Chapter 1

Explanation of the Whirlpool Galaxy from Constant Space-Time Torsion: The Case Against "Dark Matter"

by

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Abstract

Constant torsion of space-time in general relativity produces a constant angular momentum of space-time by volume integration. It is shown that this constant angular momentum gives rise straightforwardly to a potential energy proportional to inverse square distance - a negative valued centrifugal potential energy that does work on a star. The potential energy attracts the star into a logarithmic spiral orbit through a negative valued force law that is inversely proportional to the cube of distance. The orbit gradually becomes a circle with constant orbital velocity. This theory explains the main features of a whirlpool galaxy without any "dark matter". The angular momentum is a constant of motion and is conserved, i.e. does not change with time. Total energy is conserved, and consists of the kinetic energy of a star with velocity v moving in a

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plane, added to the potential energy of the spinning space-time. A whirlpool galaxy is a direct demonstration of the potential energy of spinning space-time.

Keywords: Einstein Cartan Evans field theory, dark matter, cosmology, galaxies.

1.1 Introduction

It is well known that the now obsolete Einsteinian general relativity omits consideration of space-time torsion, and in so doing uses an incorrect symmetric connection [1] - [10] with multiple sequential mathematical errors that render the theory meaningless in physics. By correctly considering the space-time torsion [1] - [10] a new cosmology has been constructed in Einstein Cartan Evans (ECE) theory, a cosmology that is based directly on Cartan geometry. In so doing the space-time torsion plays a central role. In Section 2 it is shown that a constant space-time torsion is sufficient to produce the main features of a whirlpool galaxy, in which stars move on a logarithmic spiral orbit contrary to Newtonian dynamics. The constant torsion is integrated over a volume to produce a constant angular momentum of space-time and a negative valued potential energy that does work on the star, attracting it into a logarithmic spiral orbit. The potential energy is inversely proportional to r^2, and produces a negative valued force of attraction that is inversely proportional to r^3. A Lagrangian analysis of the problem shows that the orbit due to such a force law is a logarithmic spiral as observed experimentally. The orbit gradually becomes a circle in which the orbital linear velocity is constant as observed experimentally. The angular momentum is a constant of the motion and does not change with time in this simplest theory. The angular momentum is therefore conserved. The total energy is also conserved, and is the sum of the kinetic energy due to the linear velocity of the star in a plane, and the potential energy caused by the constant torsion of space-time. In Section 3, a graphical analysis of the evolution of the whirlpool galaxy is given. The orbital equations may also be animated for direct visualization. Some discussion is given of this theory and of the main experimental features of a whirlpool galaxy. This simplest theory is soluble analytically, and is designed to produce only the main features of the galaxy. More realistic models would include a varying torsion and the dynamical equations of ECE theory solved numerically. Severe criticism of the obsolete "dark matter" speculation is summarized.

1.2 Calculation of the Logarithmic Spiral Stellar Orbit due to Constant Space-Time Torsion

The space-time torsion tensor is defined [1] - [10] in ECE theory by the Cartan - Evans dual identity:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{1.1}$$

where its covariant derivative is the curvature tensor appearing on the right hand side of Eq. (1.1). In general the torsion tensor may be integrated over a hyper-surface to give a rank two anti-symmetric tensor:

$$T^{\mu\nu} = -T^{\nu\mu} = \int_\sigma T^{\kappa\mu\nu}\, d\sigma_\kappa. \tag{1.2}$$

This tensor defines the angular momentum tensor of space-time through the following proportionality:

$$J^{\mu\nu} = \frac{c}{k} T^{\mu\nu} \tag{1.3}$$

where k is Einstein's constant and c the vacuum speed of light. Eq. (1.3) is a hypothesis that asserts that the integrated torsion tensor $T^{\mu\nu}$ is proportional to the angular momentum tensor. It is well known [11] that the angular momentum is defined by the following volume integration of the angular *momentum/angular* energy density tensor in field theory:

$$J^{\mu\nu} = \int J^{0\mu\nu}\, dV \tag{1.4}$$

and similarly:

$$T^{\mu\nu} = \int T^{0\mu\nu}\, dV. \tag{1.5}$$

Therefore the hypothesis Eq. (1.3) is one way of correcting the Einstein field equation for the presence of torsion [1] - [10].

