Spherically Symmetric Metric Manifolds and The Black Hole Catastrophe

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The usual interpretations of solutions for the gravitational field in a spherically symmetric Type I Einstein space contain mathematical anomalies. It is shown herein that the usual solutions must be modified to account for the intrinsic geometry associated with the relevant line elements, by which the geometrical relations between the components of the metric tensor are consequently invariant. A geometry is entirely determined by the form of the line element describing it. The usual solutions violate the intrinsic geometry of the associated line elements and are therefore inadmissible. Correct application of the intrinsic geometry has significant consequences for the standard models of Einstein's gravitational field.

Keywords: black hole; Schwarzschild solution

1 Introduction

The usual spherically symmetric static vacuum solution for a Type 1 Einstein space satisfying the condition $R_{\mu\nu} = 0$ is based upon a generalisation of the line element for standard Minkowski space. Minkowski's standard line element is (using $c = G = 1$),

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$0 \leq r < \infty.$$

The generalisation of this line element has the form,

$$ds^2 = A\left(\sqrt{C(r)}\right)dt^2 - B\left(\sqrt{C(r)}\right)d\sqrt{C(r)}^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2),$$

where $A\left(\sqrt{C(r)}\right), B\left(\sqrt{C(r)}\right), C(r)$ are *a priori* unknown positive-valued analytic functions that must be determined by the intrinsic geometry of the line element and associated boundary conditions, satisfying the condition $R_{\mu\nu} = 0$. Similarly, the range of the parameter r must also be determined by the intrinsic geometry of the line element. However, the usual practice is to preempt the form of the analytic function $C(r)$ by arbitrarily asserting that $C(r) = r^2$, so that the generalisation of Minkowski's line element takes the *restricted* form,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

wherein $g_{00} \equiv A(r),\ g_{11} \equiv -B(r),\ g_{22} \equiv -r^2,\ g_{33} \equiv -r^2\sin^2\theta$ become the components of the associated metric tensor.

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By setting $C(r) = r^2$ the possibility of determining its general analytic form, of which $C(r) = r^2$ is but a *particular* case, is overlooked, and the resulting line element usually obtained for $R_{\mu\nu} = 0$ has consequently been misinterpreted, owing to a number of latent assumptions introduced with the setting of $C(r) = r^2$, compounded by neglect of the intrinsic geometry of the line element, producing thereby a violation of basic elements of differential geometry and hence the drawing of false conclusions as to the geometric structure of Einstein's gravitational field.

2 Spherical symmetry of three-dimensional metrics

Following the method suggested by Palatini, and developed by Levi-Civita (Levi-Civita 1977), denote ordinary Efcleethean [2] 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point $O \in \mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation T of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric $d\ell'^2$ unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of $d\ell'^2$ of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$, except possibly at an arbitrary point, the centre of spherical symmetry $O' \in \mathbf{M}^3$.

Let a ray i emanate from an arbitrary point $O \in \mathbf{E}^3$. There is then a corresponding geodesic $i' \in \mathbf{M}^3$ issuing from the corresponding point $O' \in \mathbf{M}^3$. Let P be any point on i other than O. There corresponds a point P' on $i' \in \mathbf{M}^3$ different to O'. Let g' be a geodesic in $\mathbf{M}^3$ that is tangential to i' at P'.

Taking i as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on i' unchanged. If g' is distinct from i', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about i would cause g' to occupy an infinity of positions in $\mathbf{M}^3$ wherein g' has for each position the property of being tangential to i' at P' in the same direction, which is impossible. Hence, g' coincides with i'.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point $O \in \mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' in $\mathbf{M}^3$ with centre of symmetry at the corresponding point $O' \in \mathbf{M}^3$.

Let Q be a point in $\Sigma \in \mathbf{E}^3$ and Q' the corresponding point in $\Sigma' \in \mathbf{M}^3$. Let $d\sigma$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma' \in \Sigma'$ issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates r, θ, φ. Then

$$d\sigma^2 = r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

$$r = |\bar{OQ}|.$$

Clearly, if r, θ, φ are known, Q is determined and hence also Q' in Σ'. Therefore, θ and φ can be considered to be curvilinear coordinates for Q' in Σ' and the line element $d\sigma' \in \Sigma'$ will also be represented by a quadratic form similar to (1). To determine $d\sigma'$, consider two

[2] For the geometry due to Efcleethees, usually and abominably rendered as Euclid.

elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma_1'$ and $d\sigma_2'$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis $\bar{OQ}$ in $\mathbf{E}^3$, and so $d\sigma_1'$ and $d\sigma_2'$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $\frac{d\sigma'}{d\sigma}$ is the same for the two different directions irrespective of $d\theta$ and $d\varphi$, and so the foregoing ratio is a function of position, i.e. of r, θ, φ. But Q is an arbitrary point in Σ, and so $\frac{d\sigma'}{d\sigma}$ must have the same ratio for any corresponding points Q and Q'. Therefore, $\frac{d\sigma'}{d\sigma}$ is a function of r alone, thus

$$\frac{d\sigma'}{d\sigma} = H(r),$$

and so

$$d\sigma'^2 = H^2(r)d\sigma^2 = H^2(r)r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2}$$

where $H(r)$ is *a priori* unknown. For convenience set $R_c = R_c(r) = H(r)r$, so that (2) becomes

$$d\sigma'^2 = R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{3}$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing (3) with (1) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point Q', i.e. in terms of the relation $K = \frac{1}{R_c^2}$ since the Gaussian curvature of (1) is $K = \frac{1}{r^2}$. This is an intrinsic property of all line elements of the form (3) (Levi-Civita 1977; O'Neill 1983). Accordingly, R_c can be regarded as a radius of curvature. Therefore, in (1) the radius of curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilic.

