Reduction of the ECE Theory of Electromagnetism to the Maxwell-Heaviside Theory, Part II

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Abstract

In this paper it is shown that if the scalar potential is separable, then the vector potential is separable. The vector spin connection is shown to be independent of time. It is further shown that if the potentials are separable, then the ECE equations of electromagnetism reduce to the Maxwell-Heaviside equations of electromagnetism. This assumption may account for the small amount of experimental evidence that is not Maxwellian in character.

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1. Introduction

The ECE theory of electromagnetism is rich in non-linear behaviour [1]. In this paper we will show that if the scalar potential is a separable function, which is what is typically assumed in analysis, then the ECE equations reduce to those of Maxwell-Heaviside, with the non-linear richness vanishing. This offers an explanation for the lack of extensive experimental evidence for non-Maxwellian behaviour. This will be demonstrated in two steps. First, if it assumed that if the scalar potential is a separable function, then the vector potential is shown to be separable. Subsequently it is shown that given this, the ECE theory of electromagnetism collapses to the Maxwell-Heaviside theory.

2. Reduction of the theory

The antisymmetry equations are given by [2]

$$\frac{\partial \mathbf{A}}{\partial t} - \underline{\nabla}\phi + \omega_0 \mathbf{A} + \boldsymbol{\omega}\phi = 0 \qquad , \tag{1}$$

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \omega_j A_k + \omega_k A_j = 0 \qquad . \tag{2}$$

Equation (2) has been solved to give the vector spin connection as a differentio-algebraic function of the magnetic vector potential [3].

$$\boldsymbol{\omega} = \begin{pmatrix} \frac{1}{2A_2A_3}\left(A_1\frac{\partial A_3}{\partial y} + A_1\frac{\partial A_2}{\partial z} - A_2\frac{\partial A_3}{\partial x} - A_2\frac{\partial A_1}{\partial z} - A_3\frac{\partial A_2}{\partial x} - A_3\frac{\partial A_1}{\partial y}\right) \\ \frac{1}{2A_1A_3}\left(-A_1\frac{\partial A_3}{\partial y} - A_1\frac{\partial A_2}{\partial z} + A_2\frac{\partial A_3}{\partial x} + A_2\frac{\partial A_1}{\partial z} - A_3\frac{\partial A_2}{\partial x} - A_3\frac{\partial A_1}{\partial y}\right) \\ \frac{1}{2A_1A_2}\left(-A_1\frac{\partial A_3}{\partial y} - A_1\frac{\partial A_2}{\partial z} - A_2\frac{\partial A_3}{\partial x} - A_2\frac{\partial A_1}{\partial z} + A_3\frac{\partial A_2}{\partial x} + A_3\frac{\partial A_1}{\partial y}\right) \end{pmatrix} . \tag{3}$$

For brevity we could adopt the notation

$$\boldsymbol{\omega} = \underline{\mathfrak{L}}(\boldsymbol{A}) \tag{4}$$

where the operator $\underline{\mathfrak{L}}$ is known from equation (3).

Taking the cross product of equation (1) with $\boldsymbol{A}$ and using equation (2) we have

$$\boldsymbol{\omega} \times \boldsymbol{A} = \frac{1}{\phi}\left(\underline{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t}\right) \times \boldsymbol{A} \quad . \tag{5}$$

To prove that if ϕ is separable, then the vector potential is also separable, assume that the scalar potential can be separated into a function of space, and one of time, i.e.

$$\phi = \phi^{(t)}(t)\phi^{(\boldsymbol{r})}(\boldsymbol{r}) \quad . \tag{6}$$

If we substitute this into the electric antisymmetry equation (1) then

$$\phi^{(t)}\underline{\nabla}\phi^{(r)} = \frac{\partial \boldsymbol{A}}{\partial t} + \omega_0 \boldsymbol{A} + \boldsymbol{\omega}\phi^{(t)}\phi^{(r)} \ . \tag{7}$$

Dividing equation (7) by $\phi^{(t)}$, we have

$$\underline{\nabla}\phi^{(r)} = \frac{1}{\phi^{(t)}}\left(\frac{\partial \boldsymbol{A}}{\partial t} + \omega_0 \boldsymbol{A}\right) + \boldsymbol{\omega}\phi^{(r)} = \mathrm{f}(\mathbf{r}) \ , \tag{8}$$

a function with only spatial dependence.