Consider now the space-time angular momentum in the Z axis defined by:

$$J_Z = J^{12} = \int J^{012}\, dV. \tag{1.6}$$

This is a Z axis angular momentum generated by space-time itself. It does not exist in Einsteinian theory, and does not exist in Newtonian theory. It is a concept of the ECE unified field theory. The space-time angular momentum produces a negative valued potential energy:

$$U = -\frac{J^2}{2mr^2} \tag{1.7}$$

where m is the mass of a star moving in the spinning space-time and r is the radial distance of the star from the force centre (the centre of the space-time "whirlpool"). Work is done on the star by the spinning space-time and changes the star's potential energy from U_1 to U_2 while keeping the star's kinetic energy constant. In a whirlpool galaxy the stars move in a plane to a good approximation, so a star's kinetic energy is defined by:

$$T = \frac{1}{2} m v^2 \tag{1.8}$$

where the linear velocity in the plane is expressed [11] in terms of plane polar coordinates:

$$\mathbf{v} = \dot{r}\,\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta. \tag{1.9}$$

Therefore the kinetic energy of a star that moves in a plane with any velocity ($\mathbf{v}$) is:

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right). \tag{1.10}$$

The force on the star due to the constant angular momentum Eq. (1.7) of space-time is:

$$\int_1^2 \mathbf{F}\cdot d\mathbf{r} = U_1 - U_2 \tag{1.11}$$

and changes the star from state 1 to 2 while keeping the kinetic energy constant. This is the definition of potential energy [11]. If:

$$U_2 > U_1 \tag{1.12}$$

the force is attractive and negative valued. The initial state is chosen such that:

$$U_1 = 0. \tag{1.13}$$

The force on the star due to the spinning space-time is negative valued:

$$\mathbf{F} = -\boldsymbol{\nabla}U = -\frac{J^2}{mr^3}\mathbf{e}_r \tag{1.14}$$

and attracts the star into an orbit. It is shown as follows that this is a logarithmic spiral orbit.

The total energy of the system is:

$$E = T + U \tag{1.15}$$

and consists of the kinetic energy of the star moving at v in a plane:

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \tag{1.16}$$

and potential energy due to the spinning space-time:

$$U = -\frac{J^2}{2mr^2}. \tag{1.17}$$

The Lagrangian of the system is:

$$\mathscr{L} = E - U \tag{1.18}$$

and the Euler Lagrange equations of motion are:

$$\frac{\partial \mathscr{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{r}} \tag{1.19}$$

and

$$\frac{\partial \mathscr{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\theta}} = 0. \tag{1.20}$$

Eq. (1.19) can be rewritten [11] as:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{J^2}F(r) \tag{1.21}$$

by using a change of variable. If the potential energy Eq. (1.17) is expressed as:

$$U(r) = -\frac{J^2}{2mr^2}\left(1+\alpha^2\right) \tag{1.22}$$

the force is:

$$F(r) = -\frac{J^2}{mr^3}\left(1+\alpha^2\right) \tag{1.23}$$

and Eq. (1.21) shows that:

$$r = r_0 \exp(\alpha\theta). \tag{1.24}$$

This is a logarithmic spiral orbit as observed experimentally. The star evolves with time as follows:

$$\theta(t) = \frac{1}{2\alpha}\log\left(\frac{2\alpha J}{mr_0^2}t + C\right) \tag{1.25}$$

and

$$r(t) = \left(\frac{2\alpha J}{m}t + r_0^2 C\right)^{1/2} \tag{1.26}$$

where C is an integration constant. Both quantities (with all constants set to unity) are depicted in Figs. 1.1 and 1.2, showing their sublinear time dependence. The angular velocity is defined [11] as:

$$\omega = \dot{\theta} = \frac{d\theta}{dt} = \frac{J}{mr^2} \tag{1.27}$$

and the radial velocity is defined as:

$$v_r = \dot{r} = \frac{dr}{dt} = \frac{\alpha J}{mr}. \tag{1.28}$$

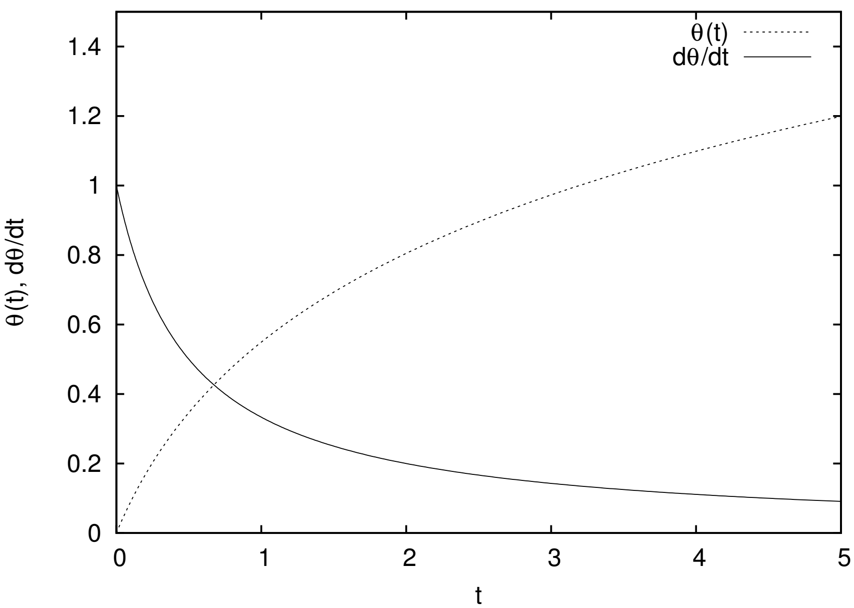

Figure 1.1: Time dependence of θ coordinate for a spiralling star.

