Chapter 4

Orbital Dynamics in Terms of Spacetime Angular Momentum

by

Myron W. Evans¹ and H. Eckardt²

Alpha Institute for Advanced Study (AIAS)

Abstract

Planar orbital theory is developed from the concept of the conserved angular momentum of spacetime in generally covariant Einstein Cartan Evans (ECE) Theory. It is shown that spacetime angular momentum is sufficient to describe all known planar orbits without using the non-relativistic Newtonian concept of attraction between two massive objects. The theory is applied to the dynamics of galaxies. Different types of planar orbits observed in galaxies can be described by the same conserved spacetime angular momentum. Example given are the relativistic Keplerian and logarithmic spiral orbits of a whirlpool galaxy. Spacetime angular momentum is defined by a volume integration over spacetime torsion and is a concept unique to ECE theory.

Keywords: Planar orbital theory, spacetime angular momentum, ECE theory, galactic dynamics.

¹e-mail: emyrone@aol.com
²e-mail: horsteck@aol.com
4.1 Introduction

The classical and non-relativistic description of planar orbits was initiated by Kepler from analysis of the orbit of Mars using observations by Brahe. From these data Kepler deduced that the orbit was an ellipse, and formulated three planar orbital laws as is well known [1]. Newton deduced that the three orbital laws could be described by the hypothesis that two masses attract in inverse square proportion to the distance between them, the constant of proportionality being the Newton constant G of universal gravitation. Later Einstein developed a generally covariant theory of gravitation using the curvature of spacetime and a field equation given independently by Einstein and Hilbert in 1915. A line element method, incorrectly attributed to Schwarzschild’s 1916 solutions of the Einstein Hilbert (EH) equations, was then used to claim that the EH field equation describes the experimentally precession of elliptical orbits in a plane. This precession is well known to be observed in the solar system and in binary pulsars for example. We refer to such orbits as relativistic Keplerian orbits, because the problem is traditionally known as the relativistic Kepler problem.

However, the point of view represented by the EH field equation is now rejected by the ECE school of physics, which has shown [2] - [11] that the EH field equation is geometrically incorrect because of its neglect of spacetime torsion and because of its use of a symmetric connection. It has been shown that the commutator of covariant derivatives [12] acting on any tensor in any spacetime always produces an anti-symmetric connection. This result is the direct consequence of the correct consideration of spacetime torsion. Therefore no deductions in physics can be based on the EH field equation. The ECE School of thought has subsequently developed physics in many new ways (www.aias.us and www.atomicprecision.com). For example the line element needed for the description of the relativistic Keplerian orbits has been deduced from a simple orbital theorem (paper 111 of the ECE series on www.aias.us) without using the EH field equation at all. Similarly, the orbits of binary pulsars have been described without use of EH type gravitational radiation (ECE paper 108). All solutions of the EH field equation have been shown to violate basic geometry and therefore to be incorrect (for example papers 93, 95, 118, 120 and 122). In consequence big bang, black hole and dark matter theory has been rejected by the ECE school of thought in physics. The concept of spacetime angular momentum has been developed from the fundamental concept of spacetime torsion in Cartan geometry [2] - [12].