Let the element of radial distance from $O \in \mathbf{E}^3$ be dr. Clearly, the radial lines issuing from O cut the surface Σ orthogonally. Combining this with (1) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$

$$d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{4}$$

Let the corresponding radial geodesic element from the point $O' \in \mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from O' cut the geodesic surface Σ' orthogonally. Combining this with (3) by the theorem of Pythagoras gives the line element in $\mathbf{M}^3$ as,

$$d\ell'^2 = dR_p^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{5}$$

where dR_p is, by spherical symmetry, also a function only of R_c. Set $dR_p = \sqrt{B(R_c)}dR_c$, so that (5) becomes

$$d\ell'^2 = B(R_c)dR_c^2 + R_c^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{6}$$

where $B(R_c(r))$ is an *a priori* unknown function.

Expression (6) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point $O' \in \mathbf{M}^3$ (Levi-Civita 1977; Eddington 1923).

Considering (4), the distance $R_p = |\bar{OQ}|$ from the point at the centre of spherical symmetry O to a point $Q \in \Sigma$, is given by

$$R_p = \int_0^r dr = r = R_c.$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature at any point in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point $O \in \mathbf{E}^3$ and the point $Q \in \Sigma$. Thus, in this case, $K = \frac{1}{R_c^2} = \frac{1}{R_p^2} = \frac{1}{r^2}$. However, this is not a general relation, since according to (5) and (6), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point $O' \in \mathbf{M}^3$ is not given by the radius of curvature, but by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c(r) = \int_0^r \sqrt{B(R_c(r))}\, \frac{dR_c(r)}{dr} dr,$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \leq r < \infty$ in (4) that it must follow that in (5) and (6) $0 \leq R_c(r) < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from (5) and (6) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the centre of spherical symmetry to some other point in a metric manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the Gaussian curvature and the radial distance from the centre of spherical symmetry, owing to the fact that R_p and R_c are identical in that special case.

It should also be noted that in writing expressions (4) and (5) it is implicit that $O \in \mathbf{E}^3$ is defined as being located at the origin of the coordinate system of (4), i.e. O is located where $r = 0$, and by correspondence O' is defined as being located at the origin of the coordinate system of (5), i.e. using (5) or (6), $O' \in \mathbf{M}^3$ is located where $R_p = 0$. Furthermore, since it is well known that a geometry is completely determined by the form of the line element describing it (Tolman 1987), expressions (4), (5) and (6) share the very same fundamental geometry because they are line elements of the same form.

Expression (6) plays an important rôle in Einstein's gravitational field.

3 The standard solution

The standard solution in the case of the static vacuum field (i.e. no deformation of the space) of a single gravitating body, satisfying Einstein's field equations $R_{\mu\nu} = 0$, is (using $G = c = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{7}$$

where m is allegedly the mass causing the field, and upon which it is routinely claimed that $2m < r < \infty$ is an exterior region and $0 < r < 2m$ is an interior region. Notwithstanding the inequalities it is routinely allowed that $r = 2m$ and $r = 0$ by which it is also routinely claimed that $r = 2m$ marks a "removable" or "coordinate" singularity and that $r = 0$ marks a "true" or "physical" singularity (Misner *et al.* 1973).

The standard treatment of the foregoing line-element proceeds from simple inspection of (7) and thereby upon the following unproven assumptions:

(a) that there is only one radial quantity defined on (7);

(b) that r can approach zero, even though the line element (7) is singular at $r = 2m$;

(c) that r is the radial quantity in (7) ($r = 2m$ is even routinely called the "Schwarzschild radius" (Misner *et al.* 1973).)

With these unstated assumptions, but assumptions nonetheless, it is usual procedure to develop and treat of black holes. However, all three assumptions are demonstrably false at an elementary level.

4 That assumption (a) is false

Consider standard Minkowski space (using $c = G = 1$) described by

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{8}$$

$$0 \leq r < \infty.$$

Comparing (8) with (4) it is easily seen that the spatial components of (8) constitute a line element of $\mathbf{E}^3$, with the point at the centre of spherical symmetry at $r_0 = 0$, coincident with the origin of the coordinate system.

In relation to (8) the *calculated* proper radius R_p of the sphere in $\mathbf{E}^3$ is,

$$R_p = \int_0^r dr = r, \tag{9}$$

and the radius of curvature R_c is

$$R_c = r = R_p. \tag{10}$$

Calculate the surface area of the sphere:

$$A = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta d\theta d\varphi = 4\pi r^2 = 4\pi R_p^2 = 4\pi R_c^2. \tag{11}$$

Calculate the volume of the sphere:

$$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2 \sin\theta dr d\theta d\varphi = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R_p^3 \tag{12}$$

$$= \frac{4}{3}\pi R_c^3.$$

Then for (8), according to (9) and (10),

$$R_p = r = R_c. \tag{13}$$

Thus, for Minkowski space, R_p and R_c are identical. This is because Minkowski space is pseudo-Efcleethean.

Now comparing (7) with (5) and (6) is is easily seen that the spatial components of (7) constitute a spherically symmetric metric manifold $\mathbf{M}^3$ described by

$$d\ell'^2 = \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, and which is therefore in one-to-one correspondence with $\mathbf{E}^3$. Then for (7),

$$R_c = r,$$

$$R_p = \int \sqrt{\frac{r}{r-2m}} dr \neq r = R_c.$$

Hence, $R_P \neq R_c$ in (7) in general. This is because (7) is non-Efcleethean (it is pseudo-Riemannian). Thus, assumption (a) is false.

5 That assumption (b) is false

On (7),

$$R_p = R_p(r) = \int \sqrt{\frac{r}{r-2m}} dr$$

$$= \sqrt{r\,(r-2m)} + 2m \ln\left|\sqrt{r} + \sqrt{r-2m}\right| + K, \qquad (14)$$

where K is a constant of integration.