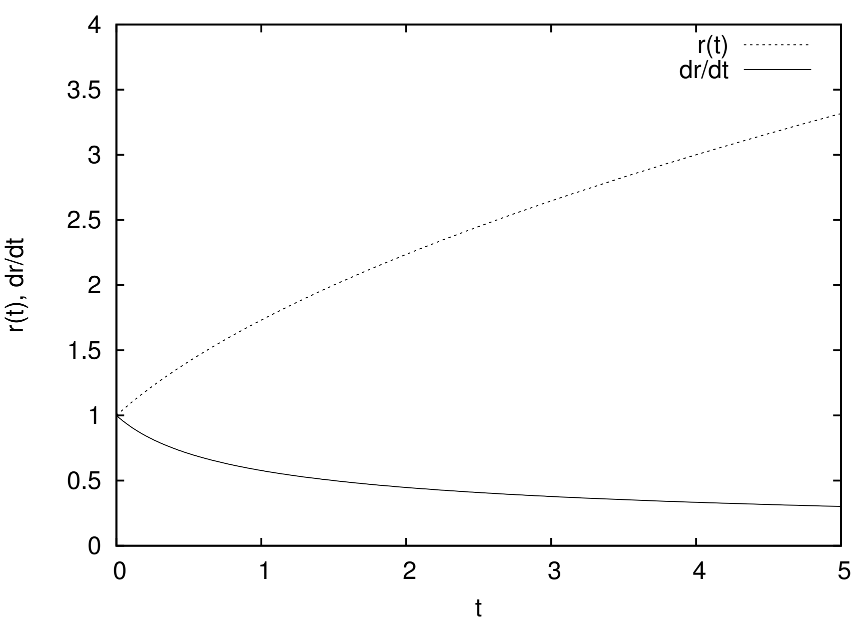

Figure 1.2: Time dependence of r coordinate for a spiralling star.

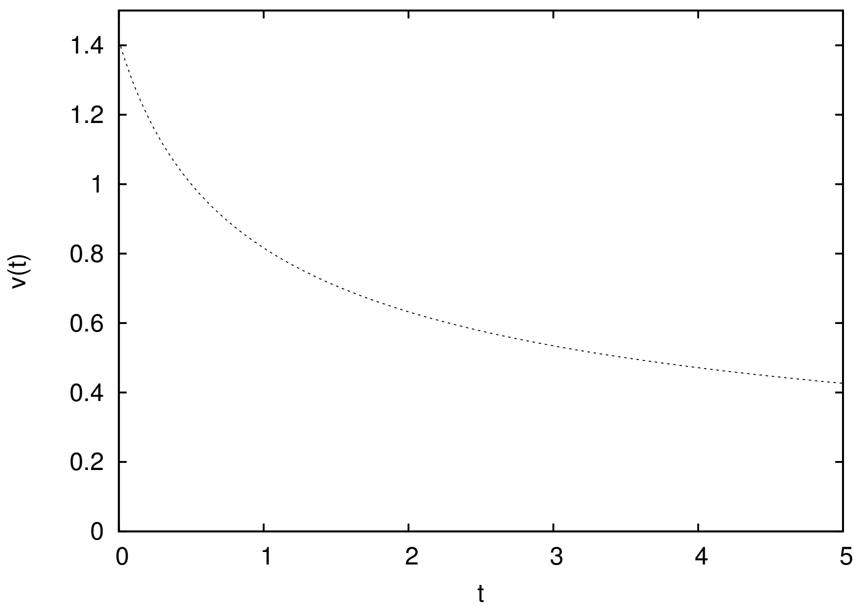

Figure 1.3: Time dependence of velocity v for a spiralling star.