In Section 2 it is shown that the single idea of conserved spacetime angular momentum is sufficient to describe all planar orbits. In mathematical terms, any closed planar orbit is due to conserved spacetime angular momentum, and not due to the non-relativistic idea of a balance between inverse square attraction and centrifugal repulsion. In physics, only certain types of planar orbit appear to exist, the best known being the elliptical orbit of Kepler and Newton. This is the non-relativistic limit of the precessing elliptical orbit of the relativistic Kepler problem worked out in papers 108 and 111 from the orbital theorem of ECE theory and without using the EH equation at all. In binary pulsars the
precessing ellipse spirals inwards, and this was worked out in ECE theory in paper 108 - again without using the incorrect EH equation and without using the incorrect idea of gravitational radiation. In papers such as 93, 95, 118 and 120 it has been shown that all metrics of the EH equation are incorrect geometrically - they violate the Cartan Evans dual identity because of their neglect of spacetime torsion [1] - [11]. In galaxies, stars orbit a galactic centre, and they do so in several different types of observed planar orbit. In a whirlpool galaxy for example, the stars form a logarithmic spiral pattern in space, and the velocity of a star in one of these spirals becomes constant, irrespective of the stellar distance from the centre of the galaxy. This is completely different from Newtonian dynamics, whereas the relativistic deviations from Newtonian dynamics in the solar system, for example, are very small. The now obsolete "standard" school of physics has compounded its many basic errors made repeatedly over many years, by attributing these logarithmic spirals of stars to a pre-Baconian concept of dark matter. Such a concept must be rejected in natural philosophy because it is not based on any post Baconian principle such as general relativity. Dark matter is the archetypical idol of the cave, it is a figment of imagination, and data are used to bolster the human fantasy. The scientific method is to use well established principles such as general relativity to describe data, and to develop theories within the framework of general relativity, a rigorously objective framework based on geometry and independent of fantasy (subjectivity or anthropomorphic distortion). In doing this a correct and self consistent geometry must be used and any self consistent geometry must include the torsion of spacetime as well as the curvature of spacetime. Therefore in Section 2 it is shown straightforwardly that only one concept is needed for any planar orbit, the conservation of spacetime angular momentum. This ECE based theory is therefore preferred by Ockham’s Razor and by the philosophy of relativity, as well as by observation, the Baconian principle of natural philosophy (physics). The great Einsteinian theory may therefore be regarded now as a monumental advance in human thought, but flawed. The flaw in it is the neglect of spacetime torsion. Einstein was correct in attributing orbits to paths or geodesics, and not to central attraction balanced by centrifugal repulsion. ECE is an extension of the Einsteinian thought to describe planar orbits as paths defined by conservation of spacetime angular momentum. This is of course a generally covariant concept as required by the need for rigorous objectivity in natural philosophy, the need to eliminate anthropomorphic distortion, Bacon’s idol of the cave. Dark matter has no basis in Keplerian, Baconian, Newtonian or Einsteinian thought, whereas ECE is an evolutionary outcome of hundreds of years of development in natural philosophy.

Finally in Section 3, the concept of free fall of an object out of orbit is discussed without using the idea of inverse square attraction. Similarly in ECE electrodynamics the Coulomb law may be given an entirely new and more rigorous philosophical basis than hitherto available. After all, physics is the philosophy of nature and there can be more than one ("standard") description of nature, especially if the "standard" description of general relativity (EH description) is riddled with errors as has been well known to critical scholars (see www.aias.us)
for almost a century. Physics is therefore a discussion between schools of philosophy, and the testing of ideas with experimental data. The simplest theory is preferred by Ockham’s Razor of philosophy, the oldest principle of physics.

4.2 Conservation of the Angular Momentum of Spacetime

Denote the spacetime angular momentum by the vector $L$. It has been shown in previous work [2] - [11] that $L$ may be obtained by a volume integration over spacetime torsion, a basic geometrical concept. The orbital angular momentum $L$ is conserved [1] for planar orbits:

$$\frac{\partial L}{\partial t} = 0$$

(4.1)

and is defined as the cross product of the radial vector $r$ and the orbital angular momentum $p$:

$$L = r \times p = \text{constant.}$$

(4.2)

In Cartesian coordinates:

$$L = m(r_X V_Y - r_Y V_X)k$$

(4.3)

where $m$ is the mass of an orbiting object and where $k$ is the unit Cartesian vector in the $Z$ axis. The orbit of an object $m$ in a plane is therefore a path in spinning spacetime, a geodesic point of view, not a Newtonian point of view. It is convenient to develop the argument in plane polar coordinates defined in Fig. 4.1.

The unit vectors in plane polar coordinates are related to the Cartesian unit vectors as follows:

$$e_r = i \cos \theta + j \sin \theta$$

(4.4)
\[ e_\theta = -i \sin \theta + j \cos \theta \] (4.5)  

and the radial vector and linear velocity are [1]:  
\[ r = re_r \] (4.6)  
\[ v = \dot{r}e_r + r\dot{\theta}e_\theta. \] (4.7)  

Therefore:  
\[ r = re_r = r_Xi + r_Yj \] (4.8)  

where  
\[ r_X = r \cos \theta, \quad r_Y = r \sin \theta. \] (4.9)  

Therefore the total orbital angular momentum in plane polar coordinates is:  
\[ L = m \begin{vmatrix} e_r & e_\theta & k \\ r & 0 & 0 \\ \dot{r} & r\dot{\theta} & 0 \end{vmatrix} = mr^2\dot{\theta}k \] (4.10)  

and the total linear velocity of the orbiting object of mass \( m \) is:  
\[ v^2 = v_X^2 + v_Y^2 = \dot{r}^2 + r^2\dot{\theta}^2. \] (4.11)  

