Non-Maxwell Static Solutions for ECE Electromagnetism

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Abstract

The ECE definitions of electric intensity and magnetic induction are assumed for the static case of single polarization, and assumed to be governed by Maxwell-like field equations. If these equations are constrained by the ECE equations of electromagnetic symmetry, no non-Maxwellian scenarios other that the trivial solution are possible for ECE electromagnetics for a single polarization given certain assumptions on the continuity of the potentials.

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1 Introduction

It has been shown in a previous publication [1], that the ECE theory of electromagnetism reduces to the standard classical electromagnetic theory when

$$\omega_0 \boldsymbol{A} = \boldsymbol{\omega}\phi \tag{1-1}$$

which is equivalent to (see Appendix I)

$$\boldsymbol{\omega} \times \boldsymbol{A} = 0. \tag{1-2}$$

With these assumptions, from a mathematical perspective, classical static electromagnetism is a subset of the static ECE equations. If the Maxwellian state is indicated by a lack of torsion, then there is no equivalence whatsoever except for the trivial state.

In what follows, it will be assumed that

$\omega_0 \boldsymbol{A} \neq \boldsymbol{\omega}\phi$ so that

$\boldsymbol{\omega} \times \boldsymbol{A} \neq \boldsymbol{0}.$

It will also be assumed that neither $\boldsymbol{\phi}, \boldsymbol{A}, \boldsymbol{\omega_0}$, nor $\boldsymbol{\omega}$ are zero globally. These special cases will be dealt with after the main discussion.

2 Static ECE Electromagnetism

The static field equations for ECE electromagnetism for a single polarization are [2]

$$\underline{\nabla} \cdot \boldsymbol{B}=0 \tag{2-1}$$

$$\underline{\nabla} \times \boldsymbol{E} = 0 \tag{2-2}$$

$$\underline{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon} \tag{2-3}$$

$$\underline{\nabla} \times \boldsymbol{B} = \mu \boldsymbol{J} \tag{2-4}$$

For static electromagnetism, the definition of electric intensity in ECE theory for a single polarization is [2]

$$\boldsymbol{E} = -\underline{\nabla}\phi - \omega_0 \boldsymbol{A} + \boldsymbol{\omega}\phi. \tag{2-5}$$

The ECE definition for the magnetic induction is [2]

$$\boldsymbol{B} = \underline{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A}. \qquad (2\text{-}6)$$

As a result of fundamental antisymmetries in Cartan geometry, new equations that constrain the above fields are introduced. The electric antisymmetry equation is for static fields [3],

$$-\underline{\nabla}\phi + \omega_0 \boldsymbol{A} + \boldsymbol{\omega}\phi = 0 \qquad (2\text{-}7)$$

and correspondingly, the magnetic antisymmetry equation is for static fields [3],

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \omega_j A_k + \omega_k A_j = 0 \qquad (2\text{-}8)$$

where i, j, k is a permutation of the coordinate indices. $\boldsymbol{E}$ is the electric intensity, $\boldsymbol{B}$ is the magnetic induction, $\boldsymbol{A}$ is the magnetic vector potential, ϕ is the electric scalar potential, $\boldsymbol{\omega}$ is the vector spin connection and ω_o is the scalar spin connection. We will assume in this analysis that the active medium is a vacuum so that complications introduced when using a more complex medium are avoided.

The electric intensity as given by equation (2-5) has two equivalent formulations when equation (2-7) is applied [3], namely

$$\boldsymbol{E} = -2\underline{\nabla}\phi + 2\boldsymbol{\omega}\phi \qquad (2\text{-}9)$$

and

$$\boldsymbol{E} = -2\omega_0 \boldsymbol{A} \qquad (2\text{-}10)$$

Similar expressions can be given for the magnetic induction by adding and subtracting (2-8) from (2-6) in turn to get, in indicial form, for each $i = 1,2,3$ with $i \neq j \neq k$

$$\frac{B_i}{2} = -\frac{\partial A_j}{\partial x_k} - \omega_j A_k \qquad (2\text{-}11)$$

$$\frac{B_i}{2} = \frac{\partial A_k}{\partial x_j} + \omega_k A_j \qquad (2\text{-}12)$$

As will be discussed in a future publication [4], there is never truly a static ECE field. The observable fields “float” upon a non-static vacuum field, and appear to demonstrate no interaction with the vacuum as long as the Maxwell-like linear field equations are satisfied. In fact, the observable potentials should be added to the vacuum solution when a potential representation is considered. This introduces a temporal component in all solutions that we shall ignore in this publication.