The total velocity of the star is therefore defined by:

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 = \left(1+\alpha^2\right)\left(\frac{J}{mr}\right)^2 \tag{1.29}$$

(see Fig. 1.3) and so the angular momentum magnitude is:

$$J = \frac{mvr}{\left(1+\alpha^2\right)^{1/2}}. \tag{1.30}$$

It is observed in a whirlpool galaxy that v in the arms of the galaxy is a constant, and that the arms are logarithmic spirals of stars, i.e. stars moving on a logarithmic spiral. The angular momentum J is constant and is defined by the lagrangian:

$$J = \frac{\partial \mathscr{L}}{\partial \dot{\theta}} = -\frac{\partial U}{\partial \dot{\theta}} = mr^2\dot{\theta} \tag{1.31}$$

and so the angular momentum of the spinning space-time is related to the potential energy by:

$$J = -\frac{\partial U}{\partial \dot{\theta}} = -\frac{\partial}{\partial \dot{\theta}}\left(-\frac{1}{2}mr^2\dot{\theta}^2\right) \tag{1.32}$$

If v and J are constants then from Eq. (1.30), r is also constant, meaning that the orbit evolves to a circle. The Newtonian attraction of the star to the heavy mass at the centre of the galaxy is balanced by the centrifugal force of the spinning space-time which attracts the stars outwards. The angular momentum is constant and given by:

$$\frac{J}{mv} = \frac{r}{(1+\alpha^2)^{1/2}} = constant. \tag{1.33}$$

For each spiral of the galaxy, the parameter α is characteristic of that spiral, and the observed rotation curve is such that the velocity v is constant over large distances from the centre, meaning that the velocity v is much greater than that expected from Kepler's equation:

$$v^2 = \frac{k}{m}\left(\frac{2}{r} - \frac{1}{a}\right), \quad F = -\frac{k}{r^2}. \tag{1.34}$$

This fact is explained in this paper by an additional v due to Eq. (1.30), i.e. due to spinning space-time. The simplest model of this paper may be elaborated in many different ways.

1.3 Approximations for Angular Momentum and Velocity

The angular momentum J of spinning space-time is given by Eq. (1.30). J is assumed to be constant throughout the spiral arms of a galaxy. Experimentally it is found that the velocity v of stars in the galaxy arms is constant too. According to Eq. (1.30) then the spiral parameter α has to be variable with r. Assuming this, the potential U Eq. (1.22) takes the form

$$U_1(r) = -\frac{J^2\left(1+\alpha^2(r)\right)}{2mr^2}. \tag{1.35}$$

Correspondingly the force can be written

$$F_1(r) = -\frac{\partial U_1}{\partial r} = -\frac{J^2\left(1+\alpha^2(r)\right)}{mr^3} + \frac{J^2}{mr^2}\alpha(r)\alpha'(r) \tag{1.36}$$

with

$$\alpha'(r) = \frac{d\alpha(r)}{dr}. \tag{1.37}$$

In order to obtain the original force law which gives the logarithmic spiral orbits we define a potential

$$U_2(r) = -\frac{J^2\left(1+\alpha^2(r)\right)}{2mr^2} + \int \frac{J^2}{mr^2}\alpha(r)\ \alpha'(r)\ dr. \tag{1.38}$$

Then we get the original force

$$F_2(r) = -\frac{\partial U_2}{\partial r} = -\frac{J^2\left(1+\alpha^2(r)\right)}{mr^3} \tag{1.39}$$

of spiral orbits. Now we make the ansatz

$$\alpha(r) := \frac{r}{r_0} \tag{1.40}$$

with a characteristic length r_0. From Eq. (1.38) we then obtain

$$U_2(r) = -\frac{J^2}{2mr^2}\left(1 + \left(\frac{r}{r_0}\right)^2\right) + \frac{J^2 \log(r)}{mr_0^2}. \tag{1.41}$$

From the Lagrange Function

$$\mathscr{L} = T - U_2 \tag{1.42}$$

and the Lagrange Equation

$$\frac{\partial \mathscr{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{r}} = 0 \tag{1.43}$$

the radial part of the equation of motion becomes

$$m\ddot{r} = mr\dot{\theta} - \frac{J^2(1+\alpha^2(r))}{mr^3} + \frac{J^2\alpha(r)\alpha'(r)}{mr^2} \tag{1.44}$$

or with Eq. (1.40):

$$m\ddot{r} = mr\dot{\theta} - \frac{J^2\left(1 + \left(\frac{r}{r_0}\right)^2\right)}{mr^3} + \frac{J^2}{mr_0^2 r} \tag{1.45}$$

There is a strong centrifugal term proportional to $1/r$ now.

From Eq. (1.30) the angular momentum for a constant $v = v_0$ becomes

$$J = \frac{mv_0 r}{\left(1 + (\frac{r}{r_0})^2\right)^{1/2}} \tag{1.46}$$

(see Fig. 1.4) which in the limit $r \to \infty$ goes towards

$$J = mv_0 r_0. \tag{1.47}$$

The orbits can be derived from the Euler Lagrange equation. We make the ansatz

$$r = r_0 \exp(\alpha(r)\,\theta) \tag{1.48}$$

and will show that this fulfills the force law Eq. (1.39). From Eq. (1.48) follows