From this, it is immediately apparent, that if ϕ is not zero, then

$$\boldsymbol{\omega} = \boldsymbol{\omega}^{(r)}(\boldsymbol{r}) \qquad \text{and} \tag{9}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} + \omega_0 \boldsymbol{A} = \boldsymbol{g}(\boldsymbol{r})\phi^{(t)} \ . \tag{10}$$

This means that

$$\frac{\boldsymbol{\partial A}}{\boldsymbol{\partial t}} = \boldsymbol{A}^{(r)}\phi^{(t)} \ , \tag{11}$$

$$\boldsymbol{A} = \boldsymbol{A}^{(r)} \int \phi^{(t)} dt \ . \tag{12}$$

Substitution of equations (9), (10) and (11) into equation (8) results in

$$\omega_0^{(r)} = \omega_o(\boldsymbol{r}) \frac{\phi^{(t)}}{\int \phi^{(t)} dt} \ . \tag{13}$$

Thus if ϕ is separable, then the vector potential is separable into a product of space-like and time-like functions with the vector spin connection not being dependent on time.

Further, if we substitute equations (9), (11), (12) and (13) into equation (1) we have that

$$\underline{\nabla}\phi^{(r)} - \boldsymbol{A}^{(r)} - \boldsymbol{\omega}^{(r)}\phi^{(r)} = \omega_o \boldsymbol{A}^{(r)} \quad . \tag{14}$$

Taking the cross product of equation (1) with $\boldsymbol{\omega}$ gives

$$\boldsymbol{\omega} \times \frac{\partial \boldsymbol{A}}{\partial t} - \boldsymbol{\omega} \times \nabla\phi + \omega_0 \boldsymbol{\omega} \times \boldsymbol{A} = 0 \,.$$

Substituting equations (6), (11), (12) and (13) into this gives

$$\boldsymbol{\omega}^{(r)} \times \boldsymbol{A}^{(r)} = \frac{\boldsymbol{\omega}^{(r)} \times \nabla\phi^{(\mathrm{r})}}{1+\omega_0^{(r)}} \,, \qquad (15)$$

but by equation (5),

$$\boldsymbol{\omega}^{(r)} \times \boldsymbol{A}^{(r)} = \frac{\nabla\phi^{(\mathrm{r})}}{\phi^{(\mathrm{r})}} \times \boldsymbol{A}^{(r)} \,. \qquad (16)$$

Comparing equations (15) and (16) gives

$$\frac{\nabla\phi^{(\mathrm{r})}}{\phi^{(\mathrm{r})}} \times \boldsymbol{A}^{(r)} = -\frac{\underline{\nabla}\phi^{(\mathrm{r})} \times \boldsymbol{\omega}^{(r)}}{1+\omega_0^{(r)}} \,. \qquad (17)$$

The solution to equation (17) is that

$$\frac{\boldsymbol{\omega}^{(r)}}{1+\omega_0^{(r)}} = -\frac{\boldsymbol{A}^{(r)}}{\phi^{(\mathrm{r})}} + k\boldsymbol{A}^{(r)}$$

where k is an arbitrary function of spatial coordinates.

This means that $\boldsymbol{\omega}$ is parallel to $\boldsymbol{A}$ **,** i.e.

$$\boldsymbol{\omega} \times \boldsymbol{A} = \boldsymbol{0} \qquad . \qquad (18)$$

Let us consider the Faraday equation, Coulomb's Law, and the Maxwell Ampere equation incorporating the limitation of equation (18).

