Chapter 6

On Metric Compatibility from Cartan's Geometry

by

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Abstract

The fundamentals of Cartan geometry are developed in the base manifold to produce a simplification of the metric compatibility condition, one which is valid in any dimension and any spacetime. This equation is used to test metrics of the Einstein field equation by computer algebra. The tetrad postulate of Cartan is proven in detail and adopted for use in the base manifold, and the meaning of the internal index of Cartan is developed for use with Einstein Cartan Evans (ECE) theory.

Keywords: ECE theory, metric compatibility, development of Cartan's geometry in the base manifold, meaning of the a index of Cartan's geometry.

6.1 Introduction

Cartan's development [1] of Riemann geometry was based originally on the use of a base manifold with torsion and curvature, labelled μ, and a Minkowski

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spacetime labelled a tangential at point P to the base manifold. The Cartan tetrad q^a_μ is defined as a mixed index rank two tensor relating a vector V^a to a vector V^μ. The tetrad in differential geometry is a vector valued one-form which allows the Cartan torsion to be defined in the first Cartan structure equation as the covariant exterior derivative of the tetrad. The torsion is a rank three mixed index tensor, and also a vector valued two-form in differential geometry. The tetrad postulate of Cartan follows from the independence of the vector field on coordinates and basis elements, and the advantage of the a index is that it can be used to develop spinors, a concept also inferred by Cartan in 1913.

In Einstein Cartan Evans (ECE) unified field theory, [2] - [11] the potential density A^a_μ of electromagnetic radiation is by hypothesis the Cartan tetrad within a proportionality $A^{(0)}$, where the vacuum is filled with a density of voltage $cA^{(0)}$ observable in the radiative corrections. The field density of electromagnetic radiation is the Cartan torsion $T^a{}_{\mu\nu}$ within the same proportionality $A^{(0)}$ and the angular energy/momentum density of spacetime $J^a{}_{\mu\nu}$ is also the Cartan torsion within the factor c/k where c is the speed of light in vacuo and where k is the Einstein constant. These hypotheses allow the successful development [2] - [11] of a generally covariant unified field theory which unifies quantum mechanics and general relativity in a causal and realist view of physics in which indeterminacy is rejected on experimental grounds. The first Cartan structure equation also allows the development of an engineering model which is based on spin connection resonance (SCR). It will be shown in this paper that the index a is necessary for the existence of SRC in nature, and the index a also indicates that the fundamental quantities of electromagnetism and gravitation are field densities. In Section 2 these deductions are made by developing the Cartan tetrad for use in the base manifold by replacing the index a by an index μ. This means that the same coordinate system and same representation space are used for the vector V^ν and V^μ. The logical consequences of this procedure are worked out in Section 2 using the well known methods of Cartan geometry. It is shown that when V^μ is specialized to x^μ, the coordinate four vector, then the tetrad linking V^μ and V^ν becomes the metric $g^\mu{}_\nu$. In this case the tetrad postulate gives a novel fundamental compatibility condition on any metric in Riemann geometry in any dimension:

$$\partial_\lambda g^\mu{}_\nu = 0. \tag{6.1}$$

Computer algebra is used to check whether solutions of the Einstein field equation obey Eq. (6.1). In previous work [2] - [11] it was found that solutions of the Einstein field equation in general violate Cartan geometry because of an arbitrary neglect of torsion.

In Section 3 these results are used to develop the meaning of the index a, and it is shown that in order for spin connection resonance (SCR) to exist, a must be the index of a spacetime which is developed with a different representation space from that labelled μ. Also, a represents the existence of field densities, the fundamental quantities of general relativity.

6.2 Development of the Cartan Tetrad in the Base Manifold

The Cartan tetrad is defined by:

$$V^a = q^a_\mu V^\mu \tag{6.2}$$

and the Cartan torsion form of differential geometry is defined by:

$$\begin{aligned} T^a{}_{\mu\nu} &= \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu, \\ T^a &= D \wedge q^a = d \wedge q^a + \omega^a_b \wedge q^b, \end{aligned} \tag{6.3}$$

where $D\wedge$ is the covariant exterior derivative and where ω^a_b is the spin connection form. In tensor notation Eq. (6.3) is:

$$T^a{}_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu. \tag{6.4}$$

In Cartan geometry forms are converted to tensors by use of the tetrad, for example the torsion tensor [2] - [11] of Riemann geometry is:

$$T^\lambda{}_{\mu\nu} = q^\lambda_a T^a{}_{\mu\nu}. \tag{6.5}$$

Summation is implied over repeated a and repeated b indices, or over repeated ν and repeated ν indices. Therefore the first Cartan structure Eq. 6.4 may be expressed more simply as:

$$T^a{}_{\mu\nu} = \partial_\nu q^a_\nu - \partial^\nu q^a_\mu + \omega^a_{\mu\nu} - \omega^a_{\mu\nu} \tag{6.6}$$

using the result:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu. \tag{6.7}$$