It may be checked that:  
\[ e_r \times e_\theta = k \] (4.12)  

and  
\[ v = (\dot{r} \cos \theta - r\dot{\theta} \sin \theta) i + (\dot{r} \sin \theta + r\dot{\theta} \cos \theta) j = v_Xi + v_Yj. \] (4.13)  

Therefore:  
\[ r_X = r \cos \theta, \quad r_Y = r \sin \theta, \] (4.14)  
\[ v_X = \dot{r} \cos \theta - r\dot{\theta} \sin \theta, \quad v_Y = \dot{r} \sin \theta + r\dot{\theta} \cos \theta, \] (4.15)  

and, self consistently,  
\[ L = mr(\cos \theta(\dot{r} \sin \theta + r\dot{\theta} \cos \theta) - \sin \theta(\ddot{r} \cos \theta - r\ddot{\theta} \sin \theta)) = mr^2\dot{\theta}. \] (4.16)  

This is a geometrical result of the definition (Eq. 4.2). Therefore total spacetime orbital angular momentum is always defined in plane polar coordinates by:  
\[ L = mr^2\dot{\theta}. \] (4.17)  

The total linear velocity of the object is defined by:  
\[ v = (v_r^2 + v_\theta^2)^{1/2} = (\dot{r}^2 + r^2\dot{\theta}^2)^{1/2} \] (4.18)
and contains time variations both of \( r \) and \( \theta \). The total spacetime orbital angular momentum of any orbit in a plane is always given by Eq. (4.17). Therefore all orbits in a plane have the same spacetime angular momentum \( L \) and are in this sense degenerate. The orbits of physical interest are those observed in astronomy, and the most well known orbit is the Keplerian ellipse defined by:

\[
    r(t) = \frac{\alpha}{1 + \epsilon \cos \theta(t)} \quad (4.19)
\]

where \( \alpha \) and \( \epsilon \) are constants. Both \( r \) and \( \theta \) are functions of time. The orbit (Eq. 4.19) is a path in spinning spacetime due to the constant of motion (Eq. 4.17). This concept exists neither in Einsteinian nor Newtonian dynamics. Differentiating (Eq. 4.19):

\[
    \dot{r}(t) = \frac{\epsilon}{\alpha} r \dot{\theta}(t) \sin \theta(t) \quad (4.20)
\]

Using Eq. (4.20) the \( X \) and \( Y \) components of total orbital velocity may be calculated using:

\[
    v_X = \dot{r} \cos \theta - r \dot{\theta} \sin \theta, \quad (4.21)
\]

\[
    v_Y = \dot{r} \sin \theta + r \dot{\theta} \cos \theta, \quad (4.22)
\]

Therefore for a Keplerian orbit:

\[
    v_X = v_\theta \sin \theta \left( \frac{\epsilon}{\alpha} r \cos \theta - 1 \right), \quad (4.23)
\]

\[
    v_Y = v_\theta \left( \frac{\epsilon}{\alpha} r \sin^2 \theta \cos \theta \right). \quad (4.24)
\]

in which the angular momentum is the constant of spacetime motion defined by Eq. (4.17). All that is needed for any planar orbit is the constant spacetime angular momentum \( 4.17 \), the observed Keplerian orbit being one example. The reason why the observed orbit is Keplerian in the ECE physics is given by the orbital theorem of paper 111 on www.aias.us. The orbital theorem gives the line element of the relativistic Kepler problem without any use of the incorrect Einstein field equation, and the Kepler orbit (the ellipse) is a well defined mathematical limit of the relativistic Kepler orbit (the precessing ellipse). The circular orbit is the limit:

\[
    \epsilon \to 0 \quad (4.25)
\]

of the elliptical orbit [1]. All these orbits are described by the same spacetime angular momentum \( L \) defined by Eq. (4.17) and are caused by conserved spacetime angular momentum. This is the simplest example of the ECE engineering model [2] - [11], the example where:

\[
    \frac{\partial L}{\partial t} = 0, \quad (4.26)
\]
\n
\[ \nabla \cdot \mathbf{L} \neq 0. \tag{4.27} \]

In general the ECE engineering model contains several other equations of generally covariant dynamics and most generally the equations must be solved numerically [2] – [11].