For some r_0, $R_p(r_0) = 0$, where r_0 is the corresponding point at the centre of spherical symmetry in $\mathbf{E}^3$ to be determined from (14). According to (14), $R_p(r_0) = 0$ when $r = r_0 = 2m$ and $K = -m \ln 2m$. Hence,

$$R_p(r) = \sqrt{r\,(r-2m)} + 2m \ln\left(\frac{\sqrt{r} + \sqrt{r-2m}}{\sqrt{2m}}\right). \qquad (15)$$

Therefore, $2m < r < \infty \Rightarrow 0 < R_p < \infty$, where $R_c = r$. The inequality is required to maintain Lorentz signature, since the line-element is undefined at $r_0 = 2m$, which is the only possible singularity on the line element. Thus, assumption (b) is false.

It follows that the centre of spherical symmetry of $\mathbf{E^3}$, in relation to (7), is located not at the point $r_0 = 0$ in $\mathbf{E^3}$ as usually assumed according to (8), but at the point $r_0 = 2m$, which corresponds to the point $R_p(r_0 = 2m) = 0$ in the metric manifold $\mathbf{M^3}$ that is described by the spatial part of (7). In other words, the point at the centre of spherical symmetry in $\mathbf{E^3}$ in relation to (7) is located at any point Q in the spherical surface Σ for which the radial distance from the centre of the coordinate system at $r = 0$ is $r = 2m$, owing to the one-to-one correspondence between all points of $\mathbf{E^3}$ and $\mathbf{M^3}$. It follows that (7) is not a generalisation of (8), as usually claimed. The manifold $\mathbf{E^3}$ of Minkowski space corresponding to the metric manifold $\mathbf{M^3}$ of (7) is not described by (8), because the point at the centre of spherical symmetry of (8), $r_0 = 0$, does not coincide with that required by (14) and (15), namely $r_0 = 2m$.

In consequence of the foregoing it is plain that the expression (7) is not general in relation to (8) and the line element (7) is not general in relation to the form (6). This is due to the incorrect way in which (7) is usually derived from (8), as pointed out in (Abrams 1980; Crothers 2005; Antoci 2001). The standard derivation of (7) from (8) unwittingly shifts the point at the centre of spherical symmetry for the $\mathbf{E^3}$ of Minkowski space from $r_0 = 0$ to $r_0 = 2m$, with the consequence that the resulting line element (7) is misinterpreted in relation to $r = 0$ in the $\mathbf{E^3}$ of Minkowski space as described by (8). This unrecognised shift actually associates the point $r_0 = 2m \in \mathbf{E^3}$ with the point $R_p(2m) = 0$ in the $\mathbf{M^3}$ of the gravitational field. The usual analysis then incorrectly associates $R_p = 0$ in $\mathbf{M^3}$ with $r_0 = 0$ in $\mathbf{E^3}$ instead of with the correct $r_0 = 2m$ in $\mathbf{E^3}$, thereby inventing a so-called "interior", as typically alleged (Misner *et al.* 1973), that actually has no relevance to the problem — a completely meaningless manifold that has nothing to do with the gravitational field and so is disjoint from the latter, as also noted in (Abrams 1980; Loinger 2002; Smoller & Temple 1998; Crothers 2006). The point at the centre of spherical symmetry of Einstein's gravitational field in $\mathbf{M^3}$ is $R_p = 0$. Thus the notion of an "interior" manifold under some other coordinate patch (such as the Kruskal-Szekeres coordinates) is incorrect. This is clarified in the next section.

6 That assumption (c) is false

Generalise (8) so that the centre of a sphere can be located anywhere in Minkowski space, relative to the origin of the coordinate system at $r = 0$, thus

$$\begin{aligned} ds^2 &= dt^2 - (d\,|r - r_0|)^2 - |r - r_0|^2\, d\Omega^2 \\ &= dt^2 - \frac{(r - r_0)^2}{|r - r_0|^2} dr^2 - |r - r_0|^2\, d\Omega^2 \\ &= dt^2 - dr^2 - |r - r_0|^2\, d\Omega^2, \\ & \qquad 0 \le |r - r_0| < \infty, \end{aligned} \tag{16}$$

which is well-defined for all real r. The value of r_0 is arbitrary. The spatial components of (16) describe a sphere of radius $D = |r - r_0|$ centred at some *point* r_0 on a common radial line through r and the origin of coordinates at $r = 0$ (i.e. centred at the *point* of orthogonal intersection of the common radial line with the spherical surface $r = r_0$). Thus, the arbitrary point r_0 is the centre of spherical symmetry in $\mathbf{E^3}$ for (16) in relation to the problem of Einstein's gravitational field, the spatial components of which is a spherically symmetric metric manifold $\mathbf{M^3}$ with line element of the form (6) and corresponding centre of spherical symmetry at the point $R_p(r_0) = 0 \;\forall\; r_0$. If $r_0 = 0$ and $r \ge 0$ is taken, (8) is recovered from (16). One does not need to make $r_0 = 0$ so that the centre of spherical symmetry in $\mathbf{E^3}$, associated with the metric manifold $\mathbf{M^3}$ of Einstein's gravitational field, coincides with the origin of the coordinate system itself, at $r = 0$. Any point in $\mathbf{E^3}$, relative to the coordinate system attached to the arbitrary point at which $r = 0$, can be regarded as a point at the centre of spherical symmetry in relation to Einstein's gravitational field. Although it is perhaps desirable to make the point $r_0 = 0$ the centre of spherical symmetry of $\mathbf{E^3}$ correspond to the point $R_p = 0$ at the centre of symmetry of $\mathbf{M^3}$ of the gravitational field, to simplify matters somewhat, this has not been done in the usual analysis of Einstein's gravitational field, despite appearances, and in consequence thereof false conclusions have been drawn owing to this fact going unrecognised in the main.

