

## Chapter 2

# The Link between Space-Time Torsion in ECE Theory and the Theory of Angular Momentum

by

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### Abstract

The link between space-time torsion and angular momentum is established by integration over a hyper-surface of the rank three angular energy momentum density tensor. The latter is assumed to be proportional by hypothesis to the rank three torsion tensor. Angular momentum theory is highly developed and appears for example in the theory of central orbits, so this link implies that ECE theory can be developed in terms of angular momentum theory, introducing several new techniques from several areas of physics.

Keywords: ECE theory, angular momentum theory, integration of a hyper-surface.

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## 2.1 Introduction

In the recently developed [1] - [10] Einstein Cartan Evans (ECE) field theory the role of space-time torsion is of key importance. It appears for example in the Cartan-Bianchi identity [11] and the Cartan-Evans dual identity [1] - [10]. A valid theory of relativity must obey both identities, and on these grounds the Einstein field equation has been rejected as incorrect. The torsion defines the acceleration due to gravity,  $\mathbf{g}$ , and the static electric field strength  $\mathbf{E}$ . More generally the torsion defines the electromagnetic field and the complete field of gravitation, encompassing the Newtonian  $\mathbf{g}$  and other fields such as the gravitomagnetic field. Such phenomena as the equinoctial precession have been shown to be due to the gravitomagnetic field. The ECE equations of electrical engineering and cosmology are based on the Cartan Bianchi identity and the Cartan Evans dual identity, together with the Cartan Maurer structure equation [1] - [11] that defines torsion as the covariant exterior derivative of the Cartan tetrad. In cosmology, the torsion defines central orbits, and more generally, orbits of all kinds. It is therefore important to develop the theory of spacetime torsion. In Section 2, the rank three spacetime torsion tensor  $T^{\kappa\mu\nu}$  [1] - [10] is assumed by hypothesis to be proportional to the rank three angular *energy/momentum* tensor density,  $J^{\kappa\mu\nu}$ , which is defined [12] from the canonical energy momentum density tensor as is well known in field theory. The tensor  $J^{\kappa\mu\nu}$  can be integrated over a hypersurface [12] to give a rank two tensor  $J^{\mu\nu}$ .

Integration over volume [12] gives the well known rank two, antisymmetric, angular momentum tensor:

$$J^{\mu\nu} = -J^{\nu\mu} = \int J^{0\mu\nu} dV. \quad (2.1)$$

The theory of angular momentum is well developed [1] - [10], [12] and the procedure given in this paper therefore links ECE theory with angular momentum theory, simplifying the derivation of the ECE engineering model [1] - [10] and introducing concepts about angular momentum from several other areas of physics, both classical and quantum.

## 2.2 The Link Between Torsion and Angular Momentum

The ECE theory and engineering model can be simplified and strengthened by developing it in terms of rank two tensors derived from the general rank three tensor of spacetime torsion. The complete information is still contained in the rank three tensor, but for practical purposes the two index tensors are sufficient in general. A three index tensor density may always be integrated over a hypersurface to give a rank two tensor [12]. For example, the conserved charge in the usual theory of the Noether Theorem [12] is defined by:

$$Q_\nu = \int_\sigma J_\nu^\mu d\sigma_\mu \quad (2.2)$$

where the integral is over a spacelike hypersurface  $\sigma_\mu$ . This integration gives a rank one tensor form a rank two tensor. In the same way as a two-dimensional surface can be defined in 3-D Euclidean space, a three dimensional hypersurface can be defined in a 4 - D spacetime. Thus:

$$Q_\nu = \int J_\nu^0 dV \quad (2.3)$$

in which:

$$\mu = 0 \quad (2.4)$$

and

$$dV = d\sigma_0 \quad (2.5)$$

where  $V$  denotes volume in cubic metres.