In a planar whirlpool galaxy the stars are arranged on a logarithmic spiral of type:

\[ r(t) = r_0 \exp(b\theta(t)) \tag{4.28} \]

so by differentiation:

\[ \dot{r} = bV_\theta. \tag{4.29} \]

The dynamical problem is defined by:

\[ L = mr^2 \dot{\theta}, \tag{4.30} \]

\[ v_X = \dot{r} \cos \theta - r \dot{\theta} \sin \theta, \tag{4.31} \]

\[ v_Y = \dot{r} \sin \theta + r \dot{\theta} \cos \theta, \tag{4.32} \]

so for a logarithmic spiral orbit:

\[ v_X = v_0 (b \cos \theta - \sin \theta), \tag{4.33} \]

\[ v_Y = v_0 (b \sin \theta + \cos \theta), \tag{4.34} \]

and the total velocity is:

\[ v = r \dot{\theta} (1 + b^2)^{\frac{1}{2}}. \tag{4.35} \]

By experimental observation the velocity \( v \) of a star becomes constant as \( r \) becomes very large:

\[ v \to v_0, \tag{4.36} \]

\[ r \to \infty. \tag{4.37} \]

So:

\[ \dot{\theta} (1 + b^2)^{\frac{1}{2}} \to 0. \tag{4.38} \]

The relativistic Kepler orbit is a precessing ellipse described [1] to a good approximation by:

\[ r(t) = \alpha \left( 1 + \epsilon \cos((1 - \frac{\beta}{\alpha})\theta(t)) \right)^{-1} \tag{4.39} \]

where \( \alpha \) and \( \beta \) are constants. In this case:

\[ v_X = v_0 \left( \frac{c r}{\alpha} (1 - \frac{\beta}{\alpha}) \sin((1 - \frac{\beta}{\alpha})\theta) \cos \theta - \sin \theta, \right) \tag{4.40} \]
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\[ v_Y = v_\theta \left( \frac{er}{\alpha} \left( 1 - \frac{\beta}{\alpha} \right) \sin((1 - \frac{\beta}{\alpha})\theta) \sin \theta + \cos \theta \right). \]  

The orbit or path is a precessing ellipse caused again by spinning spacetime, whose angular momentum (4.17) is a constant of motion.

For the three orbital examples considered it is seen that the X and Y components of the total linear velocity of an object of mass m in a planar orbit are related mathematically. For the ellipse (Kepler orbit):

\[ v_X = v_\theta \left( \frac{er}{\alpha} \sin \theta \cos \theta - \sin \theta \right), \]
\[ v_Y = v_\theta \left( \frac{er}{\alpha} \sin^2 \theta + \cos \theta \right). \]

For the precessing ellipse (relativistic Kepler orbit):

\[ v_X = v_\theta \left( \frac{er}{\alpha} \left( 1 - \frac{\beta}{\alpha} \right) \sin((1 - \frac{\beta}{\alpha})\theta) \cos \theta - \sin \theta \right), \]
\[ v_Y = v_\theta \left( \frac{er}{\alpha} \left( 1 - \frac{\beta}{\alpha} \right) \sin((1 - \frac{\beta}{\alpha})\theta) \sin \theta + \cos \theta \right), \]

and for the logarithmic spiral:

\[ v_X = v_\theta (b \cos \theta - \sin \theta), \]
\[ v_Y = v_\theta (b \sin \theta + \cos \theta). \]

The transverse component of velocity in plane polar coordinates is:

\[ v_\theta = r \dot{\theta} \]

For all three types of orbit the angular momentum is:

\[ L + mr^2 \dot{\theta} \]

and is conserved:

\[ \frac{\partial L}{\partial \dot{\theta}} = 0. \]

The Kepler orbit can transmute or evolve into a logarithmic spiral orbit by:

\[ \frac{er}{\alpha} \sin \theta \rightarrow b \]

keeping the angular momentum the same (Eq. 4.49). The change or mutation from one type of orbit to another occurs by changing the X and Y components of the total linear velocity for constant L. This mutation of orbits may occur suddenly or over time. Eq. (4.51) means that:

\[ r_Y = r \sin \theta = \frac{ab}{\epsilon} = \text{constant}. \]
Similarly the relativistic Kepler orbit (precessing ellipse) may mutate into a logarithmic spiral orbit when:

\[
\frac{er}{\alpha} (1 - \frac{\beta}{\alpha}) \sin((1 - \frac{\beta}{\alpha})\theta) \rightarrow b
\]  

(4.53)

i.e.

\[
(1 - \frac{\beta}{\alpha}) \frac{\alpha b}{\epsilon} = \text{constant}.
\]  

(4.54)

In binary pulsars (paper 108 of the ECE series on www.aias.us) the precessing ellipse also spirals inwards, and this is a fourth experimentally observed type of orbit which again is described by the conservation of spacetime angular momentum. In ECE physics therefore no use is made of the geometrically incorrect Einstein field equation, or of any concepts based on the Einstein field equation.