From Faraday’s Law, equation (2-2) using equations (2-9) and (2-10) becomes

$$\underline{\nabla} \times \boldsymbol{\omega}\phi = 0 \qquad (2\text{-}13)$$

and

$$\underline{\nabla} \times \omega_0 \boldsymbol{A} = 0 \,. \tag{2-14}$$

We can write

$$\underline{\nabla}\psi = \omega_0 \boldsymbol{A} \tag{2-15}$$

and

$$\underline{\nabla}\zeta = \boldsymbol{\omega}\phi \tag{2-16}$$

with two new scalar potentials ψ and ζ. Substituting (2-15) and (2-16) into (2-7) gives

$$\underline{\nabla}\zeta = \underline{\nabla}\phi - \underline{\nabla}\psi. \tag{2-17}$$

Gauss's Law acting on (2-6) becomes

$$\nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{A}) = 0 \tag{2-18}$$

Upon substituting equations (2-15), (2-16) and) into (2-18) gives

$$\nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{A}) = \nabla \cdot \left(\frac{\underline{\nabla}\zeta}{\phi} \times \frac{\underline{\nabla}\psi}{\omega_0}\right).$$

Noting (2-17),

$$\nabla \cdot \left(\frac{\underline{\nabla}\zeta}{\phi} \times \frac{\underline{\nabla}\psi}{\omega_0}\right) = \nabla \cdot \left(\frac{\underline{\nabla}\phi - \underline{\nabla}\psi}{\phi} \times \frac{\underline{\nabla}\psi}{\omega_0}\right).$$

Since

$$\underline{\nabla}\psi \times \underline{\nabla}\psi = 0$$

we have

$$\nabla \cdot \left(\frac{\underline{\nabla}\phi - \underline{\nabla}\psi}{\phi} \times \frac{\underline{\nabla}\psi}{\omega_0}\right) = \nabla \cdot \left(\underline{\nabla}Ln(\phi) \times \frac{\underline{\nabla}\psi}{\omega_0}\right).$$

Using the vector identity

$$\underline{\nabla} \cdot (\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{b} \cdot \underline{\nabla} \times \boldsymbol{a} - \boldsymbol{a} \cdot \underline{\nabla} \times \boldsymbol{b}$$

we have

$$\nabla \cdot \left(\underline{\nabla}Ln(\phi) \times \frac{\underline{\nabla}\psi}{\omega_0}\right) = \frac{\underline{\nabla}\psi}{\omega_0} \cdot \underline{\nabla} \times \underline{\nabla}Ln(\phi) - \underline{\nabla}Ln(\phi) \cdot \underline{\nabla} \times \frac{\underline{\nabla}\psi}{\omega_0}.$$

The first term on the right hand side vanishes identically. Further, since

$\underline{\nabla} \times \varphi \boldsymbol{a} = \varphi \underline{\nabla} \times \boldsymbol{a} + \underline{\nabla}\varphi \times \boldsymbol{a}$ for a scalar φ, we have

$$\underline{\nabla} \times \frac{\underline{\nabla}\psi}{\omega_0} = \underline{\nabla}\left(\frac{1}{\omega_0}\right) \times \underline{\nabla}\psi + \frac{1}{\omega_0}\underline{\nabla} \times \underline{\nabla}\psi = -\frac{\underline{\nabla}\omega_0 \times \underline{\nabla}\psi}{{\omega_0}^2}$$

We thus have that

$$\nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{A}) = -\underline{\nabla}Ln(\phi) \cdot \frac{\underline{\nabla}\omega_0 \times \underline{\nabla}\psi}{{\omega_0}^2} = 0 \,. \qquad \text{(2-19)}$$

Because

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b})$$

equation (2-19) has the following alternate forms

$$\frac{\underline{\nabla}\phi \cdot (\underline{\nabla}\omega_0 \times \underline{\nabla}\psi)}{\phi{\omega_0}^2} = \frac{\underline{\nabla}\omega_0 \cdot (\underline{\nabla}\psi \times \underline{\nabla}\phi)}{\phi{\omega_0}^2} = \frac{\underline{\nabla}\psi \cdot (\underline{\nabla}\phi \times \underline{\nabla}\omega_0)}{\phi{\omega_0}^2} = 0 \,. \qquad \text{(2-20)}$$

An examination of (2-20) suggests three possible solutions if neither ω_0 nor ϕ are zero.

i. $\underline{\nabla}\phi$, $\underline{\nabla}\omega_0$, and $\underline{\nabla}\psi$ (i.e. $\boldsymbol{A}$) are all parallel, because all cross-product terms are zero,
ii. $\underline{\nabla}\phi$, $\underline{\nabla}\omega_0$, and $\underline{\nabla}\psi$ are mutually perpendicular, since all dot product term are zero, or
iii. one or more of the gradients are zero.

The first option has

$$\underline{\nabla}\phi \times \boldsymbol{A} = 0 \,.$$

Therefore, according to (2-7), $\boldsymbol{\omega}$ is parallel to A and $\boldsymbol{\omega} \times \boldsymbol{A} = 0$ from which the fields are Maxwellian.

