -\frac{\partial A_1}{\partial x_2}\frac{1}{A_2} = -\frac{\partial A_1}{\partial x_3}\frac{1}{A_3}, \tag{21}$$

$$\omega_2 = -\frac{\partial A_2}{\partial x_1}\frac{1}{A_1} = -\frac{\partial A_2}{\partial x_3}\frac{1}{A_3}, \tag{22}$$

$$\omega_3 = -\frac{\partial A_3}{\partial x_1}\frac{1}{A_1} = -\frac{\partial A_3}{\partial x_2}\frac{1}{A_2} \tag{23}$$

which can be written as

$$A_j \frac{\partial A_i}{\partial x_k} = A_k \frac{\partial A_i}{\partial x_j} \tag{24}$$

with (i, j, k) being permutations of $(1, 2, 3)$. solving Eqs. (24) as a boundary value problem and inserting the solution into (21)-(23) gives a full solution of the magnetic constraint (4) in terms of the vector potential $\mathbf{A}$.

So far we have handled Eqs. (4) and (6) from the given set (3)-(6). Next we have to handle the electric conditions (3) and (5). The only variables remaining to be determined are Φ and ω_0. From inserting (3) into (1) we have [3]

$$\begin{aligned}\mathbf{E} &= -\boldsymbol{\nabla}\Phi + \boldsymbol{\omega}\Phi \\ &= -\frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A}\end{aligned} \tag{25}$$

which gives us two vector equations

$$\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\Phi = 0, \tag{26}$$

$$\frac{\partial \mathbf{A}}{\partial t} + \omega_0 \mathbf{A} = 0. \tag{27}$$

Restricting to the static case, from the latter equation follows

$$\omega_0 = 0. \tag{28}$$

Since Eq. (26) is a vector eqation and we only need to determine one variable, we take the divergence of (26):

$$\boldsymbol{\nabla}^2\Phi - \boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\Phi) = 0 \tag{29}$$

or

$$\boldsymbol{\nabla}^2\Phi - \boldsymbol{\omega}\cdot\boldsymbol{\nabla}\Phi - (\boldsymbol{\nabla}\cdot\boldsymbol{\omega})\Phi = 0. \tag{30}$$

This is identical with the Coulomb law for zero charge density. Solving this differential equation for Φ (with $\boldsymbol{\omega}(\mathbf{A})$ already given) completes the solution for the static ECE vacuum equations.

3 Solutions of the vacuum equations

According to Equation set (24) the vacuum solution for the $\mathbf{A}$ field is decoupled from the electric potential and the spin connections. We first consider the solutions of this equation set. According to computer algebra (we used Mathematica [8]) there are three solutions with a number of integration constants C_i, k_i and β:

$$\mathbf{A}^{(1)} = \mathbf{k}\,\frac{1}{k_3}\,(C_5 + C_6\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)) \tag{31}$$

$$\boldsymbol{A}^{(2)} = \mathbf{k}\,\frac{1}{k_3}\,(C_5 + \tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)\,(C_6 + C_7\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t))) \tag{32}$$

$$\begin{aligned}\mathbf{A}^{(3)} = \mathbf{k}\,\tfrac{1}{k_3}\,(C_5 + \tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)\,(C_6 + \tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)\\ \cdot (C_7 + C_8\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t))))\end{aligned} \tag{33}$$

Details can be found in the appendices. In the form presented here some constants C_i have been renamed to clarify their physical meaning. The first three constants are written as the wave vector $\mathbf{k}$ and the fourth constant which is a phase factor has been interpreted as time dependence βt so that we get a proper argument of the tanh function. In principle all constants could be time dependent as shown in the appendices.

Comparing the three solutions, it is obvious that these are parts of a more general solution of type

$$\mathbf{A}^{(m)} = \mathbf{k}\frac{1}{k_3}\sum_{n=0}^{m} D_n(t)\,(\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t))^n \tag{34}$$

with (possibly time-dependent) canstants D_n. This is a basis set of the function space spanning the solutions of Eq. (24). We will return to this point later.

Next we will derive the solutions of the other variables. The spin connection $\boldsymbol{\omega}$ follows directly from (21)-(23):

$$\boldsymbol{\omega}^{(1)} = -\mathbf{k}\,\frac{C_6\,(sech(\mathbf{k}\cdot\mathbf{x} - \beta t))^2}{C_5 + C_6\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)}, \tag{35}$$

the corresponding $\boldsymbol{\omega}^{(2,3)}$, can be found in Appendix A.