Now on (16),

$$R_c = |r - r_0|,$$

$$R_p = \int_0^{|r-r_0|} d\,|r - r_0| = \int_{r_0}^{r} \frac{(r - r_0)}{|r - r_0|} dr = |r - r_0| \equiv R_c, \tag{17}$$

and so $R_p = R_c$ on (16), since (16) is pseudo Efcleethean. Setting $D = |r - r_0|$ for convenience, generalise (16) thus,

$$ds^2 = A(C(D))dt^2 - B(C(D))d\sqrt{C(D)}^2 - C(D)d\Omega^2, \tag{18}$$

where $A(C(D)) > 0, B(C(D)) > 0, C(D) > 0$. Then for $R_{\mu\nu} = 0$, metric (18) has the solution,

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(D)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(D)}}\right)^{-1} d\sqrt{C(D)}^2 - C(D)d\Omega^2, \tag{19}$$

where α is a function of the mass generating the gravitational field (Eddington 1923; Abrams 1980; Crothers 2005; Loinger 1998). Then for (19),

$$R_c = R_c(D) = \sqrt{C(D)},$$

$$R_p = R_p(D) = \int \sqrt{\frac{\sqrt{C(D)}}{\sqrt{C(D)} - \alpha}}\; d\sqrt{C(D)} = \int \sqrt{\frac{R_c(D)}{R_c(D) - \alpha}}\; dR_c(D)$$

$$= \sqrt{R_c(D)\,(R_c(D) - \alpha)} + \alpha \ln\left(\frac{\sqrt{R_c(D)} + \sqrt{R_c(D) - \alpha}}{\sqrt{\alpha}}\right), \tag{20}$$

where $R_c(D) \equiv R_c\,(|r - r_0|) = R_c(r)$. Clearly r is a parameter, located in Minkowski space according to (16).

Now $r = r_0 \Rightarrow D = 0$, and so by (20), $R_c(D = 0) = \alpha$ and $R_p(D = 0) = 0$. One must ascertain the admissible form of $R_c(D)$ subject to the conditions $R_c(D = 0) = \alpha$ and $R_p(D = 0) = 0$ and $dR_c(D)/dD > 0$ (Abrams 1980; Crothers 2005), along with the requirements that $R_c(D)$ must produce (7) from (19) at will, must yield Schwarzschild's (Schwarzschild 1916) original solution at will (which is *not* the line element (7) with r down to zero), must produce Brillouin's (Brillouin 1923) solution at will, must produce Droste's (Droste 1917) solution at will, must yield Minkowski space when $\alpha = 0$, must approach Minkowski space asymptotically, and must yield an infinite number of equivalent metrics (Eddington 1923). The only admissible form satisfying these conditions is (Crothers 2005),

$$R_c = R_c(D) = (D^n + \alpha^n)^{\frac{1}{n}} \equiv (|r - r_0|^n + \alpha^n)^{\frac{1}{n}} = R_c(r), \tag{21}$$

$$D > 0, \quad r \in \Re, \quad n \in \Re^+, \quad r \neq r_0,$$

where r_0 and n are entirely arbitrary constants.

Choosing $r_0 = 0,\ r > 0,\ n = 3$, gives

$$R_c(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \tag{22}$$

and putting (22) into (19) gives Schwarzschild's original solution, defined on $0 < r < \infty$.

Choosing $r_0 = 0,\ r > 0,\ n = 1$, gives

$$R_c(r) = r + \alpha, \tag{23}$$

and putting (23) into (19) gives Marcel Brillouin's solution, defined on $0 < r < \infty$.

Choosing $r_0 = \alpha,\ r > \alpha,\ n = 1$, gives

$$R_c(r) = (r - \alpha) + \alpha = r, \tag{24}$$

and putting (24) into (19) gives line element (7), but defined on $\alpha < r < \infty$, as found by Johannes Droste in May 1916. Note that according to (24) (and in general by (21)), r is not a radial quantity in the gravitational field, because $R_c(r) = (r - \alpha) + \alpha = D + \alpha$ is really the radius of curvature in (7), defined for $0 < D < \infty$.

Thus, assumption (c) is false.

It follows from this that the usual line element (7) is a restricted form of (19), and by (21), with $r_0 = \alpha = 2m$, $n = 1$ gives $R_c = |r - 2m| + 2m$, which is well defined on $-\infty < r < \infty$, i.e. on $0 \leq D < \infty$, so that when $r = 0$, $R_c(0) = 4m$ and $R_P(0) > 0$. In the limiting case of $r = 2m$ the line element becomes undefined, and then $R_c(2m) = 2m$ and $R_p(2m) = 0$. The latter two relationships hold for any value of r_0.

Thus, if one insists that $r_0 = 0$ to match (8), it follows from (21) that,

$$R_c = \left(|r|^n + \alpha^n\right)^{\frac{1}{n}},$$

and if one also insists that $r > 0$, then

$$R_c = \left(r^n + \alpha^n\right)^{\frac{1}{n}}, \tag{25}$$

and for $n = 1$,

$$R_c = r + \alpha,$$

which is the simplest expression for R_c in (19) (Abrams 1980; Crothers 2005; Brillouin 1923).

Expression (25) has the centre of spherical symmetry of $\mathbf{E}^3$ located at the point $r_0 = 0\ \forall\ n \in \Re^+$, corresponding to the centre of spherical symmetry of $\mathbf{M}^3$ for Einstein's gravitational field at the point $R_p(0) = 0\ \forall\ n \in \Re^+$. Then taking $\alpha = 2m$ it follows that $R_p(0) = 0$ and $R_c(0) = \alpha = 2m$ for all values of n.

There is no such thing as an interior solution for the line element (19) and consequently there is no such thing as an interior solution on (7), and so there can be no black holes.