The convention used in Cartan geometry to define the inverse tetrad q^μ_a from the tetrad q^a_μ is:

$$q^\sigma_a q^a_\mu = \delta^\sigma_\mu \tag{6.8}$$

where:

$$\delta^\sigma_\mu = \begin{cases} 1 & \text{if } \sigma = \mu, \\ 0 & \text{if } \sigma \neq \mu. \end{cases} \tag{6.9}$$

Note carefully that summation over repeated indices is not implied in the special case of Eq. (6.8). The complete vector field DV is defined by:

$$DV = D_\mu V^\nu dx^\nu \otimes \partial_\nu = D_\mu V^a dx^\mu \otimes \hat{q}_a \tag{6.10}$$

because it is independent of coordinates and basis elements. The basis elements in the space labelled a are denoted $\hat{q}_a$. The basis elements in the space labelled μ

are denoted ∂_ν [1] - [11]. The components of space a are V^a and the components of space μ are V^μ Eq. (6.10) implies the tetrad postulate:

$$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0 \tag{6.11}$$

where the connections appearing in Eq. (6.11) are defined by the covariant derivatives respectively in space a and μ

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \tag{6.12}$$

and

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda. \tag{6.13}$$

The connection in space a is the spin connection $\omega^a_{\mu b}$. Note carefully that $\Gamma^\nu_{\mu\lambda}$ is a connection which describes the presence of torsion, and that this can never be the Christoffel connection. By consideration of the fundamental equation [1] - [11]:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \tag{6.14}$$

it is seen that:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \tag{6.15}$$

because of the one to one correspondence between the antisymmetric commutator:

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] \tag{6.16}$$

and the antisymmetric torsion:

$$T^\lambda{}_{\mu\nu} = -T^\lambda{}_{\nu\mu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{6.17}$$

in Eq. (6.14). The connection is therefore always antisymmetric in Riemann and Cartan geometry:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \tag{6.18}$$

and the unfortunately well used Christoffel connection is geometrically erroneous:

$$\Gamma^\lambda_{\mu\nu} =?\ \Gamma^\lambda_{\nu\mu}. \tag{6.19}$$

This deduction invalidates gravitational physics based on the Christoffel connection, so such concepts of Einstein field equation, big bang, black holes and dark matter are meaningless.

It is convenient to give the proof of the tetrad postulate in detail, because this proof leads to Eq. (6.1), a new fundamental result of Riemann geometry.

The proof proceeds as follows. Consider the covariant derivative of Riemann geometry in the base manifold with basis elements ∂_ν:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \tag{6.20}$$

and consider the covariant derivative in the space labelled a with basis elements $\hat{q_a}$:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b. \tag{6.21}$$

The complete vector field is:

$$DV = D_\mu V^\nu dx^\mu \otimes \partial_\nu = D_\mu V^a dx^\mu \otimes \hat{q}_a \tag{6.22}$$

where the basis elements and components are related by tetrads:

$$\hat{q}_a = q^\sigma_a \partial_\sigma, \tag{6.23}$$

$$V^a = q^a_\nu V^\nu. \tag{6.24}$$

Therefore in Eq. 6.22:

$$\begin{aligned} DV &= (\partial_\mu (q^a_\nu V^\nu) + \omega^a_{\mu b} q^b_\lambda V^\lambda) dx^\mu \otimes (q^\sigma_a \partial_\sigma) \\ &= q^\sigma_a (\partial_\mu (q^a_\nu V^\nu) + \omega^a_{\mu b} q^b_\lambda V^\lambda) dx^\mu \otimes \partial_\sigma \end{aligned}. \tag{6.25}$$

The Cartan convention is:

$$q^\sigma_a q^a_\nu = \delta^\sigma_\nu \tag{6.26}$$

where:

$$\delta^\sigma_\nu = \begin{cases} 1 & \text{if } \sigma = \nu, \\ 0 & \text{if } \sigma \neq \nu. \end{cases} \tag{6.27}$$