$$\frac{1}{r} = \frac{1}{r_0}\exp(-\alpha(r)\,\theta) \tag{1.49}$$

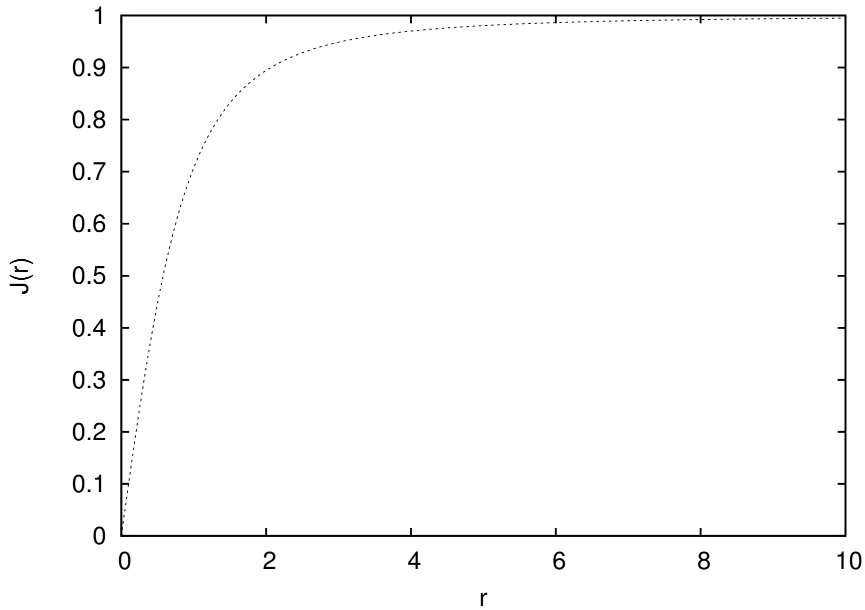

Figure 1.4: Angular momentum $J(r)$ for radius-dependent α.

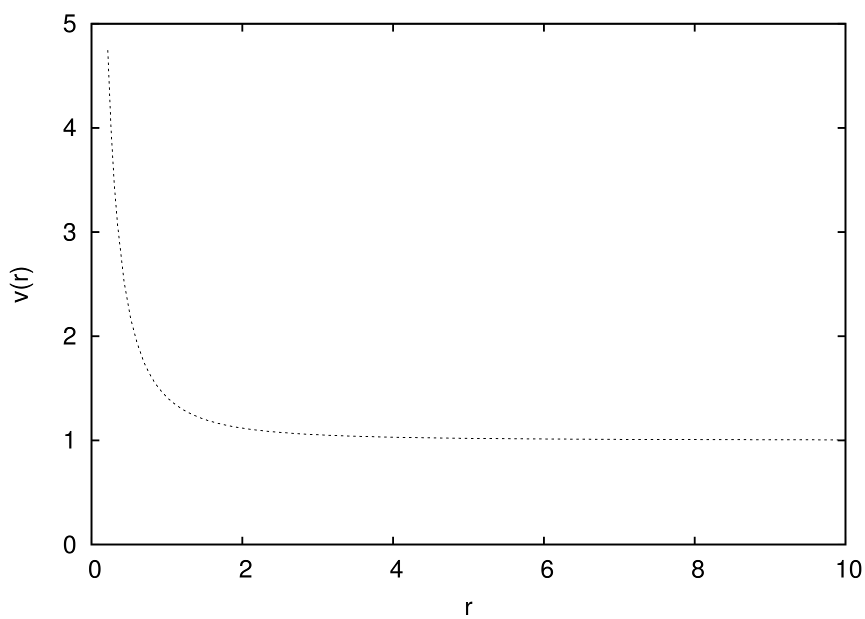

Figure 1.5: Velocity $v(r)$ for radius-dependent α.

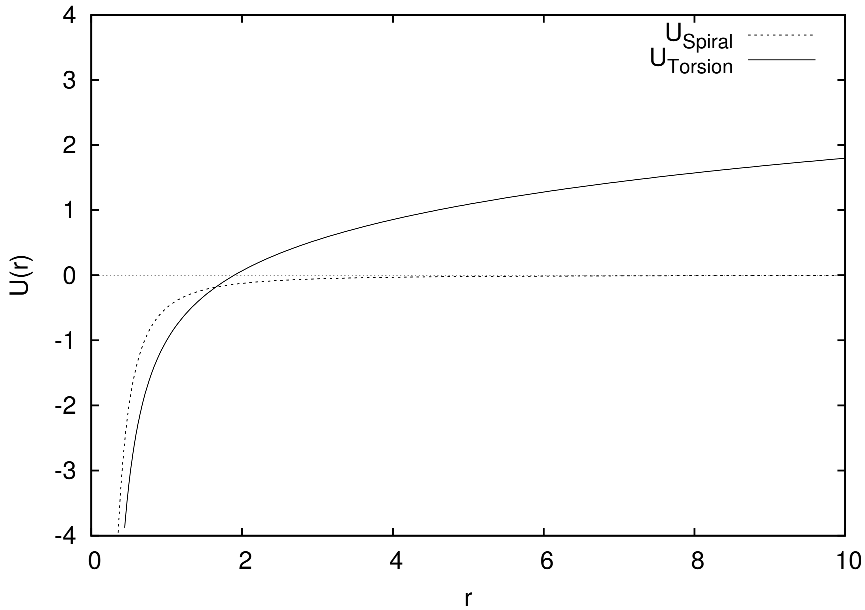

Figure 1.6: Potential for radius-dependent α, model of Eq. (1.40).