Faraday's equation gives

$$\underline{\nabla} \times (\boldsymbol{\omega}\phi - \omega_0 \boldsymbol{A}) = 0 \,. \qquad (19)$$

Coulombs equation becomes

$$\underline{\nabla} \cdot (\boldsymbol{\omega}\phi - \omega_0 \boldsymbol{A}) = \frac{\rho}{\varepsilon} - \underline{\nabla} \cdot \left(-\underline{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t}\right) \qquad (20)$$

and the Maxwell-Ampere equation, in potential form, noting (18), is

$$\underline{\nabla} \times \underline{\nabla} \times \boldsymbol{A} - \frac{1}{\mathrm{c}^2}\frac{\partial}{\partial \mathrm{t}}\left(-\frac{\partial \boldsymbol{A}}{\partial \mathrm{t}} + (\boldsymbol{\omega}\phi - \omega_0 \boldsymbol{A}) - \underline{\nabla}\phi\right) = \mu \boldsymbol{J} \,. \qquad (21)$$

This simplifies to

$$\frac{\partial}{\partial t}(\boldsymbol{\omega}\phi - \omega_0 \boldsymbol{A}) = c^2\left(\left(-\nabla^2\boldsymbol{A} + \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \mu J\right) + \underline{\nabla}\left(\underline{\nabla}\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right)\right). \quad (22)$$

Substituting equations (9), (11), (12) and (13) into this equation, and dividing by $\frac{\partial\phi^{(t)}}{\partial t}$ gives

$$\boldsymbol{\omega}^{(r)}\phi^{(r)} - \omega_0^{(r)}\boldsymbol{A}^{(r)} =$$
$$c^2\left(-\nabla^2\boldsymbol{A}^{(r)}\int\phi^{(t)}dt + \frac{\boldsymbol{A}^{(r)}}{c^2}\frac{\partial\phi^{(t)}}{\partial t} - \mu J\right)\left(\frac{1}{\frac{\partial\phi^{(t)}}{\partial t}}\right) + c^2\underline{\nabla}\left(\underline{\nabla}\cdot\boldsymbol{A}^{(r)}\int\phi^{(t)}dt + \frac{\phi^{(r)}}{c^2}\frac{\partial\phi^{(t)}}{\partial t}\right)\left(\frac{1}{\frac{\partial\phi^{(t)}}{\partial t}}\right). \quad (23)$$

The right hand side of this expression is not a function of spatial coordinates alone. This then requires for equation (25) to be valid that both the right hand side and the left hand side of the equation must both be zero.

The left hand side being zero gives us that upon multiplying by $\frac{\partial\phi^{(t)}}{\partial t}$

$$\frac{\partial}{\partial t}(\boldsymbol{\omega}\phi - \omega_0\boldsymbol{A}) = \boldsymbol{0}$$

or

$$\boldsymbol{\omega}\phi - \omega_0\boldsymbol{A} = \boldsymbol{f}(\boldsymbol{r}). \quad (24)$$

For the right hand side of equation (23) to be zero, a sufficient condition is

$$-\nabla^2\boldsymbol{A} + \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \mu\boldsymbol{J} = 0 \qquad \text{and} \quad (25)$$

$$\nabla\left(\nabla\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = 0\ . \quad (26)$$

Equation (25) is the Maxwellian wave equation for the vector potential and equation (26) is the gradient of the Lorenz constraint of classical electromagnetism [4,5].