The scalar spin connection can be computed from the explicit time dependence of $\mathbf{A}$ by using Eq. (27). In principle this equation - being a vector equation - is over-determined, but as shown in Appendix A all three components are consistent, giving three solutions from which the first is

$$\omega_0^{(1)} = \beta\,\frac{C_6\,(sech(\mathbf{k}\cdot\mathbf{x} - \beta t))^2}{C_5 + C_6\tanh(\mathbf{k}\cdot\mathbf{x} - \beta t)} \tag{36}$$

which is similar to the components of the vector spin connection. We have assumed that the coefficients are independent of time. Finally the pontential Φ can be derived similarly from Eq. (26). Again all three components of this equation lead to the same result, achieving self-consistency. However, as an important finding, a part of the solution (denoted by $F^{(i)}$) is always the same, regardless of the choice of $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$, or $\mathbf{A}^{(3)}$, for example:

$$\Phi^{(1,i)} = \frac{\cosh(\mathbf{k}\cdot\mathbf{x} - \beta t)}{C_5 \cosh(\mathbf{k}\cdot\mathbf{x} - \beta t) + C_6 \sinh(\mathbf{k}\cdot\mathbf{x} - \beta t)}\, F^{(i)} \tag{37}$$

where $F^{(i)}$ is an arbitrary function depending on only one coordinate surface:

$$\begin{aligned} F^{(1)} &= F(y, z, t), \\ F^{(2)} &= F(x, z, t), \\ F^{(3)} &= F(x, y, t). \end{aligned} \tag{38}$$

Choosing $F = const.$ makes the solution unique. To obtain an impression of the nature of vacuum solutions, some of them are graphed in Figs. 1-4. The constants C_i were chosen 1 or -1 and the wave vector was positioned in z direction: $\mathbf{k} = (0, 0, k_3)$. Then there are only z components of $\mathbf{A}$ and $\boldsymbol{\omega}$. Fig. 1 shows that A_3 varies the more the higher the degree of the solution is. A similar result holds for the vector spin connection (Fig. 2), however the second solution is a pole, indicating that there are discontinuities in the structure of the vacuum. The same holds for the scalar spin connection (Fig. 3) which is similar to ω_3. Most interesting is the potential, see Fig. 4. It is pole-like or diverges for $z \rightarrow \pm\infty$. This may be quite unusual from the classical view, but we know from experiments that the energy density of the vacuum is very high. Such a structure may give rise e.g. to sponaneous creation of elementary particles; see sect. 5.5 for further discussion.

Besides this particular solution, we tried to find a more general solution. As already shown in Eq. (34), a general series of tanh functions fulfils this. We are lead to the conjecture that a general series expansion of the form

$$\mathbf{A}^{(m)} = \mathbf{k}\frac{1}{k_3}\sum_{n=0}^{m} D_n(t) f^n(\mathbf{k}\cdot\mathbf{x} - \beta t) \tag{39}$$

for any complete function set f^n with constants D_n is also a solution for $\mathbf{A}$. This has been shown in Appendix B by assuming this form of $\mathbf{A}$ and proving all equations. The Computer Algebra tool was only able to prove this for a finite series, but the general result follows easily by induction. In particular we get:

$$\boldsymbol{\omega}^{(m)} = -\mathbf{k}\,\frac{f' \sum_{n=1}^{m} n D_n f^{n-1}(\mathbf{k}\cdot\mathbf{x} - \beta t)}{\sum_{n=0}^{m} D_n f^n(\mathbf{k}\cdot\mathbf{x} - \beta t)}, \tag{40}$$

$$\omega_0^{(m)} = \beta\,\frac{f' \sum_{n=1}^{m} n D_n f^{n-1}(\mathbf{k}\cdot\mathbf{x} - \beta t)}{\sum_{n=0}^{m} D_n f^n(\mathbf{k}\cdot\mathbf{x} - \beta t)}, \tag{41}$$

$$\Phi^{(m)} = \frac{\Phi_0}{\sum_{n=0}^{m} D_n f^n(\mathbf{k}\cdot\mathbf{x} - \beta t)}. \tag{42}$$

f' is the derivative of f according to its argument. As an example the general solution for the tanh function is depicted in Fig. 5 for several maximum indices m (all constants set to unity). It is seen that for large m the solution is a smooth step function with growing height. The vector and scalar spin connection only differ in sign and a factor indicating the space-like and time-like character of both connetcions. The potential contains the original function series in the