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{\alpha(r)}{r_0}\exp(-\alpha(r)\,\theta) \tag{1.50}$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{\alpha^2(r)}{r} \tag{1.51}$$

The Euler Lagrange equation is

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{J^2}F(r), \tag{1.52}$$

so

$$F(r) = -\frac{J^2}{mr^3}\left(1+\alpha^2(r)\right) \tag{1.53}$$

q.e.d.

The orbital velocity Eq. (1.29) is

$$v^2 = \left(\frac{J}{mr}\right)^2\left(1+\alpha^2(r)\right). \tag{1.54}$$

Inserting the above approach Eq. (1.40) for α leads to

$$v^2 = \left(\frac{J}{mr}\right)^2\left(1+\left(\frac{r}{r_0}\right)^2\right) \tag{1.55}$$

which again has a constant limit for large r:

$$v_0 = \frac{J}{m r_0} \tag{1.56}$$

and is consistent with Eq. (1.47). It is shown in Fig. 1.5. The logarithmic potential U_2 which is induced by constant angular momentum is compared in Fig. 1.6 with the pure spiral potential being proportional to $1/r^2$.

As an alternative approach, let's start directly with the condition that Eq. (1.30) is exactly constant:

$$J = \frac{m v_0 r}{\left(1+\alpha^2(r)\right)^{1/2}} = J_0 = const. \tag{1.57}$$

From this condition we obtain $\alpha(r)$ directly:

$$\alpha(r) = \sqrt{\frac{r^2}{r_0^2} - 1} \tag{1.58}$$

with

$$r_0 := \frac{J_0}{m v_0}. \tag{1.59}$$

Computeralgebra then delivers quite simple expressions for the potential and force law:

$$U_2 = m\, v_0^2 \left(\log(r) - \frac{1}{2}\right), \tag{1.60}$$

$$F_2 = -\frac{m\, v_0^2}{r}. \tag{1.61}$$

This is the potential and force law for spiralling orbits where angular momentum and orbital velocity are strictly constant. The velocity condition changes the spiral $1/r^3$ force law to a $1/r$ force law. The potential is a logarithmic function with a constant shift (see Fig. 1.7, in comparison to pure spiral potential). It is known that a $1/r$ force is longer reaching than a Newtonian $1/r^2$ force. This explains in a natural way why galaxies are developing spiral structures outside the central bulge region. Work done on the star implies a negative valued potential energy by convention, and an attractive force by convention. This attracts the star outwards from the centre of the galaxy. To obtain a constant velocity as observed, the second positive valued term of Eq. (1.38) is needed. This means that the galaxy develops spirals of stars which reach a constant velocity - the graph of velocity against r is a plateau.

In a third approach we start with the expression for the asymptotically contant velocity which according to Eq. (1.54) is

$$v^2 = \left(\frac{J_0}{mr}\right)^2 \left(1+\alpha^2(r)\right). \tag{1.62}$$

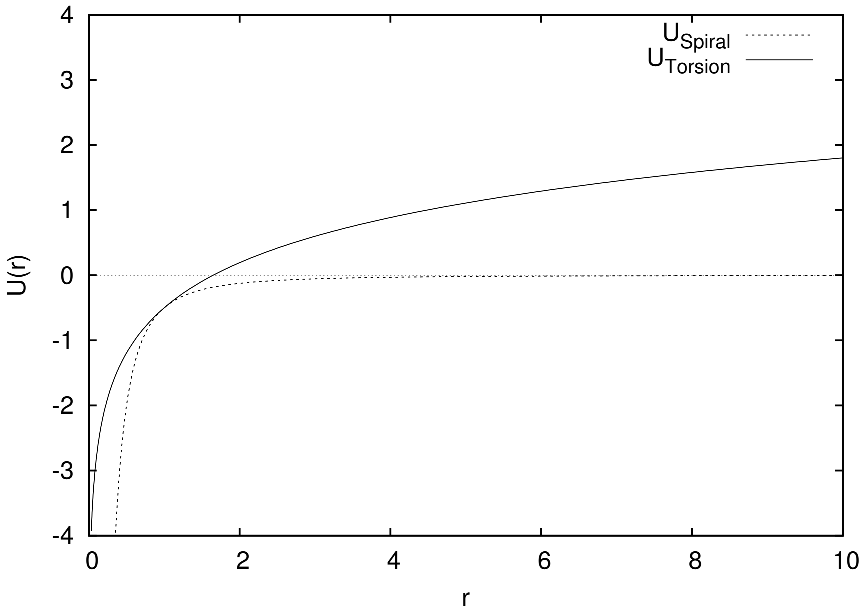

Figure 1.7: Potential for radius-dependent α, model of Eq. (1.58).