The rank three angular momentum/energy density tensor is defined [12] as:

$$J^{\mu\rho\sigma} = -\frac{1}{2}(T^{\mu\rho}x^\sigma - T^{\mu\sigma}x^\rho) \quad (2.6)$$

where  $T^{\mu\rho}$  (not to be confused with torsion) denotes the symmetric canonical energy momentum density tensor and  $x^\mu$  is the coordinate vector:

$$x^\mu = (ct, X, Y, Z). \quad (2.7)$$

As argued in previous work [1] - [10]:

$$T^{\kappa\mu\nu} = \frac{k}{c} J^{\kappa\mu\nu} \quad (2.8)$$

where  $T^{\kappa\mu\nu}$  is the spacetime torsion tensor,  $k$  is Einstein's constant:

$$k = \frac{8\pi G}{c^2} = 1.86595 \times 10^{-26} \text{ m kgm}^{-1} \quad (2.9)$$

and  $c$  is the vacuum speed of light. The angular momentum tensor is defined in field theory [12] by:

$$J^{\mu\nu} = \int_\sigma J^{\kappa\mu\nu} d\sigma_\kappa. \quad (2.10)$$

Similarly define:

$$T^{\mu\nu} = \int_\sigma T^{\kappa\mu\nu} d\sigma_\kappa \quad (2.11)$$

which is an anti-symmetric tensor with the units of torsion ( $m^{-1}$ ) multiplied by cubic metres. Thus:

$$J^{\mu\nu} = \frac{c}{k} T^{\mu\nu}. \quad (2.12)$$

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The electromagnetic field tensor in ECE theory is in general a rank three tensor, but may be expressed as a rank two tensor: and related directly to angular momentum:

$$F^{\mu\nu} = \frac{A^0}{V} T^{\mu\nu} = A^{(0)} \int T^{0\mu\nu} dV. \quad (2.13)$$

The theory of angular momentum is highly developed, and angular momentum operators [1] - [10], [12] are infinitesimal generators within  $\hbar$ , the reduced Planck constant. The electromagnetic field tensor is the following volume integral over the electromagnetic tensor density, the latter being the most general expression of the electromagnetic field in relativity:

$$F^{\mu\nu} = \int F^{0\mu\nu} dV. \quad (2.14)$$

Similarly the Hodge dual field tensor is:

$$\tilde{F}^{\mu\nu} = \int \tilde{F}^{0\mu\nu} dV. \quad (2.15)$$

The homogeneous and inhomogeneous field equations of ECE theory are based on the Cartan Bianchi identity:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^\kappa{}_\mu{}^{\mu\nu} \quad (2.16)$$

and the Cartan Evans dual identity:

$$D_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu}. \quad (2.17)$$

The two identities may be integrated over volume on both sides as follows:

$$\int D_\mu \tilde{T}^{0\mu\nu} dV = \int \tilde{R}^0{}_\mu{}^{\mu\nu} dV \quad (2.18)$$

and

$$\int D_\mu T^{0\mu\nu} dV = \int R^0{}_\mu{}^{\mu\nu} dV. \quad (2.19)$$

The covariant derivatives are defined by [1] - [10]:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \partial_\mu \tilde{T}^{\kappa\mu\nu} + \omega^\kappa{}_{\mu\lambda} \tilde{T}^{\lambda\mu\nu} \quad (2.20)$$

and

$$D_\mu T^{\kappa\mu\nu} = \partial_\mu T^{\kappa\mu\nu} + \omega^\kappa{}_{\mu\lambda} T^{\lambda\mu\nu}. \quad (2.21)$$

Therefore:

$$\partial_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{j}^{\kappa\nu} \quad (2.22)$$

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \quad (2.23)$$

where the currents are defined by:

$$\tilde{j}^{\kappa\nu} = \tilde{R}^\kappa{}_\mu{}^{\mu\nu} - \omega^\kappa{}_{\mu\lambda} \tilde{T}^{\lambda\mu\nu} \quad (2.24)$$

$$j^{\kappa\nu} = R^\kappa{}_\mu{}^{\mu\nu} - \omega^\kappa{}_{\mu\lambda} T^{\lambda\mu\nu}. \quad (2.25)$$