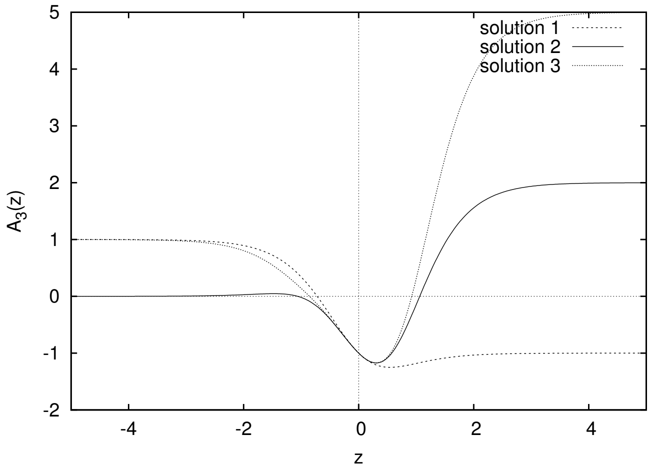

Figure 1: Three solutions for vector potential component A_3.

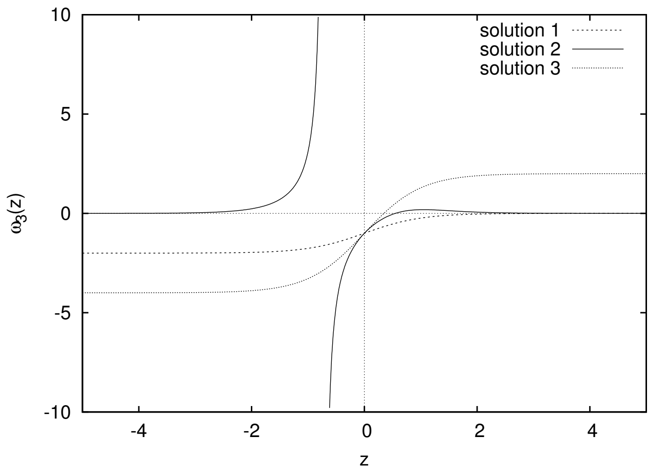

Figure 2: Three solutions for vector spin connection component ω_3.

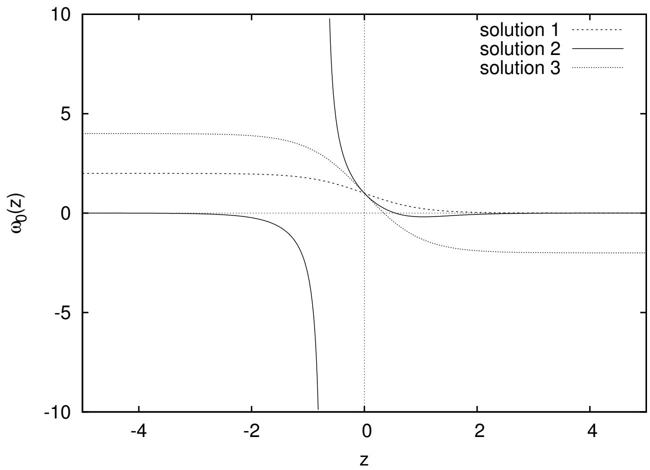

Figure 3: Three solutions for calalar spin connection component ω_0.

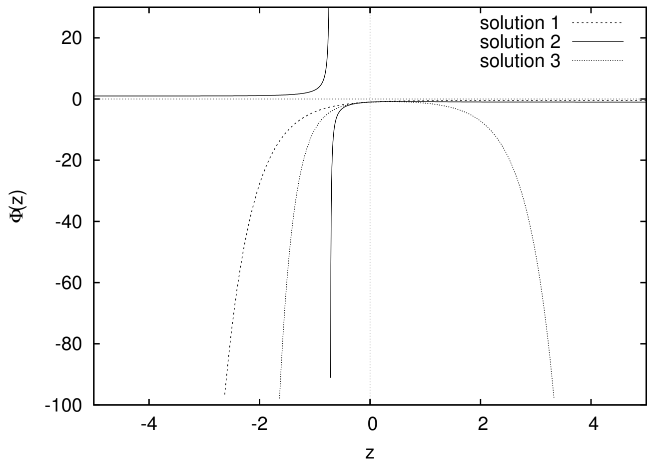

Figure 4: Three solutions for scalar vacuum potential Φ.