It is observed that in the limit $r \to \infty$ we have $v \to v_0$, so

$$\frac{1+\alpha^2(r)}{r^2} \to \frac{1}{r_0^2} \tag{1.63}$$

with

$$r_0 = \frac{J_0}{m v_0} \tag{1.64}$$

as in Eq. (1.59). The force Eq. (1.53) therefore becomes

$$F_2 = -\frac{m\,v_0^2}{r} \tag{1.65}$$

which is self-consistently the same result as Eq. (1.61).

1.4 Simulation of Dynamical Galaxy Behaviour

Finally we describe a method for numerical solution of the galaxy problem. According to [12] the equations of motion can be derived from the kinetic energy Eq. (1.16) and potential energy Eq. (1.60) via the Lagrange function Eq. (1.42). Besides the radial and angular coordinate, we introduce additional variables to

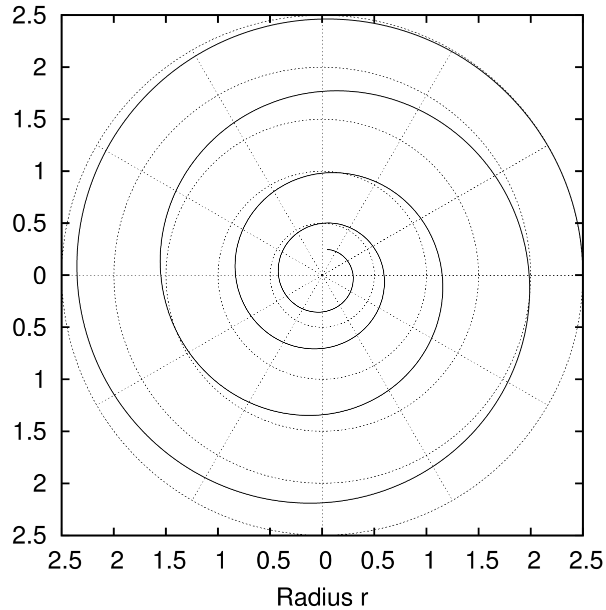

Figure 1.8: Orbit for $-1/r^3$ potential.

obtain differential equations of first order which can be solved by the Runge-Kutta method:

$$\dot{r} = v_r, \tag{1.66}$$

$$\dot{\theta} = \omega, \tag{1.67}$$

$$\dot{v}_r = r\omega^2 + \frac{1}{m}F(r), \tag{1.68}$$

$$\dot{v}_\theta = -2\frac{v_r}{r}\omega. \tag{1.69}$$

The first approach we analyse is the $1/r^3$ force law which should give spiralling orbits. The problem is that such orbits are only obtained for a negative force

$$F(r) = -\frac{1}{r^3}. \tag{1.70}$$

Then the orbits spiral inwards, not outwards as in galaxies. Taking the positive value of $F(r)$ does not give spiral orbits since the equations of motion do not exhibit mirror symmetry in space. Therefore we restrict to the statement that a time reversal $t \to -t$ together with a sign reversal of the force $F(r) \to -F(r)$ gives at least conceptually the desired behaviour. Fig. 1.8 shows the orbits of Eq. (1.70), Fig. 1.9 the velocity, angular momentum and total energy. The velocity is not constant because a force component in diretction of θ would be

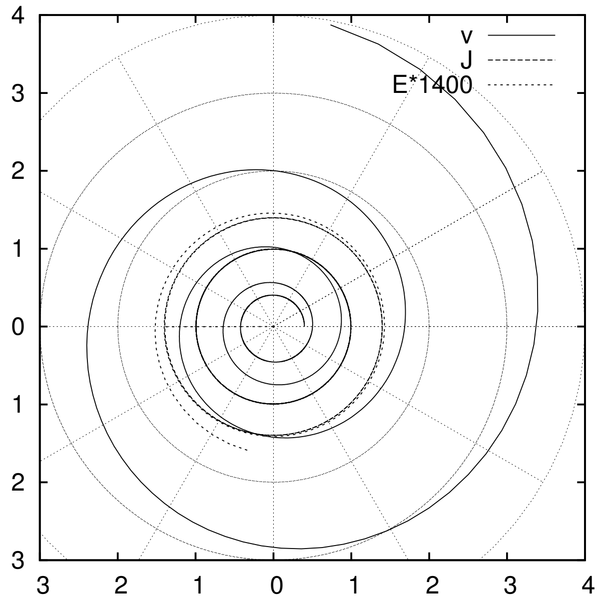

Figure 1.9: Velocity, angular momentum and total energy for $-1/r^3$ potential.

required to achieve this. Angular momentum and total energy are conserverd, with exception near to the end of the calculation where numerical errors become significant. This is because the radius of curvature of the orbit becomes very small near to the center.