7 That the manifold cannot be extended

That the singularity at $R_p(r_0) \equiv 0$ is insurmountable is clear by the following ratio,

$$\lim_{r \to r_0^{\pm}} \frac{2\pi R_c(r)}{R_p(r)} = \lim_{r \to r_0^{\pm}} \frac{2\pi\left(|r - r_0|^n + \alpha^n\right)^{\frac{1}{n}}}{R_p(r)} = \infty,$$

since $R_p(r_0) = 0$ and $R_c(r_0) = \alpha$ are invariant.

Hagihara (1931) has shown that all radial geodesics that do not run into the boundary at $R_c(r_0) = \alpha$ (i.e. that do not run into the boundary at $R_p(r_0) = 0$) are geodesically complete.

Doughty (1981) has shown that the acceleration a of a test particle approaching the centre of mass at $R_p(r_0) = 0$ is given by,

$$a = \frac{\sqrt{-g_{00}}\left(-g^{11}\right)|g_{00,1}|}{2g_{00}}.$$

By (19) and (21), this gives,

$$a = \frac{\alpha}{2R_c^{\frac{3}{2}}\sqrt{R_c(r) - \alpha}}.$$

Then clearly as $r \to r_0^{\pm}$, $a \to \infty$, independently of the value of r_0.

J. Smoller and B. Temple (1998) have shown that the Oppenheimer-Volkoff equations do not permit gravitational collapse to form a black hole and that the alleged interior of the Schwarzschild spacetime (i.e. $0 \le R_c(r) \le \alpha$) is therefore *disconnected* from Schwarzschild spacetime and so does not form part of the solution space.

N. Stavroulakis (1974; 1997; 2002; 2006) has shown that an object cannot undergo gravitational collapse into a singularity, or to form a black hole.

Suppose $0 \le \sqrt{C(D(r))} < \alpha$. Then (19) becomes

$$ds^2 = -\left(\frac{\alpha}{\sqrt{C}} - 1\right)dt^2 + \left(\frac{\alpha}{\sqrt{C}} - 1\right)^{-1} d\sqrt{C}^2 - C(d\theta^2 + \sin^2\theta d\varphi^2),$$

which shows that there is an interchange of time and length. To amplify this set $r = \bar{t}$ and $t = \bar{r}$. Then

$$ds^2 = \left(\frac{\alpha}{\sqrt{C}} - 1\right)^{-1} \frac{\dot{C}^2}{4C} d\bar{t}^2 - \left(\frac{\alpha}{\sqrt{C}} - 1\right) d\bar{r}^2 - C(d\theta^2 + \sin^2\theta d\varphi^2),$$

where $C = C(\bar{t})$ and the dot denotes $d/d\bar{t}$. This is a time dependent metric and therefore bears no relation to the problem of a static gravitational field.

Thus, the Schwarzschild manifold described by (19) with (21) (and hence (7)) cannot be extended.

8 That the Riemann tensor scalar curvature invariant is everywhere finite

The Riemann tensor scalar curvature invariant (the Kretschmann scalar) is given by $f = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. In the general case of (19) with (21) this is

$$f = \frac{12\alpha^2}{R_c^6(r)} = \frac{12\alpha^2}{\left(|r - r_0|^n + \alpha^n\right)^{\frac{6}{n}}}.$$

A routine attempt to justify the standard assumptions on (7) is the *a posteriori* claim that the Kretschmann scalar must be unbounded at a singularity (Misner *et al.* 1973; Kruskal

1960). Nobody has ever offered a proof that General Relativity necessarily requires this. That this additional *ad hoc* assumption is false is clear from the following ratio,

$$f(r_0) = \frac{12\alpha^2}{\left(|r_0 - r_0|^n + \alpha^n\right)^{\frac{6}{n}}} = \frac{12}{\alpha^4} \;\forall\; r_0.$$

In addition,

$$\lim_{r\to\pm\infty} \frac{12\alpha^2}{\left(|r - r_0|^n + \alpha^n\right)^{\frac{6}{n}}} = 0,$$

and so the Kretschmann scalar is everywhere finite.

9 That the Gaussian curvature is everywhere finite

The Gaussian curvature K of (19) is,

$$K = K(R_c(r)) = \frac{1}{R_c^2(r)},$$

where $R_c(r)$ is given by (21). Then,

$$K(r_0) = \frac{1}{\alpha^2} \quad \forall \quad r_0,$$

and

$$\lim_{r\to\pm\infty} K(r) = 0,$$

and so the Gaussian curvature is everywhere finite.

Furthermore,

$$\lim_{\alpha\to 0} \frac{1}{\alpha^2} = \infty,$$

since when $\alpha = 0$ there is no gravitational field and empty Minkowski space is recovered, wherein R_p and R_c are identical and $0 \le R_p < \infty$. A centre of spherical symmetry in Minkowski space has an infinite Gaussian curvature because Minkowski space is pseudo-Efcleethean.

10 Translation of the centre of symmetry

The usual interpretations confound the location of the centre of spherical symmetry of the gravitational field with the origin of a coordinate system associated with the parameter r whose corresponding centre of spherical symmetry is not at its origin of coordinates. In Efcleethean 3-space the equation of a sphere of radius D and centre C located at the extremity of the fixed vector $\vec{\mathbf{r}}_0$, may be written

$$(\vec{\mathbf{r}}(u) - \vec{\mathbf{r}}_0) \bullet (\vec{\mathbf{r}}(u) - \vec{\mathbf{r}}_0) = D^2. \tag{26}$$