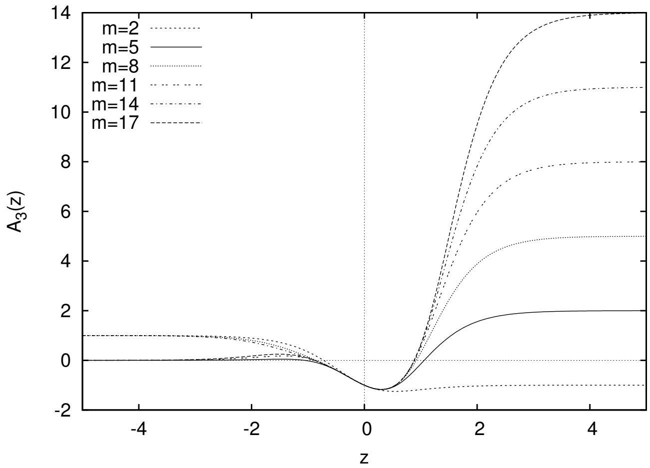

Figure 5: Vector potential component A_3 for several degrees m of series expansion.

denominator. If the the series is oscillating (which normally is the case), the potential has a high number of resonances.

Another important class of solutions is obtained by a Fourier series:

$$\mathbf{A}^{(m)} = \mathbf{k}\frac{1}{k_3}\sum_{n=0}^{m} D_n \exp\left(n \cdot i(\mathbf{k}\cdot\mathbf{x} - \beta t)\right). \tag{43}$$

This is a Fourier series in one dimension only because there is no variation in the wave vector $\mathbf{k}$. In other words, this is a plane wave with fixed direction. A variation in direction, however, can occur if the set of first four constants $(\mathbf{k}, \beta)$ or the D_n's is time-dependent which is possible for the general solution (see Appendices). This is further discussed in sections 5.2 and 5.3.

4 Energy and momentum density

4.1 Standard theory

The classical expression for the energy density of the electromagnetic field is

$$u(\mathbf{r}, t) = \frac{1}{2}\epsilon_0 \mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2, \tag{44}$$

and the power flux is described by the Pointing vector

$$\mathbf{S}(\mathbf{r}, t) = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}. \tag{45}$$

Physical units of these quantities are

$$[u] = \frac{J}{m^3}, \tag{46}$$

$$[\mathbf{S}] = \frac{W}{m^2}. \tag{47}$$

Both are based on the electromagnetic force fields **E** and **B** and are therefore not suitable in situations of potentials without fields. The solution to this difficulty is to formulate the energy density and power flux in terms of the potentials. This problem has been studied by Ribaric and Sustersic [4]. They found that other expressions can be defined in terms of potentials which give the same total energy taken as an integral over space as the original terms (44, 45). A practical solution has been reported by Puthoff [5]. For a given current density **J**, charge density ρ, scalar potential Φ and vector potential **A** the energy density can be written

$$u(\mathbf{r}, t) = u_A - u_\Phi + \rho\Phi \tag{48}$$

with

$$u_A(\mathbf{r}, t) = \frac{1}{2\mu_0}\sum_i \left(\frac{1}{c^2}\left(\frac{\partial A_i}{\partial t}\right)^2 + |\boldsymbol{\nabla} A_i|^2 \right) \tag{49}$$

and

$$u_\Phi(\mathbf{r}, t) = \frac{1}{2}\epsilon_0 \left(\frac{1}{c^2}\left(\frac{\partial \Phi}{\partial t}\right)^2 + |\boldsymbol{\nabla}\Phi|^2 \right). \tag{50}$$

Correspondingly, the power flux is defined by

$$\mathbf{S}(\mathbf{r}, t) = \mathbf{S}_A - \mathbf{S}_\Phi + \Phi\mathbf{J}, \tag{51}$$

$$\mathbf{S}_A(\mathbf{r}, t) = -\frac{1}{\mu_0}\sum_i \left(\frac{\partial A_i}{\partial t}\right)\boldsymbol{\nabla} A_i, \tag{52}$$

$$\mathbf{S}_\Phi(\mathbf{r}, t) = -\epsilon_0 \left(\frac{\partial \Phi}{\partial t}\right)\boldsymbol{\nabla}\Phi. \tag{53}$$