The trajectory dynamics of these orbits are the plots or computer animations of \( r \) and \( \theta \) against time \( t \). For the logarithmic spiral orbit for example:

\[
r = r_0 \exp(b\theta)
\]  

(4.55)

and

\[
L = mr^2 \dot{\theta} = \text{constant}.
\]  

(4.56)

From Eq. (4.56):

\[
e^{2b\theta} d\theta = \left( \frac{L}{mr_0^2} \right) dt.
\]  

(4.57)

Integrating the left hand side of Eq. (4.57) with respect to \( \theta \), and the right hand side of Eq. (4.57) with respect to \( t \):

\[
e^{2b\theta} = \left( \frac{L}{mr_0^2} \right) t + C
\]  

(4.58)

i.e.

\[
\theta(t) = \frac{1}{2b} \log_e \left( \frac{2bLt}{mr_0^2} + 2bC \right)
\]  

(4.59)

where \( C \) is a constant of integration [1]. Similarly:

\[
r(t) = \left( \frac{2bL}{m} t + r_0^2 bC \right)^{\frac{1}{2}}.
\]  

(4.60)

It is seen that as \( t \) increases both \( r \) and \( \theta \) increase, and from Eq. (4.55), as \( \theta \) increases, \( r \) increases. So the stars in the logarithmic spiral arms of a whirlpool galaxy are moving OUTWARDS from the force centre. This motion is due to conserved spacetime angular momentum \( L \), a concept of general relativity developed from proper consideration of spacetime torsion in Cartan geometry [2].
- [11]. In the geometrically incorrect and philosophically meaningless "standard" gravitational physics, the stars on the whirlpool galaxy are attracted outwards by "dark matter", an ad hoc concept without scientific meaning.

For Keplerian orbits [1]:

\[
\frac{d\theta}{dt} = \frac{L}{mr^2} = \frac{L}{ma^2} (1 + \epsilon \cos \theta)^2
\]

because:

\[
\frac{1}{r} = \frac{1}{\alpha} (1 + \epsilon \cos \theta),
\]

so:

\[
\frac{d\theta}{(1 + \epsilon \cos \theta)^2} = \frac{L}{ma^2} dt.
\]

Integrating the left hand side with respect to \( \theta \), and the right hand side with respect to \( t \):

\[
\int_0^{\theta} \frac{d\theta}{(1 + \epsilon \cos \theta)^2} = \frac{Lt}{ma^2}
\]

\[
= \frac{1}{(1 - \epsilon^2)} \left( \frac{2}{(1 - \epsilon^2)^{\frac{3}{2}}} \tan^{-1} \left( \frac{(1 - \epsilon)\tan^2 \frac{\theta}{2}}{1 + \epsilon \cos \theta} \right) - \frac{\epsilon \sin \theta}{1 + \epsilon \cos \theta} \right) := f(\theta)
\]

so:

\[
t = \frac{ma^2}{L} f(\theta)
\]

a function which can be plotted and animated on a computer. For a circle \( \epsilon \) is zero, so

\[
\theta = \left( \frac{L}{ma^2} \right) t
\]

meaning that the angle \( \theta \) increases linearly with time and vice versa. In these orbits:

\[
v_\theta = r \dot{\theta} = \frac{L}{r}
\]

so for a logarithmic spiral orbit for example:

\[
v_x r = b \cos \theta - \sin \theta
\]

\[
v_y r = b \sin \theta + \cos \theta
\]

and for an ellipse:

\[
b = \frac{er_y}{\alpha} = \frac{er}{\alpha} \sin \theta.
\]

In galaxies, many different types of orbit are observed and if this is a planar orbit, each one is described by the conservation of spacetime angular momentum, a simple and powerful new concept of ECE general relativity.
CHAPTER 4. ORBITAL DYNAMICS IN TERMS OF SPACETIME...