Now use:

$$T^{\mu\nu} = \int_\sigma T^{\kappa\mu\nu} d\sigma_\kappa \quad (2.26)$$

and

$$j^\nu = \int_\sigma j^{\kappa\nu} d\sigma_\kappa \quad (2.27)$$

and similarly for the homogeneous equation 2.24. The hypersurface is defined by:

$$\sigma_\kappa = (\sigma_0, -\boldsymbol{\sigma}) = (V, -\boldsymbol{\sigma}) \quad (2.28)$$

where the volume is defined by its zero'th element:

$$V = \sigma_0. \quad (2.29)$$

Therefore:

$$T^{\mu\nu} = \int T^{0\mu\nu} dV \quad (2.30)$$

and

$$j^\nu = \int j^{0\nu} dV. \quad (2.31)$$

Thus:

$$T^{0\mu\nu} = \frac{T^{\mu\nu}}{V}, \quad (2.32)$$

$$j^{0\nu} = \frac{j^\nu}{V}. \quad (2.33)$$

With these definitions, Eqs. (2.22) and (2.23) reduce to:

$$\partial_\mu \tilde{T}^{\mu\nu} = \tilde{j}^\nu, \quad (2.34)$$

$$\partial_\mu T^{\mu\nu} = j^\nu, \quad (2.35)$$

where:

$$j^\nu = V(R^0{}_\mu{}^{\mu\nu} - \omega^0{}_{\mu\lambda} T^{\lambda\mu\nu}), \quad (2.36)$$

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$$\tilde{j}^\nu = V(\tilde{R}^0{}_\mu{}^{\mu\nu} - \omega^0{}_{\mu\lambda}\tilde{T}^{\lambda\mu\nu}). \quad (2.37)$$

In electrodynamics:

$$\partial_\mu \tilde{F}^{\mu\nu} = A^{(0)}\tilde{j}^\nu, \quad (2.38)$$

$$\partial_\mu F^{\mu\nu} = A^{(0)}j^\nu, \quad (2.39)$$

and if there is no magnetic monopole:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0, \quad (2.40)$$

$$\partial_\mu F^{\mu\nu} = A^{(0)}j^\nu. \quad (2.41)$$

In vector notation, Eq. (2.40) becomes the homogeneous field equations:

$$\nabla \cdot \mathbf{B} = 0 \quad (2.42)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \quad (2.43)$$

and Eq. (2.41) becomes the inhomogeneous field equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (2.44)$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad (2.45)$$

These are the generally covariant equations of classical electrodynamics. The charge density in coulombs per cubic metre is:

$$\rho = c\epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 0} - \omega^0{}_{\mu\lambda}T^{\lambda\mu 0}) \quad (2.46)$$

where the units of  $cA^{(0)}$  are volts (joules per coulomb) and where the S.I. units of vacuum permittivity,  $\epsilon_0$ , are  $J^{-1}C^2m^{-1}$ . The current density is defined as:

$$\mathbf{J} = J_X \mathbf{i} + J_Y \mathbf{j} + J_Z \mathbf{k} \quad (2.47)$$

where:

$$J_x = \epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 1} - \omega^0{}_{\mu\lambda}T^{\lambda\mu 1}) \quad (2.48)$$

$$J_y = \epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 2} - \omega^0{}_{\mu\lambda}T^{\lambda\mu 2}) \quad (2.49)$$

$$J_z = \epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 3} - \omega^0{}_{\mu\lambda}T^{\lambda\mu 3}) \quad (2.50)$$