For second option, from equation (2-7)
$\phi\boldsymbol{\omega} \times \boldsymbol{A} = \underline{\nabla}\phi \times \boldsymbol{A}$
gives us that $\boldsymbol{\omega}\phi$, $\underline{\nabla}\phi$ and $\boldsymbol{A}$ are co-planar but perpendicular to $\boldsymbol{\omega} \times \boldsymbol{A}$ (see Figure 1).

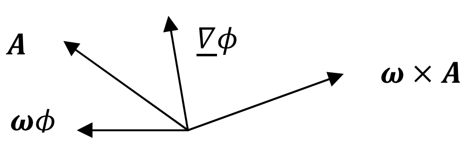

Figure 1

Since $\underline{\nabla}\phi$ is perpendicular to $\underline{\nabla}\psi$ and both must be perpendicular to $\boldsymbol{\omega} \times \boldsymbol{A}$ and equation (2-17) requires that

$\boldsymbol{\omega}\phi = \underline{\nabla}\phi - \underline{\nabla}\psi$ (2-21)

which must also be perpendicular to $\boldsymbol{\omega} \times \boldsymbol{A}$**,** then the only possible direction for $\boldsymbol{\omega}\phi$ is perpendicular to both $\underline{\nabla}\phi$ and $\underline{\nabla}\psi$ with the three terms forming the mutually perpendicular axes of a 3D coordinate system. This is an impossibility because of perpendicularity with $\boldsymbol{\omega} \times \boldsymbol{A}$ unless one of the terms is zero. This leads us to the third option, unless

$\boldsymbol{\omega}\phi = 0$
in which case
$\boldsymbol{\omega} \times \boldsymbol{A} = 0.$

The third option

a. $\underline{\nabla}\phi = 0$ has $\boldsymbol{\omega} \times \boldsymbol{A} = \underline{\nabla}\phi \times \boldsymbol{A} = 0$ from which the fields are Maxwellian.
b. $\underline{\nabla}\psi = 0$ has $\boldsymbol{A} = 0$ from which both $\boldsymbol{E}$ and $\boldsymbol{B}$ are zero.
c. $\underline{\nabla}\omega_0 = 0$ has $\omega_0 = constant$ with the result that $\underline{\nabla} \times \boldsymbol{A} = 0$ from (2-14).

For option "c", Gauss's Law has
$\underline{\nabla} \cdot (\boldsymbol{\omega} \times \boldsymbol{A}) = \boldsymbol{A} \cdot (\underline{\nabla} \times \boldsymbol{\omega}) = 0,$
but
$\underline{\nabla} \times \boldsymbol{\omega} = \frac{1}{\phi}\underline{\nabla} \times \boldsymbol{\omega}\phi - \frac{\underline{\nabla}\phi}{\phi} \times \boldsymbol{\omega}.$
Therefore, because of Faraday's Law,
$\boldsymbol{A} \cdot (\underline{\nabla} \times \boldsymbol{\omega}) = -\frac{\underline{\nabla}\psi}{\omega_0} \cdot \left(\frac{\underline{\nabla}\phi}{\phi} \times \boldsymbol{\omega}\right) = 0 .$
Either one or more of the terms are zero, or they are mutually perpendicular. These options have all been dealt with in the above discussion.

Consider now the special case of one or more of $\phi, \boldsymbol{A}, \omega_0,$ and $\boldsymbol{\omega}$ being zero globally. Firstly, it has been argued [5] that $\omega_0 = 0$ and $\boldsymbol{\omega} = 0$ are not possible solutions to the ECE equations of electromagnetism because the electromagnetic field, even the vacuum state, is a continually rotating spacetime. The only possible exception would be when both the scalar and vector magnetic potentials are zero.

However, if $\phi = 0$ globally then $\underline{\nabla}\phi = 0$ then $\boldsymbol{E} = 0$ so that $\boldsymbol{A} = 0$ and $\boldsymbol{B} = 0$.
If $\boldsymbol{A} = 0$ then $\boldsymbol{E} = 0$ and $\boldsymbol{B} = 0$.
Thus if any of the potentials vanish, the solution is the zero potential state.

It has been shown also that in general, for potentials with continuous second derivatives, that the only static solutions to the ECE engineering equations are equivalent to those given by traditional electromagnetic theory [1]. This fails to be true under two conditions, namely when $\underline{\nabla}\phi \neq 0$ but $\phi = 0$, and when at least one of the potentials is multi-valued.

Let us examine the situation where $\underline{\nabla}\phi \neq 0$ but $\phi = 0$.

By equation (1-1) if $\boldsymbol{\omega}$ is real, and non-singular, then

$$\boldsymbol{E} = -2\underline{\nabla}\phi, \qquad (2\text{-}22)$$

that is to say, $\boldsymbol{E}$ will be double the value of the corresponding Maxwellian value at that particular point in space.