According to Jackson [6] the change in energy is defined by the divergence of the flux and the electric energy of the charges:

$$\frac{\partial u}{\partial t} = -\boldsymbol{\nabla} \cdot \mathbf{S} - \mathbf{J} \cdot \mathbf{E}. \tag{54}$$

In vacuo we have $\rho = \mathbf{J} = \mathbf{E} = 0$. The Pointing flux is connected with the momentum density $\mathbf{g}$ of the electromagnetic field by Einstein's mass-energy equivalent

$$\mathbf{g}c^2 = \mathbf{S} \tag{55}$$

and the equation of motion for the field is

$$\frac{\partial \mathbf{g}}{\partial t} = -\boldsymbol{\nabla} \cdot \boldsymbol{\Theta} \tag{56}$$

(written with tensor divergence [7]) where $\boldsymbol{\Theta}$ is the Maxwell stress tensor.

4.2 ECE theory

In ECE theory there are additional terms contained in the fields according to the spin connections, see Eqs. (1-2). We apply some reasonable replacement rules for genral relativity:

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \omega_0, \tag{57}$$

$$\boldsymbol{\nabla} \rightarrow \boldsymbol{\nabla} + \boldsymbol{\omega}. \tag{58}$$

Therefore we add such terms to the definitions (48-53) analogously:

$$u_{A_{ECE}}(\mathbf{r}, t) = \frac{1}{2\mu_0} \sum_i \left(\frac{1}{c^2} |\omega_0 A_i|^2 + |\omega_i A_i|^2 \right), \tag{59}$$

$$u_{\Phi_{ECE}}(\mathbf{r}, t) = \frac{1}{2} \epsilon_0 \left(\frac{1}{c^2} (\omega_0 \Phi)^2 + \sum_i |\omega_i \Phi|^2 \right), \tag{60}$$

$$u(\mathbf{r}, t) = u_A + u_{A_{ECE}} - u_\Phi - u_{\Phi_{ECE}}, \tag{61}$$

and

$$\mathbf{S}_{A_{ECE}}(\mathbf{r}, t) = -\frac{1}{\mu_0} \sum_i (\omega_0 A_i) \boldsymbol{\omega} A_i, \tag{62}$$

$$\mathbf{S}_{\Phi_{ECE}}(\mathbf{r}, t) = -\epsilon_0 \, (\omega_0 \Phi) \, \boldsymbol{\omega} \Phi, \tag{63}$$

$$\mathbf{S}(\mathbf{r}, t) = \mathbf{S}_A + \mathbf{S}_{A_{ECE}} - \mathbf{S}_{\boldsymbol{\Phi}} - \mathbf{S}_{\Phi_{ECE}}. \tag{64}$$

This procedure should be considered as tentative. For example all cross-terms in the products have been omitted. Note that there is a Pointing flux even if $\mathbf{A}$ and Φ are not time-dependent.

When inserting the general solutions (39-42) with abbreviations

$$g := \sum_{n=0}^{m} D_n f^n(\mathbf{k} \cdot \mathbf{x} - \beta t), \tag{65}$$

$$g' := f' \sum_{n=1}^{m} n D_n f^{n-1}(\mathbf{k} \cdot \mathbf{x} - \beta t) \tag{66}$$

into the above expressions of energy density and power flux, we obtain:

$$u_A(\mathbf{r}, t) = \frac{k^2(k^2 + (\frac{\beta}{c})^2)^2}{2\mu_0 k_3^2} g'^2 \tag{67}$$

$$u_\Phi(\mathbf{r}, t) = \epsilon_0 \frac{(k^2 + (\frac{\beta}{c})^2)\Phi_0^2}{2g^4} g'^2 \tag{68}$$

$$u_{A_{ECE}}(\mathbf{r}, t) = \frac{k_1^4 + k_2^4 + k_3^4 + k^2(\frac{\beta}{c})^2}{2\mu_0 k_3^2} g'^2 \tag{69}$$

$$u_{\Phi_{ECE}}(\mathbf{r}, t) = \epsilon_0 \frac{(k^2 + (\frac{\beta}{c})^2)\Phi_0^2}{2g^4} g'^2 \tag{70}$$