The next example is more realistic. We added a Neutonian potential with a repulsive term γ/r

$$F(r) = \frac{\gamma}{r} - \frac{1}{r^2} \tag{1.71}$$

where γ has been adopted to a suitable value of 0.15775. In this combination we obtain outward-spiralling orbits with a limit of a straight line as is predicted by logarithmic spirals (Fig. 1.10). This is an indication that our result Eq. (1.61) is able to predict the correct orbitals in connection with Newtonian attraction in the inner region of a galaxy. From Fig. 1.11 the velocity components v_r and ω are shown. One sees that for $t \to \infty$ the angular component goes to zero and the radial component dominates.

It has to be noted finally that in none of the simulations the velocity is constant. The velocity is a dependent variable and is completely determined by the Eqs. (1.66-1.69). To enforce constancy of velocity we have to define

$$\omega = \frac{J_0}{mr^2} \tag{1.72}$$

from constancy of angular momentum J_0. Then ω is no more an independent

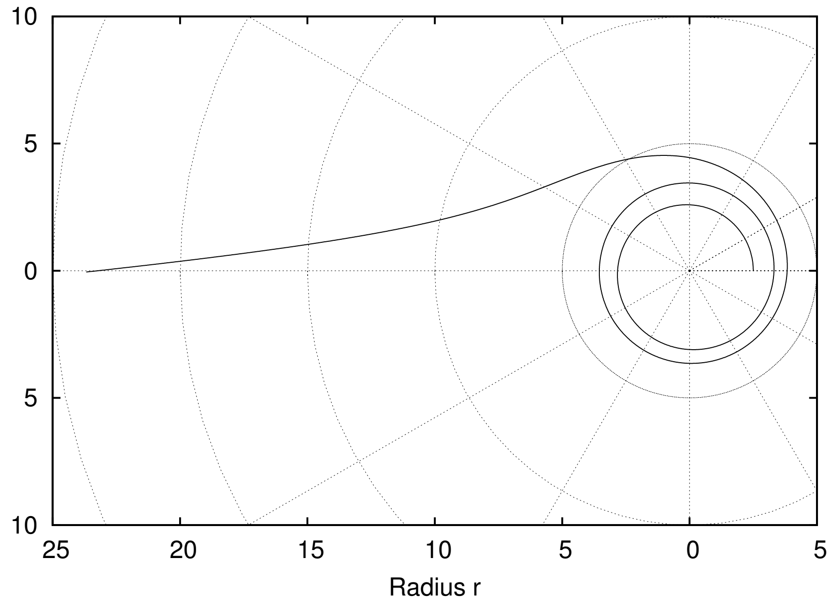

Figure 1.10: Orbit for potential of type $\gamma/r - 1/r^3$.

variable but defined by r. Therefore the equation system Eqs. (1.66- 1.69 would have to be modified.

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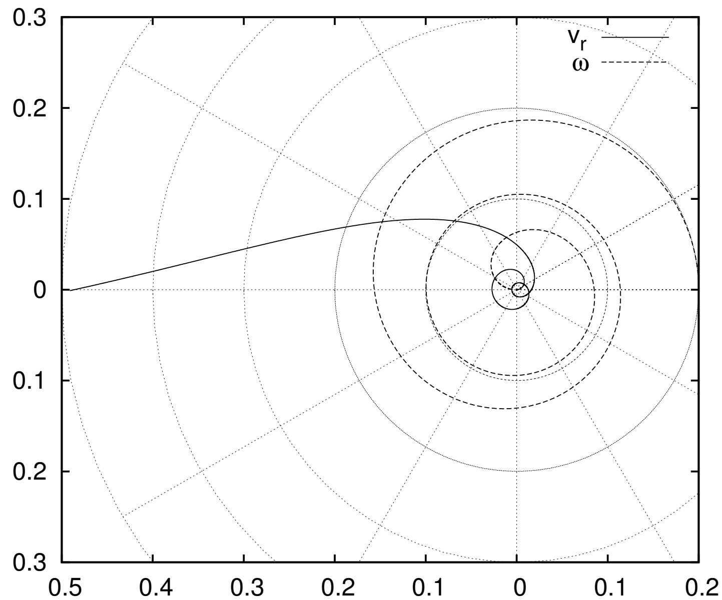

Figure 1.11: Velocity components for potential of type $\gamma/r - 1/r^3$.

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