where u is some parameter upon which $\vec{\mathbf{r}}$ depends. The centre of the sphere is not at the origin of the coordinate system unless $\vec{\mathbf{r}}_0 = \vec{\mathbf{0}}$. The usual interpretations treat the origin of the parametric coordinate system as the centre of a non-Efcleethean sphere (for the gravitational field) when the centre of the said sphere is not in fact located at the origin of the parametric coordinate system or in the gravitational field. The centre of spherical symmetry of the source of the gravitational field is actually a *centre of mass* and as such is not a physical object. The black hole results from the construction of a completely different and irrelevant manifold, with the claim that the singularity at $r = \alpha$ on (7) is a "coordinate" singularity, and the claim that the origin of the parametric coordinate system at $r = 0$ is the "true" singularity for the gravitational field given by (7), in the belief that the origin of the parametric coordinate system is the location of the centre of spherical symmetry for Einstein's gravitational field. The inadmissibility of this usual conception, from which the black hole is actually obtained, can be readily seen by claiming that the "true" centre of the sphere described by (26) is not at the extremity of the fixed non-zero vector $\vec{\mathbf{r}}_0$, but at the origin of the coordinate system to which the vectors are referred, and then construct a "transformation of coordinates", in the fashion of the Kruskal-Szekeres procedure, that actually retains the centre of the sphere at the extremity of the fixed non-zero vector $\vec{\mathbf{r}}_0$, but interprets the "centre" of the sphere to be at the origin of coordinates, so that its actual centre, the centre of mass, is a "removable" coordinate artifact.

Note that if $\vec{\mathbf{r}}(u)$ and $\vec{\mathbf{r}}_0$ are collinear, then the vector notation can be dropped, whereupon equation (26) takes the scalar form

$$(r(u) - r_0)(r(u) - r_0) = D^2,$$

where D is now more readily seen to be precisely that quantity appearing in expressions (18) to (21), being the parametric *distance* $|r(u) - r_0|$ between the parametric points $r(u)$ and r_0 on the real line (the radial line in Minkowski space that passes through the origin of the coordinate system and the points $r(u)$ and r_0 on that radial line), where r_0 is the location of the centre of symmetry in Minkowski space corresponding to the centre of symmetry $R_p(r_0) = 0$ in the gravitational field.

11 Some additional general comments

The infinite acceleration at $R_p(r_0) = 0$ (i.e. where the radius of curvature $R_c(r_0) = \alpha$) is not physical. Also, nobody ever takes the centre of mass in Newton's theory as a physical object. It is not a physical object in Einstein's theory either. The fact that the usual line-element is a solution to $R_{\mu\nu} = 0$ $(\mu, \nu = 0, 1, 2, 3)$ is a clear statement that the mass comprising the source is taken to be a centre of mass, just as in Newton's theory of gravitation. There is no treatment or consideration of the distribution of the mass of the source of the field, so solutions to $R_{\mu\nu} = 0$ cannot be taken to have any meaning at the centre of mass (just as is the case in Newtonian gravitation). They only apply where the distribution of the mass and energy of the source can be ignored, that is, when considering the region exterior to the source, and hence of the source as a centre of mass (the mass concentrated at the centre of spherical symmetry).

It arises in the case of the alleged black holes the curious situation that there is an infinite acceleration where, according to the proponents of the black hole, there is no matter, and another infinite acceleration where the alleged physical singularity is located, which is supposed to be an infinitely dense "point-mass" (see section 6 herein). In any event, point-masses are physically meaningless, they are only centres of mass, and as such are mathematical artifices, not real objects.

Outside the source of a massive object, one has a solution for $R_{\mu\nu} = 0$. But inside the source, $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$, where the energy-momentum tensor $T_{\mu\nu}$ is not zero and κ is a non-zero constant. The real issue here is how to determine the tensor $T_{\mu\nu}$. In the case of a star, nobody knows what to write for $T_{\mu\nu}$, and for the interaction of two bodies this seems impossible. However, Schwarzschild, realising this fact, addressed, as a matter of mathematical tractability, the idealised situation for a homogeneous and incompressible fluid (it is interesting that P-M. L. Robitaille (2006; 2007) has recently and independently proposed a stellar model in terms of an incompressible fluid, based upon other considerations). Schwarzschild could then easily write an expression for $T_{\mu\nu}$. His solution (Schwarzschild 1916) and the generalisation of his solution (Crothers 2005), reveal some compelling results. First, there is no singularity anywhere, as to be expected, since the field outside the source described by $R_{\mu\nu} = 0$ does not apply to the interior of the source of the field. This is precisely the same qualitative feature of Newton's gravitation. Outside Newton's gravitating body, the field is described in relation to a centre of mass, but on approaching the source, the field expression gives way, at the surface, to a different description of the gravitational field, for the interior of the source. This is precisely the situation in Schwarzschild's case for the ideal fluid. Furthermore, Schwarzschild has shown that the constant that appears in his "mass-point" solution is not determined by a far field comparison to the Newtonian potential, but is in fact determined by the solution for the interior of the source of the field. Thus, the usual claim that the constant appearing in the "point-mass" solution is $2GM/c^2$, is incorrect. Schwarzschild himself did not ever make such a claim, and rightly so. The proponents of the black hole obtain their constant by comparison to the Newtonian potential in the far field. But in Newton's theory, gravitational potential is conceived of in terms of *two* masses interacting, because Newton's gravitational potential is conceived of as the work per unit mass in the gravitational field. This concept makes no sense without the possibility of arbitrary introduction of a mass into the gravitational field of some other mass. Indeed, Newton's force of gravitation is defined in terms of two interacting masses to begin with. This is not the case in Einstein's theory. One cannot rightly get Newton's potential from a solution to $R_{\mu\nu} = 0$ because $R_{\mu\nu} = 0$ is a clear statement that there is no matter or energy in the Universe outside the source of the gravitational field, and so a mass cannot, in principle, be arbitrarily inserted into the gravitational field. Thus, one cannot get a far field Newtonian approximation to Einstein's gravitational field in the way claimed by the relativists. One cannot get a far field Newtonian potential for Schwarzschild's ideal fluid solution either, because outside the source the field is described by $R_{\mu\nu} = 0$, but the constant appearing in the line element for $R_{\mu\nu} = 0$ is determined in full by the distribution of matter and energy in the source, from the line-element for that distribution, and there is no matter anywhere in the Universe outside the source of the gravitational field (and no possibility of introducing any matter or energy into that field, owing to the condition $R_{\mu\nu} = 0$). The solution for $R_{\mu\nu} = 0$ does not include the source mass at the centre of mass because it is not even in that gravitational field (and it is not a physical object). Schwarzschild's ideal sphere of fluid is in the gravitational field, without singularity. That singularities in the field are inadmissible Einstein was himself aware, and he repeatedly objected to all attempts to attach physical meaning to a singularity in the field. According to Einstein (1967),