Eq. (6.25) is therefore:

$$\begin{aligned} DV &= q^\sigma_a q^a_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_\sigma + ... \\ &= \delta^\sigma_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_\sigma + ... \\ &= \delta^0_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_0 + \delta^1_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_1 \\ &+ \delta^2_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_2 + \delta^3_\nu \partial_\mu V^\nu dx^\mu \otimes \partial_3 + ... \\ &= \delta^0_0 \partial_\mu V^0 dx^\mu \otimes \partial_0 + \delta^1_1 \partial_\mu V^1 dx^\mu \otimes \partial_1 \\ &+ \delta^2_2 \partial_\mu V^2 dx^\mu \otimes \partial_2 + \delta^3_3 \partial_\mu V^3 dx^\mu \otimes \partial_3 \\ &= \partial_\mu V^0 dx^\mu \otimes \partial_0 + \partial_\mu V^1 dx^\mu \otimes \partial_1 \\ &+ \partial_\mu V^2 dx^\mu \otimes \partial_2 + \partial_\mu V^3 dx^\mu \otimes \partial_3 + ... \\ &= \partial_\mu V^\nu dx^\mu \otimes \partial_\nu = ... \end{aligned} \tag{6.28}$$

so:

$$DV = (\partial_\mu V^\nu + q^\nu_a (\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda) V^\lambda) dx^\mu \otimes \partial_\nu. \tag{6.29}$$

From Eqs. (6.20), (6.22) and (6.29):

$$\Gamma^{\nu}_{\mu\lambda} = q^{\nu}_{a}(\partial_{\mu} q^{a}_{\lambda} + \omega^{a}_{\mu b} q^{b}_{\lambda}). \tag{6.30}$$

The connection may therefore be expanded in terms of the tetrad and the spin connection. This result follows from the basic property (Eq. 6.29) and so is very fundamental and applies in all areas of physics. Now multiply both sides of Eq. (6.30) by q^{a}_{ν} and use the Cartan convention again:

$$q^{a}_{\nu} q^{\nu}_{a} = 1 \tag{6.31}$$

to obtain the tetrad postulate:

$$D_{\mu} q^{a}_{\lambda} = \partial_{\mu} q^{a}_{\lambda} + \omega^{a}_{\mu b} q^{b}_{\lambda} - \Gamma^{\nu}_{\mu\lambda} q^{a}_{\nu} = 0 \tag{6.32}$$

Q.E.D. Eq. (6.32) uses the rule [1] - [11] for the covariant derivative of a mixed index rank two tensor q^{a}_{λ}. This is also a vector valued one-form of differential geometry.

Now consider the tetrad postulate in the base manifold by using the special case:

$$a = \nu. \tag{6.33}$$

Then:

$$V^{\nu} = q^{\nu}_{\mu} V^{\mu} \tag{6.34}$$

where V^{ν} and V^{μ} are any vectors. In general q^{ν}_{μ} is a tetrad of the base manifold, and in general this tetrad may also relate two different vectors. In Eq. 6.34 it relates different components of the same vector. In the special case:

$$V^{\mu} = x^{\mu} \tag{6.35}$$

where:

$$x^{\mu} = (ct, X, Y, Z) \tag{6.36}$$

the tetrad becomes the mixed index metric:

$$q^{\nu}_{\mu} = g^{\nu}_{\mu}. \tag{6.37}$$

This metric is defined by:

$$g^{\nu}_{\mu} = g^{\nu\alpha} g_{\alpha\mu} \tag{6.38}$$

where

$$x_{\mu} = g_{\mu\nu} x^{\nu} \tag{6.39}$$

In the special case (6.33), Eq. (6.21) becomes:

$$D_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \omega^{\nu}_{\mu b} V^{b}. \tag{6.40}$$

However, it is known that:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \tag{6.41}$$

so:

$$\omega^\nu_{\mu b} V^b = \Gamma^\nu_{\mu\lambda} V^\lambda. \tag{6.42}$$

Therefore:

$$b = \lambda. \tag{6.43}$$

The tetrad postulate (Eq. 6.32) becomes:

$$D_\mu g^\kappa_\nu = \partial_\mu g^\kappa_\nu + \Gamma^\kappa_{\mu\lambda} g^\lambda_\nu - \Gamma^\lambda_{\mu\nu} g^\kappa_\lambda = 0. \tag{6.44}$$

In Cartan geometry the differential form and the tensor or vector are defined using the tetrad. For example:

$$T^a{}_{\mu\nu} = q^a_\lambda T^\lambda{}_{\mu\nu} \tag{6.45}$$

and

$$\omega^a_{\mu b} = \omega^\lambda_{\mu b} q^b_\lambda. \tag{6.46}$$

Therefore using Eq. (6.37) we obtain:

$$\Gamma^\kappa_{\mu\nu} = g^\lambda_\nu \Gamma^\kappa_{\mu\lambda}, \tag{6.47}$$

$$\Gamma^\kappa_{\mu\nu} = g^\kappa_\lambda \Gamma^\lambda_{\mu\nu}. \tag{6.48}$$

The tetrad postulate (Eq. 6.44) therefore simplifies to:

$$\partial_\mu g^\kappa_\nu = 0 \tag{6.49}$$

which is a new fundamental result of Riemann geometry.