To investigate this particular case further, consider $\boldsymbol{E}$ as a function of the scalar potential, then by equation (2-9)

$$\boldsymbol{E}(\phi + \delta\phi) = -2\underline{\nabla}(\phi + \delta\phi) + 2\underline{\boldsymbol{\omega}}(\phi + \delta\phi)(\phi + \delta\phi) \qquad (2\text{-}23)$$

If we expand $\underline{\boldsymbol{\omega}}$ about ϕ, then to first order in $\delta\phi$,

$$\boldsymbol{E}(\phi + \delta\phi) = -2\underline{\nabla}(\phi) - 2\underline{\nabla}(\delta\phi) + 2\boldsymbol{\omega}\phi + 2\boldsymbol{\omega}\delta\phi + 2\frac{\partial\boldsymbol{\omega}}{\partial\phi}\phi\delta\phi \qquad (2\text{-}24)$$

Writing

$$\frac{\partial\boldsymbol{E}}{\partial\phi} = \lim_{\delta\phi\to0}\frac{\boldsymbol{E}(\phi+\delta\phi)-\boldsymbol{E}(\phi)}{\delta\phi} = \lim_{\delta\phi\to0}\left(-2\nabla\big(Ln(\delta\phi)\big) + \boldsymbol{\omega} + 2\frac{\partial\boldsymbol{\omega}}{\partial\phi}\phi\right) \qquad (2\text{-}25)$$

At $\phi = 0$,

$$\frac{\partial\boldsymbol{E}}{\partial\phi} = \lim_{\delta\phi\to0}(-2\nabla(Ln(\delta\phi)\,) + \boldsymbol{\omega}) \qquad (2\text{-}26)$$

if $\frac{\partial\boldsymbol{\omega}}{\partial\phi}$ is finite.

In general, for $\frac{\partial\boldsymbol{E}}{\partial\phi}$ to be finite at $\phi = 0$, $\boldsymbol{\omega}$ must be infinite.

For $\boldsymbol{B}$ to be finite, then by equation (2-6)

$$\boldsymbol{\omega} \times \boldsymbol{A} = 0 \qquad (2\text{-}27)$$

indicating a Maxwellian type of solution.

The situation where the potentials are multi-valued will be dismissed for the moment on the grounds that they can be described with sufficient accuracy by an appropriate series of orthogonal functions, and so have well defined derivatives to any desired levels.

3 Discussion

It has been shown in the previous discussion that all static ECE electromagnetic fields are either trivial (the vacuum state, or the even lower state of zero potentials) or Maxwellian. Enough continuity of the variables had to be assumed to provide values for the Maxwell field equations. This requires that ϕ and $\boldsymbol{A}$ have spatial first derivatives that are defined everywhere and are continuous so that its second degree derivatives exist. The products $\boldsymbol{\omega}\phi$ and $\omega_0\boldsymbol{A}$ need to be defined everywhere and continuous to that their spatial derivatives can be formed.

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Appendix I

Proof that $\boldsymbol{\omega} \times \boldsymbol{A} = 0$ is equivalent to $\omega_0 \boldsymbol{A} = \boldsymbol{\omega}\phi$.

From the electric antisymmetry equation (2-7)

$\boldsymbol{\omega} \times \boldsymbol{A} = 0$ is the same as

$\underline{\nabla}\phi \times \boldsymbol{A} = 0$.

If none of the variables in these two expressions are zero, then $\boldsymbol{\omega}$, $\underline{\nabla}\phi$, and $\boldsymbol{A}$ are parallel. We can then write

$\boldsymbol{\omega} = k\,\boldsymbol{A}$.

If we substitute this into Faraday's Law,

$\underline{\nabla} \times \boldsymbol{\omega}\phi = \underline{\nabla} \times k\,\boldsymbol{A}\phi = \underline{\nabla}(k\,\phi) \times \boldsymbol{A} + k\,\phi\underline{\nabla} \times \boldsymbol{A} = 0$

or

$\underline{\nabla} \times \boldsymbol{A} = -\underline{\nabla}Ln(k\,\phi) \times \boldsymbol{A}$. (A-1)

The second form for the electric intensity gives Faraday's Law as

$\underline{\nabla} \times (\omega_0 \boldsymbol{A}) = \underline{\nabla}\omega_0 \times \boldsymbol{A} + \omega_0 \underline{\nabla} \times \boldsymbol{A} = 0$

or

$\underline{\nabla} \times \boldsymbol{A} = -\underline{\nabla}Ln(\omega_0) \times \boldsymbol{A}$.

Comparing this to (A-1) we see that

$k\,\phi = \omega_0$

from which follows

$\omega_0 \boldsymbol{A} = k\,\phi \boldsymbol{A} = \boldsymbol{\omega}\phi$.