> *"A field theory is not yet completely determined by the system of field equations. Should one admit the appearance of singularities? Should one postulate boundary*

conditions? As to the first question, it is my opinion that singularities must be excluded. It does not seem reasonable to me to introduce into a continuum theory points (or lines, etc.) for which the field equations do not hold. Moreover, the introduction of singularities is equivalent to postulating boundary conditions (which are arbitrary from the point of view of the field equations) on 'surfaces' which closely surround the singularities. Without such a postulate the theory is too vague. In my opinion the answer to the second question is that postulation of boundary conditions is indispensable."

Since $R_{\mu\nu} = 0$ excludes all matter and energy, one cannot derive from it a black hole and then assert that black holes can collide or otherwise interact. A second black hole cannot be arbitrarily introduced into the field of a black hole, described by $R_{\mu\nu} = 0$, when $R_{\mu\nu} = 0$ is itself a statement that there is no other mass or energy in the Universe, and since the centre of mass (i.e. the black hole singularity) is itself not even in the field described by $R_{\mu\nu} = 0$. Thus, the concepts of black holes colliding, merging, or being components of binary systems, are spurious, even if, for the sake of argument, black holes are predicted by General Relativity. For the gravitational interaction of two or more bodies, one needs either a non-zero energy-momentum tensor that describes the configuration for two or more interacting comparable masses and a solution to the field equations for that tensor, or an existence theorem that proves that Einstein's field equations admit of solutions for such configurations, even if those solutions cannot yet be found (McVittie 1978). Furthermore, one cannot simply assert, by an analogy with Newton's theory, that black holes can collide, merge, or be components of a binary system (McVittie 1978).

It is also frequently claimed that the escape velocity of a black hole is that of light in vacuo. If this is true then light could escape from the black hole. The existence of an escape velocity does not mean that an object cannot leave the surface of another object. It only means that it cannot escape to infinity, but will be pulled back to the host object if its initial velocity is less than the escape velocity. But according to black hole theory, no object and no light can even leave the black hole. That is not a statement that relates to the concept of an escape velocity. It is also claimed that Newton's theory predicts some kind of primitive black hole. That too is not correct. First, the Michell-Laplace dark body does not involve the concept of gravitational collapse or the concept of a singularity (the black hole has a singularity that allegedly results from irresistible gravitational collapse). Second, it has an escape velocity, namely the speed of light in vacuo (a black hole has no escape velocity). Third, in Newton's mechanics there is no upper limit to a speed (in Einstein's theory there is an upper speed limit). Fourth, there is always a class of observers that can see the Michell-Laplace dark body: an observer merely has to be within the distance a radially moving object can travel before being pulled back to the host body (but nothing can even leave a black hole and so there is no class of observers that can see it, however close an observer is to the alleged event horizon). Fifth, it is routinely claimed that the "Schwarzschild" radius $R = 2GM/c^2$ occurs in the case of the Michell-Laplace dark body. In the case of the Michell-Laplace dark body $R = 2GM/c^2$ is a true measurable radius, but in the case of Einstein's gravitational field it is not a radius at all, but an immeasurable radius of curvature (and a minimum radius of curvature at that) by virtue of its relationship to the Gaussian curvature (it is not a distance, let alone a "radial" distance, in Einstein's gravitational field). Thus, the Michell-Laplace dark body is not related to a black hole at all.

In addition the solution to $R_{\mu\nu} = 0$ does not even generalise Special Relativity, because $R_{\mu\nu} = 0$ excludes all matter and energy (the "point-mass" source is not in the field). But Special Relativity permits the presence of arbitrarily large masses and energies (but not infinitely large). Thus, the solutions to $R_{\mu\nu} = 0$ are not generalisations of the dynamical system of Special Relativity, but are only generalisations of a geometry, namely, of Minkowski space. All motion in the field described by solutions to $R_{\mu\nu} = 0$ is kinematic in nature. It is incorrect to conceive of moving points in that system of kinematics as photons or as massive particles, howsoever small or large, because $R_{\mu\nu} = 0$ excludes all such matter and energy. The idea of photons in the field described by $R_{\mu\nu} = 0$ is inadmissible, and is directly related to a misinterpretation of the Special Relativistic upper limiting speed c, in vacuo. This is merely an upper speed limit for the motion of points in relation to solutions to $R_{\mu\nu} = 0$, nothing more. One cannot associate that speed limit with a photon in the solutions to $R_{\mu\nu} = 0$, since photons (being energy carriers) cannot exist in those gravitational fields by the very statement $R_{\mu\nu} = 0$. The presence of the upper speed limit c is taken by the relativists to mean that there are photons that can have that speed, by the definition of c in relation to Special Relativity. But that is a false concept for $R_{\mu\nu} = 0$, which only has a system of kinematics (only a geometry) in which there is an upper speed limit called c, for the motion of a point. Indeed, Minkowski space describes a system of kinematics wherein there is an upper limit c to the speed of a point. Length contraction and time dilation are purely kinematic features (geometric features) of Minkowski space, both of which result from the hypothesis of an upper speed limit for a moving point. The dynamics of Special Relativity take place in Minkowski space on the assumption that mass and energy can merely be inserted into Minkowski space, in similar fashion to the assumption that mass and energy can be merely inserted into the 3-D Efcleethean geometry (i.e. the kinematic system) of Newton's universe, and the assignment of the upper speed limit to the speed of light in vacuo. Furthermore, Special Relativity forbids the existence of infinite density, yet black hole theory routinely claims that the black hole singularity is a point containing the mass of the black hole, and hence it must have infinite density. The black hole event horizon does not define, according to that theory, a region throughout which the mass of the black hole is distributed (recall that black holes are allegedly formed by an irresistible gravitational collapse into a singularity).