4.3 The Origin of Kinetic and Potential Energy and Force, and Free Fall Limit

As shown in ref. [1], Eq. (4.1) for the Keplerian orbit is equivalent to:

\[
\theta(r) = \int \frac{(\frac{1}{r})dr}{(2m(E + \frac{\kappa}{r} - \frac{L^2}{2mr^2}))^{\frac{3}{2}}}
\]  

(4.71)

and to:

\[
E = \frac{1}{2}(mr^2 + \frac{L^2}{mr^2}) + U
\]

(4.72)

where:

\[
U = -\frac{k}{r} = -\int F(r) \, dr
\]

(4.73)

and:

\[
F(r) = -\frac{k}{r^2}.
\]

(4.74)

These equations are interpreted as being due to conserved angular momentum and the results of astronomical observation. From Eq. (4.62):

\[
\dot{r}(t) = \frac{\epsilon}{\alpha m} L \sin \theta(t)
\]

(4.75)

and therefore in Eq. (4.72):

\[
E = \frac{1}{2} \frac{L^2}{m} \left((\frac{\epsilon}{\alpha})^2 \sin^2 \theta + \frac{1}{r^2}\right) + U(r).
\]

(4.76)

The quantity that is known as "kinetic energy" is therefore proportional to \(L\) as follows:

\[
T = \frac{1}{2} \frac{L^2}{m} \left((\frac{\epsilon}{\alpha})^2 \sin^2 \theta + \frac{1}{r^2}\right)
\]

(4.77)

and the quantity known as "potential energy" is proportional to \(L^\frac{1}{2}\) as follows:

\[
U = -\frac{k}{r} = -k(\frac{L}{m\theta})^\frac{1}{2}.
\]

(4.78)

The quantity known as "force" is proportional to \(L\) as follows:

\[
F = -\kappa \frac{L}{m\theta}.
\]

(4.79)

These quantities are therefore all due to spacetime angular momentum.
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In the free fall limit, the conserved angular momentum goes to zero:
\[ L \rightarrow 0, \quad \dot{\theta} \rightarrow 0, \]  
(4.80)
and the kinetic energy goes to zero:
\[ T \rightarrow 0. \]
(4.81)
The total energy goes to:
\[ E \rightarrow U. \]
(4.82)
In the limit (4.80):
\[ \left( \frac{L}{m\dot{\theta}} \right)^\frac{1}{2} \rightarrow \frac{1}{r} \]  
(4.83)
and the acceleration due to gravity in free fall is proportional to \( L \):
\[ g = \frac{MG}{r^2} = -\frac{MG}{m} \left( \frac{L}{\dot{\theta}} \right). \]
(4.84)
Therefore all the familiar features of Newtonian dynamics are recovered from one source, conserved spacetime angular momentum.

This analysis is generally covariant and starts with the geometrical fact that spacetime torsion is non-zero. The torsion is integrated to give spacetime angular momentum. An orbit is a path or geodesic that is a consequence of spinning spacetime. In a planar orbit the angular momentum \( L \) is a constant of motion. In the familiar Newtonian analysis the angular momentum \( L \) is a consequence of an already existing \( r \) and \( p \), but in the correctly covariant analysis given here, \( r \) and \( p \) are defined by the existence of spacetime torsion. In the Einsteinian analysis there is no spacetime torsion and the Newtonian dynamics are recovered in the Einsteinian analysis from curvature. As described in the introduction the Einsteinian analysis is incorrect [2] - [11] due to its neglect of torsion and use of a symmetric connection. These are basic geometrical errors which were accepted uncritically throughout the twentieth century. Effectively it was asserted arbitrarily that torsion is zero. The Einsteinian line element used to describe the relativistic Kepler problem is incorrectly attributed to Schwarzschild, who did not in fact derive it. When there are large deviations from the Newtonian dynamics, as in the logarithmic spiral trajectories of whirlpool galaxies for example, the Einsteinian method fails completely, and fictitious "dark matter" was invoked arbitrarily. The ECE method on the other hand places torsion as central to physics and shows that all planar orbits, including the logarithmic spiral, are consequences of conserved spacetime torsion integrated over volume to give conserved spacetime angular momentum.

**ACKNOWLEDGMENTS**
The British Government is thanked for the award of a Civil List pension in 2005 and armorial bearings in 2008 for services to Britain in science, and the staff of A.I.A.S. / T.G.A. for many interesting discussions.
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