$$\mathbf{S}_A(\mathbf{r}, t) = \mathbf{k} \frac{\beta k^2}{\mu_0 k_3^2} g'^2 \tag{71}$$

$$\mathbf{S}_\Phi(\mathbf{r}, t) = \epsilon_0 \mathbf{k} \frac{\beta \Phi_0^2}{g^4} g'^2 \tag{72}$$

$$\mathbf{S}_{A_{ECE}}(\mathbf{r}, t) = -\mathbf{k} \frac{\beta k^2}{\mu_0 k_3^2} g'^2 \tag{73}$$

$$\mathbf{S}_{\Phi_{ECE}}(\mathbf{r}, t) = -\epsilon_0 \mathbf{k} \frac{\beta \Phi_0^2}{g^4} g'^2 \tag{74}$$

It can be seen that the energy densities of ECE and standard theory are equal or very similar. u_A and u_Φ have been graphed in Figs. 6 and 7 for illustration. The vector potential contributes smooth densities while the scalar potential leads to infinities due to its diverging character. Interestingly the power flux is exactly opposite in ECE and standard theory. Within the approximations made, there is no energy transfer in vacuo.

5 Discussion

In the last section we want to discuss some further points found for the background or vacuum potential.

5.1 Lindstrom constraint

In section 2.1, Eqs. (9-10), it was pointed out that the Lindstrom constraint normally being used to simplify the equations of electrodynamics is not suited to describe the vacuum vector potential consistently. Alternatively, we can try to retain the Lindstrom condition. Then we have to assume the validity of both equations, it follows

$$\boldsymbol{\omega} \times \mathbf{A} = 0, \tag{75}$$

$$\boldsymbol{\nabla} \times \mathbf{A} = 0. \tag{76}$$

Eq. (75) could alternatively be used to compute $\boldsymbol{\omega}$ but analysis shows that only two components of $\boldsymbol{\omega}$ can be determined, they depend on the third component which remains unspecified. Therefore the method of basing the solution on Eqs. (21)-(23) is preferrable as it leads to unique results. It should be noted that the Lindstrom constraint can even be derived from (21)-(23). Therefore it is not a restriction on the solution in case of the background potential.

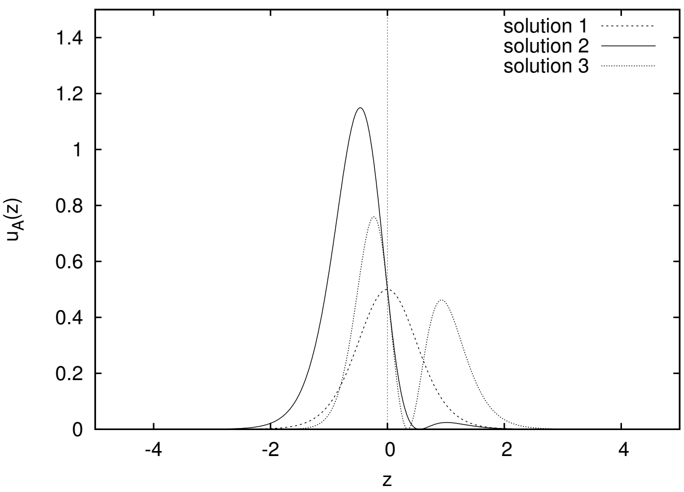

Figure 6: Energy density u_A for three solutions of tanh type.

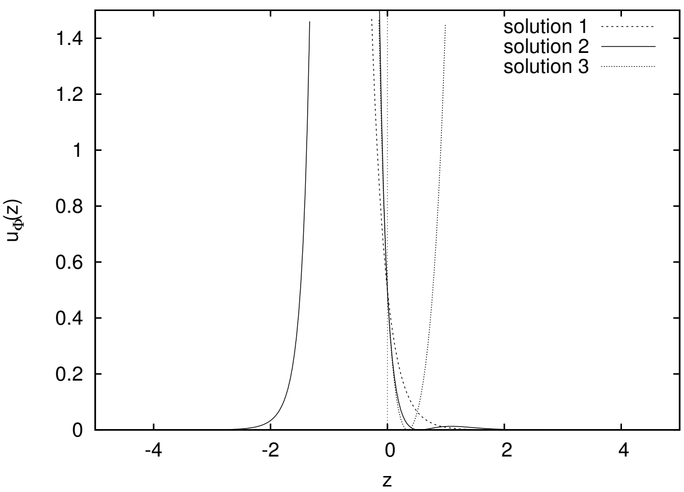

Figure 7: Energy density u_Φ for three solutions of tanh type.