This result was used with computer algebra to test some metrics [12] with off-diagonal components which are exact solutions of a particular field equation, the Einstein field equation of 1915. Metrics with diagonal components all obey Eq. (6.49) because:

$$\partial_\mu g^{00} g_{00} = ... = \partial_\mu g^{33} g_{33} = 0 \tag{6.50}$$

and $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$. So there is no functional dependence on μ of g^κ_ν in Eq. (6.50). This means that for all diagonal metrics:

$$\partial_\mu (g^{\kappa\alpha} g_{\alpha\nu}) = 0 \tag{6.51}$$

automatically, even if those metrics are known for other reasons [2] - [11] to be incorrect geometrically. From previous work [2] - [11] using computer algebra it is known that off diagonal metrics which are solutions of the Einstein field equation in the presence of matter are ALL incorrect geometrically because they do not account for torsion and use an incorrect, symmetric, connection symmetry. All metrics of the Einstein field equation (diagonal or off-diagonal) are incorrect for this reason, and there is by now a proliferation of incorrect metrics in the meaningless literature [12] of twentieth century gravitational physics.

Eq. (6.49) was used to test a small sample of incorrect off-diagonal metrics from [12] as follows:

1. spherically symmetric metric with off diagonal components;
2. Kerr metric;
3. Goedel metric;
4. Eddington-Finkelstein metric;
5. anti-Mach plane wave metric;
6. Petrov metric;
7. homogeneous non-null electromagnetic metric;
8. homogeneous perfect fluid Cartesian metric;
9. Petrov type N metric;
10. metric describing the collision of plane waves.

It was found by computer algebra that Eq. (6.49) is fortuitously obeyed by these incorrect metrics, chosen to contain off-diagonal components. The reason for this is that the mixed index metric is a type of unit metric which never has functional dependence on μ.

Eq. (6.49) is therefore a special case in which the covariant derivative may be replaced by the ordinary derivative. The use of the incorrect connection symmetry of the obsolete physics does not therefore have any effect on Eq. (6.49) because the latter does not use the connection. This overall result checks the correctness of the derivation of Eq. 6.49 and also checks the correctness of the computer algebraic method [2] - [11]. Eq. (6.49) is an illustration of the power of Cartan geometry and the generality of the tetrad concept. The latter is closely related to the spinor concept, also introduced by Cartan in 1913.

Having derived Eq. (6.49) from the base manifold tetrad postulate (Eq. 6.44), the self-consistency of Eq. (6.49) may be checked as follows using the methods of Cartan geometry. The first Cartan structure equation [1] - [11] defines the torsion form from the tetrad form, and in tensor notation this equation is:

$$T^a{}_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu . \tag{6.52}$$

In the base manifold for the special case (6.33), it follows that the first Cartan structure equation specializes to:

$$T^{\kappa}{}_{\mu\nu} = \partial_\mu g^\kappa_\nu - \partial_\nu g^\kappa_\mu + \Gamma^\kappa_{\mu\lambda} g^\lambda_\nu - \Gamma^\kappa_{\nu\lambda} g^\lambda_\mu. \tag{6.53}$$

This may be simplified to:

$$T^{\kappa}{}_{\mu\nu} = \partial_\mu g^\kappa_\nu - \partial_\nu g^\kappa_\mu + \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu}. \tag{6.54}$$

However it is known that the torsion tensor is:

$$T^{\kappa}{}_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu} \tag{6.55}$$

so it follows that:

$$\partial_\mu g^\kappa_\nu - \partial_\nu g^\kappa_\mu = 0. \tag{6.56}$$

This result is consistent with Eq. (6.49), Q.E.D. Therefore the methods used in the analysis of this paper are self consistent and consistent with Cartan geometry.