The index  $\lambda$  is restricted to 0 in Eq. (2.48) to 2.50 because of Eqs. (2.34) and (2.35) so:

$$\rho = c\epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 0} - \omega^0{}_{\mu 0}T^{0\mu 0}) \quad (2.51)$$

$$J_X = \epsilon_0 A^{(0)}(R^0{}_\mu{}^{\mu 1} - \omega^0{}_{\mu 0}T^{0\mu 1}) \quad (2.52)$$

$$J_Y = \epsilon_0 A^{(0)} (R^0_{\mu}{}^{\mu 2} - \omega^0_{\mu 0} T^{0\mu 2}) \quad (2.53)$$

$$J_Z = \epsilon_0 A^{(0)} (R^0_{\mu}{}^{\mu 3} - \omega^0_{\mu 0} T^{0\mu 3}) \quad (2.54)$$

It is seen that this method of deducing the vector structure of the ECE engineering model uses:

$$\kappa = 0 \quad (2.55)$$

throughout and so simplifies previous work, in which  $\kappa$  was varied. Charge and current densities are defined by combinations of curvature, spin connection and torsion.

The field potential equations of the ECE engineering model are of key importance because they introduce the spin connection of general relativity [1] - [10], allowing spin connection resonance to occur. These field potential equations are based on the Cartan Maurer structure equation:

$$T^a_{\mu\nu} = \partial_\mu q_\nu^a - \partial_\nu q_\mu^a + \omega^a_{\mu b} q_\nu^b - \omega^a_{\nu b} q_\mu^b. \quad (2.56)$$

Define the tensor:

$$T_{\mu\nu} = \int_\sigma T^a_{\mu\nu} d\sigma_a \quad (2.57)$$

where the hypersurface is:

$$\sigma_a = (\sigma_0, -\boldsymbol{\sigma}) \quad (2.58)$$

in the Minkowski spacetime labelled by a. When:

$$a = 0 \quad (2.59)$$

then:

$$T_{\mu\nu} = \int T^0_{\mu\nu} dV \quad (2.60)$$

and similarly:

$$q_\mu = \int q_\mu^0 dV. \quad (2.61)$$

Thus:

$$T^0_{\mu\nu} = \frac{T_{\mu\nu}}{V}, q_\mu^0 = \frac{q_\mu}{V}. \quad (2.62)$$

Therefore Eq. (2.56) simplifies to:

$$T_{\mu\nu} = \partial_\mu q_\nu - \partial_\nu q_\mu + \omega_{\mu b} q_\nu^b - \omega_{\nu b} q_\mu^b. \quad (2.63)$$

Finally define:

$$\omega_\mu q_\nu = \omega_{\mu b} q_\nu^b. \quad (2.64)$$

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This means that the  $b$  index is restricted to 0 because:

$$q_\nu = Vq_\nu^0 \quad (2.65)$$

and

$$\omega_\mu = \frac{\omega_{\mu 0}}{V}. \quad (2.66)$$

Therefore:

$$T_{\mu\nu} = \partial_\mu q_\nu - \partial_\nu q_\mu + \omega_\mu q_\nu - \omega_\nu q_\mu \quad (2.67)$$

$$= (\partial_\mu + \omega_\mu)A_\nu - (\partial_\nu + \omega_\nu)A_\mu \quad (2.68)$$

and:

$$F_{\mu\nu} = (\partial_\mu + \omega_\mu)A_\nu - (\partial_\nu + \omega_\nu)A_\mu \quad (2.69)$$

where:

$$\partial_\mu = \left( \frac{1}{c} \frac{\partial}{\partial t}, \nabla \right), \quad (2.70)$$

$$A_\nu = \left( \frac{\phi}{c}, -\mathbf{A} \right), \quad \omega_\mu = \left( \frac{\omega_0}{c}, -\boldsymbol{\omega} \right). \quad (2.71)$$

Therefore we obtain the electric and magnetic fields of the ECE engineering model in terms of potentials and spin connections, Q.E.D.:

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} + \boldsymbol{\omega}\phi - \omega_0\mathbf{A}, \quad (2.72)$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}. \quad (2.73)$$

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