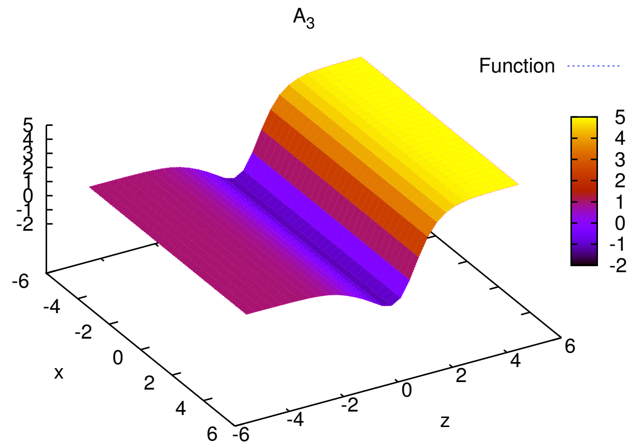

Figure 8: Surface plot of $A_3^{(3)}$.

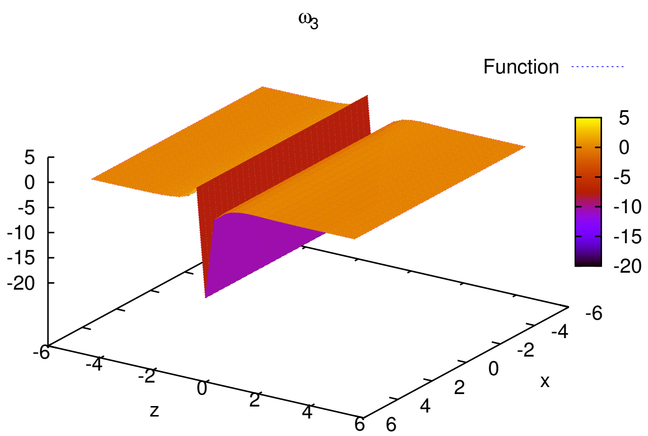

Figure 9: Surface plot of $\omega_3^{(2)}$.

5.2 Relations of time dependent constants

As shown in the appendices, all integration constants can be time-dependent. In Appendix A the condition

$$\frac{k_1'}{k_1} = \frac{k_2'}{k_2} = \frac{k_3'}{k_3} \tag{77}$$

was derived. A time-dependent **k** would not guarantee energy conservation. This had to be accounted for by an additional condition like

$$k(t)^2 = k_1(t)^2 + k_2(t)^2 + k_3(t)^2 = const. \tag{78}$$

If one component of **k** is time-independent, it follows from Eq. (77) that all components are time-independent. Therefore we surmise that **k** cannot be time-dependent in general. Then introducing additional conditions like (78) is not required.

5.3 Character of solutions

From Eqs. (40)-(41) it can directly be seen that the ratio of the vector and scalar spin connection is

$$\frac{\boldsymbol{\omega}}{\omega_0} = \frac{\mathbf{k}}{\beta} \tag{79}$$

and similarly

$$\Phi \propto \frac{1}{A_i} \tag{80}$$

for all components A_i. This shows some interconnection of the potentials and spin connections. The scalar spin connection can be written as

$$\omega_0 = \beta \frac{\partial \log g}{\partial t}. \tag{81}$$

The vector potential **A** is rotation free (see Eq. (76)) and therefore could be written as a gradient of a scalar function but this would be a "potential of a potential" and purely mathematical. In principle **A** can be defined as an arbitrary function f multiplied by a directional vector **k**:

$$\mathbf{A} = \mathbf{k}\ f(\mathbf{k} \cdot \mathbf{x} - \beta t). \tag{82}$$

This is a representation for plane waves as is depicted in Figs. 8-9 for **A** and $\boldsymbol{\omega}$. Since f can be arbitrarily nonlinear, these are anharmonic waves in general. The propagation speed (phase velocity) v can be obtained from the relation (restricted to one dimension)

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial t}\frac{dt}{dx} = \frac{1}{v}\frac{\partial A}{\partial t}. \tag{83}$$