6.3 The Existence of Spin Connection Resonance

Spin connection resonance (SCR) is an Euler Bernoulli resonance [2] - [11] which follows from the first (and second) Cartan structure equation used in the Cartan Bianchi identity in differential geometry. For SCR to exist the use of the a index is necessary. This inference follows from the novel Eqs. (6.49) and (6.53) of Section 2. Without a finite $\partial_\mu g^\kappa_\nu$ term, SCR disappears. In ECE electromagnetic theory the a index is that of a mathematical representation space that is different from the representation space of the μ index. For example the complex circular representation may be used for a:

$$V^a = (V^{(0)}, V^{(1)}, V^{(2)}, V^{(3)}) \tag{6.57}$$

and the Cartesian representation for μ:

$$V^a = (V_0, V_X, V_X, V_Z). \tag{6.58}$$

Both are representations in four dimensions. A static tetrad may be defined as linking V^a and V^μ as follows:

$$V^a = q^a_\mu V^\mu. \tag{6.59}$$

Cartesian components of this static tetrad are also Cartesian components of the unit vectors of the complex circular representation. The space part of this representation is [2] - [11]:

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \tag{6.60}$$

$$\boldsymbol{e}^{(1)*} = \boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}), \tag{6.61}$$

$$\boldsymbol{e}^{(3)} = \boldsymbol{k}, \tag{6.62}$$

and has $O(3)$ symmetry as follows, where $*$ denotes complex conjugate:

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = i\boldsymbol{e}^{(3)*}, \tag{6.63}$$

$$\boldsymbol{e}^{(3)} \times \boldsymbol{e}^{(1)} = i\boldsymbol{e}^{(2)*}, \tag{6.64}$$

$$\boldsymbol{e}^{(2)} \times \boldsymbol{e}^{(3)} = i\boldsymbol{e}^{(1)*}. \tag{6.65}$$

Therefore there exist components such as $e_x^{(1)}$ and so on. The static tetrad in vector format is the unit vector of the complex circular representation:

$$\boldsymbol{q}^{(1)} = \boldsymbol{e}^{(1)}, \tag{6.66}$$

$$\boldsymbol{q}^{(2)} = \boldsymbol{e}^{(2)}, \tag{6.67}$$

$$\boldsymbol{q}^{(3)} = \boldsymbol{e}^{(3)}. \tag{6.68}$$

So using two different representations of the same three dimensional space makes a profound difference to the geometry and physics.

By hypothesis the electromagnetic potential density of ECE theory is defined as being proportional to the dynamic tetrad, which is the static tetrad multiplied by the electromagnetic phase. This procedure produces complex conjugate plane waves as follows:

$$\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(0)}\boldsymbol{q}^{(1)} \exp\,(i(\omega t - \kappa Z)), \tag{6.69}$$

$$A^{(2)} = A^{(0)}\boldsymbol{q}^{(2)} \exp\,(-i(\omega t - \kappa Z)), \tag{6.70}$$

where ω is the angular frequency of the electromagnetic plane wave at instant t, and where κ is the wave-vector of the pane wave at point Z. Therefore there exist components such as:

$$A_X^{(1)} = A_X^{(2)*} = \frac{A^{(0)}}{\sqrt{2}} e^{i(\omega t - \kappa Z)}, \tag{6.71}$$

$$A_Y^{(1)} = A_Y^{(2)*} = -\frac{iA^{(0)}}{\sqrt{2}} e^{i(\omega t - \kappa Z)}. \tag{6.72}$$

Note carefully that for these components:

$$\partial_\mu A^a_\nu \neq 0 \tag{6.73}$$

so spin connection resonance appears [2] - [11]. For the electromagnetic potential density therefore:

$$D_\mu A^a_\nu = 0, \partial_\nu A^a_\nu \neq 0 \tag{6.74}$$

and the electromagnetic field density is:

$$F^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu. \tag{6.75}$$

In summary therefore the existence of SCR may be traced to the fact that three dimensional space may be represented in two different ways, using the complex circular and cartesian bases. There are also other ways of representing three dimensional space, notably the Pauli spinors, vector spherical harmonics [2] - [11] and so on.

Finally the index a also represents the fundamental potential and field densities of ECE theory, the potential and field are obtained by integration as follows [2] - [11] over a four-volume:

$$A_\mu = \int A^a_\mu \, dV_a, \tag{6.76}$$

$$F_{\mu\nu} = \int F^a{}_{\mu\nu} \, dV_a. \tag{6.77}$$

Therefore the a index has two major roles: 1) to introduce a distinct representation space in which SCR may exist; 2) to introduce potential and field densities of general relativity.

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