That it is necessary but insufficient that a line element used to model Einstein's static vacuum gravitational field must satisfy the field equations $R_{\mu\nu} = 0$ and be asymptotically Minkowski, is clearly illustrated by the following counter-examples. Consider the line element

$$ds^2 = \left(1 - \frac{\alpha}{r-\alpha}\right) dt^2 - \left(1 - \frac{\alpha}{r-\alpha}\right)^{-1} dr^2 - (r-\alpha)^2 (d\theta^2 + \sin^2\theta d\varphi^2). \tag{27}$$

This line element satisfies $R_{\mu\nu} = 0$ and is asymptotically Minkowski, which can be easily verified. Neglecting the intrinsic geometry of the line element and instead applying the usual method of "inspection", it is apparent, in accordance with the usual interpretation of expression (7), that it is singular at $r = 2\alpha$ and at $r = \alpha$. However, at $r = 2\alpha$, $g_{00} = 0$ and at $r = \alpha$, $g_{00} = -\infty$, so it seems that there is an "event horizon" with a "Schwarzschild radius" $r = 2\alpha$ and a "singularity" with a "radius" of $r = \alpha$. In addition, the line element seems to be well-defined at $r = 0$ — nothing remarkable occurs at $r = 0$. Alternatively, from another perspective, it seems to have *two* event horizons and no singularity, "assuming" as usual that r can go down to zero.

Now consider the line element

$$ds^2 = \left(1 + \frac{\alpha}{r + \alpha}\right) dt^2 - \left(1 + \frac{\alpha}{r + \alpha}\right)^{-1} dr^2 - (r + \alpha)^2 (d\theta^2 + \sin^2\theta d\varphi^2). \tag{28}$$

This line element satisfies $R_{\mu\nu} = 0$ and is asymptotically Minkowski, which can be easily verified. Neglecting the intrinsic geometry of the line element and instead applying the usual method of "inspection", it is apparent, in accordance with the usual interpretation of expression (7), that it is singular nowhere, "assuming" as usual that r can go down to zero.

Nonetheless, line elements (27) and (28) are in actual fact valid and equivalent models for Einstein's static vacuum field for the *centre of mass* configuration, entirely consistent with Karl Schwarzschild's original solution (Schwarzschild 1916). To see that this is so requires application of expression (21) and the intrinsic geometry of the line element. Still, there are no possibilities for the "black hole".

12 Conclusions

Using the spherical-polar coordinates, the general solution to $R_{\mu\nu} = 0$ is (19) with (21), which is well-defined on

$$-\infty < r < \infty, \quad r \neq r_0,$$

where r_0 is entirely arbitrary, and corresponds to

$$0 < R_p(r) < \infty, \quad \alpha < R_c(r) < \infty,$$

for the gravitational field. The only singularity that is possible occurs at $g_{00} = 0$. It is impossible to get $g_{11} = 0$ because there is no value of the parameter r by which this can be attained. No interior exists in relation to (19) with (21), which contain the usual metric (7) as a particular case, and there are no curvature-type singularities in Einstein's gravitational field.

The radius of curvature $R_c(r)$, given by expression (21), does not in general determine the radial geodesic distance to the centre of spherical symmetry of Einstein's gravitational field and is only to be interpreted in relation to the Gaussian curvature by the equation $K = 1/R_c^2(r)$. The radial geodesic distance from the point at the centre of spherical symmetry to the spherical geodesic surface with Gaussian curvature $K = 1/R_c^2(r)$ is given by the proper radius, $R_p(R_c(r))$. The centre of spherical symmetry in the gravitational field is invariantly located at the point $R_p(r_0) = 0$.

Expression (19) with (21) (and hence (7)) describes only a *centre of mass* located at $R_p(r_0) = 0$ in the gravitational field, $\forall\ r_0$. As such it does not take into account the distribution of matter and energy in a gravitating body, since $\alpha(M)$ is indeterminable in this limited situation. One cannot generally just utilise a potential function in comparison with the Newtonian potential to determine α by the weak field limit because α is subject to the distribution of the matter of the source of the gravitational field. The value of α must be calculated from a line element describing the interior of the gravitating body, satisfying $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} \neq 0$. The interior line element is necessarily different to the exterior line element of an object such as a star. A full description of the gravitational field of a star therefore requires two line elements (Schwarzschild 1916; Crothers 2005), not one as is routinely assumed, and when this is done, there are no singularities anywhere. The standard assumption that one line element is sufficient is false. Outside a star, (19) with (21) describes the gravitational field in relation to the centre of mass of the star, but α is nonetheless determined by the interior metric, which, in the case of the usual treatment of (7), has gone entirely unrecognised, so that the value of α is instead determined by a comparison with the Newtonian potential in a weak field limit.

Black holes are not predicted by General Relativity. The Kruskal-Szekeres coordinates do not describe a coordinate patch that covers a part of the gravitational manifold that is not otherwise covered - they describe a completely different pseudo-Riemannian manifold that has nothing to do with Einstein's gravitational field (Abrams 1980; Loinger 2002; Crothers 2006). The manifold of Kruskal-Szekeres is not contained in the fundamental one-to-one correspondence between the $\mathbf{E}^3$ of Minkowski space and the $\mathbf{M}^3$ of Einstein's gravitational field, and is therefore a spurious augmentation.

It follows in similar fashion that expansion of the Universe and the Big Bang cosmology are inconsistent with General Relativity, as is easily demonstrated (Crothers 2005; Crothers 2007; Crothers 2007).

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