Inserting the form of (82) it follows

$$kf' = -\frac{1}{v}\beta f' \tag{84}$$

or

$$|v| = \frac{\beta}{k} \tag{85}$$

which is identical to the classical dispersion relation for electromagnetic waves in vacuo. However, the time dependence of constants in the solution (39) also allows other forms of time dependence, for example a "product wave function"

$$A = k\ f(kx) \cdot g(\beta t). \tag{86}$$

The dispersion relation then becomes

$$|v| = \frac{\beta}{k} \cdot \frac{g'}{f'} \tag{87}$$

which allows arbitrary travelling velocities for waves depending on the forms of f and g. The use of product wave functions is similar to quantum mechanics where we have instantaneous interaction effects which appear plausible in the light of the equation above. Since the vacuum solutions are not constrained by the ECE (or Maxwell-Heaviside) field equations, there is more freedom for wave forms. This subject may be promising for new types of superluminar communication mechanisms and should be investigated further. Another type of wave to be investigated is standing waves in vacuo.

A further natural interpretation would be a fluctuating background field. It is known from observations [9] - although not widely recognized - that such fluctuations do exist, resulting in a variance of vacuum speed of light. The origin of these fluctuations is speculative at the current state of knowlegde, it may be from global motion of galaxies or local distortions of background fields due to matter of massive stars. Further investigation of these effects would probably require usage of fluid dynamics models.

5.4 Resonant Coulomb law

It was shown that the resonant Coulomb (Eq. (29)) follows from the vacuum equations. This is remarkable for two reasons. First, one of the ECE field equations, the Coulomb law, here appears although the field equations vanish identically because of the condition $\mathbf{E} = \mathbf{B} = 0$. This shows that the Coulomb law is deeply anchored in geometry and is a very fundamental law of nature. Secondly there are resonances possible even in the vacuum. As was shown elsewhere [10] the resonances of the Coulomb law arise from a variable spin connection and do not necessarily need an oscillatory charge density as in classical Euler-Bernoulli resonances. Charge density is not present in vacuo.

5.5 Topological charge density

In classical field theory the Coulomb or Newtonian gravitational law can be written in the form of the Poisson equation:

$$\boldsymbol{\nabla}^2 \Phi = -\frac{\rho}{\epsilon_0} \tag{88}$$

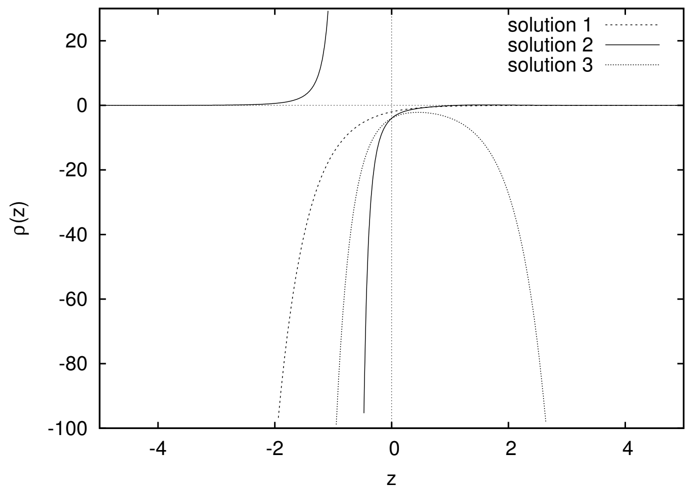

Figure 10: Topological charge density ρ for three solutions of tanh type.

(here for the electromagnetic case). In vacuo there is no charge density, therefore we would expect

$$\boldsymbol{\nabla}^2\Phi = 0. \tag{89}$$

Comparing this with Eq. (29), the spin connection terms appear as a topological charge density:

$$\rho_{top} = -\boldsymbol{\nabla}\cdot(\boldsymbol{\omega}\Phi). \tag{90}$$

As Φ has diverging regions so has the topological charge density. It has to be stressed that this density is not made up of real charges. The topological charge density consists of certain structures of space-time. However, it could be the origin of real particle processes which are known from quantum electrodynamics: pairs of particles can appear spontaneously.

An example of topological charge densities is shown in Fig. 10. The picture is very similar to the corresponding potential (Fig. 4). The form looks similar to atomic or molecular charge densities. This could give a hint to virtual particles, although ECE theory is completey confined to the wave or field model, in contrast to the particle model of quantum mechanics.

6 Acknowledgments

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