If we substitute (24) into equation (20), we can write with a scalar potential $\zeta^{(r)}$:

$$\nabla^2\zeta^{(r)} = \frac{\rho}{\varepsilon_0} - \underline{\nabla}\cdot\left(-\underline{\nabla}\phi - \frac{\partial A}{\partial t}\right)$$

where the left hand side is a result of equation (19). Due to (24), $\zeta^{(r)}$ is only a function of spatial coordinates. The only way for this to be valid is

$$\frac{\rho}{\varepsilon_0} - \underline{\nabla}\cdot(-\underline{\nabla}\phi - \frac{\partial\boldsymbol{A}}{\partial t}) = 0 \quad (27)$$

and equation (24) becomes

$$\boldsymbol{\omega}\phi - \omega_0\boldsymbol{A} = \mathbf{0}\ . \tag{28}$$

Equation (28) is the required condition for the ECE equations to reduce to the Maxwell-Heaviside as developed earlier [4]. The only assumption used in this derivation was that that ϕ is separable into the product of a spatially dependent function and a time dependent function. The implication, since spatial and temporal derivatives were used extensively, is that the field variables must be well defined (i.e. derivatives are single valued everywhere).

It remains to consider the effects of the scalar potential or one of the components of the magnetic vector potential becoming zero at some point in space and time, in terms of potential singularities. Consider $\phi = 0$ in equation (5), and note that by equation (18) $\boldsymbol{\omega} \times \boldsymbol{A} = \mathbf{0}$. For this to be true, one of two things must happen.

i. $\underline{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t} = 0$ or
ii. $\underline{\nabla}\phi$, $\boldsymbol{A}$, $\frac{\partial \boldsymbol{A}}{\partial t}$, and $\boldsymbol{\omega}$ are all parallel.

Case (i) can be rejected because we are making the implicit assumption that ϕ and $\boldsymbol{A}$ are somewhat independent.

Case (ii) means that equation (5) can be written

$$\lim_{\epsilon\to\infty}\left(\frac{0}{\epsilon}\right) \equiv 0.$$

This means that there is no singularity issue caused by ϕ going to zero.

By equation (3), there is potentially a singularity when any component of the magnetic vector potential is zero. Take for example

$$A_3 = 0,$$

at some point in space and at some time with the other two components of $\boldsymbol{A}$ being finite. If the components of $\boldsymbol{\omega}$ remain finite, then substitution of this into equation (2) gives

$$\omega_3 = \frac{1}{A_2}\left(\frac{\partial A_3}{\partial y} + \frac{\partial A_2}{\partial x}\right) = \frac{1}{A_1}\left(\frac{\partial A_3}{\partial x} + \frac{\partial A_1}{\partial z}\right).$$

A similar equation can be written with each of the other two A_i in turn being zero. This is just a statement of the magnetic antisymmetry equation in two dimensions, and presents no singularity issue.

3. Discussion

If the traditional scalar potential is separable, then the vector potential is separable. This being the case, the ECE equations of electromagnetism reduce to those of the Maxwell-Heaviside theory. Non-Maxwellian effects can then only be expected when the scalar potential cannot be expressed as a separable function, not a common occurrence. This is certainly borne out by experiment, where very few non-Maxwellian effects have been observed in electrical engineering directly.

Condition Which May Trigger a Non-Maxwellian Event

We have identified many situations under which that ECE system becomes Maxwellian and is forced to stay there because of its stable nature. We shall now present the conditions for the opposite to be true, i.e. the ECE system to become non-Maxwellian state, and stay there.

Whether the state of an electromagnetic field is Maxwellian or not prior to "turning on the switch" is still open for discussion. Let us assume the worst, and assume that the state is Maxwellian, (or has become Maxwellian by one of the mechanisms discussed above). To achieve a non-Maxwellian ECE state, none of the potentials can ever be zero, nor can they ever become separable nor continuous. This means that the system has to be placed in a state of potential that is either negative or positive, and remains that way, and that the potentials become discontinuous making their derivative multi-valued, or perhaps "near infinite". A pulsed potential, with extremely fast rise and collapse times, would have this property, for example. A multivalued potential is closely connected with non-conservative fields of field theory. Such fields can be used to extract energy by reaching the same point of definiton space on different paths. However it is not easy to find such fields in nature.

References

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