Chapter 7

Einstein Cartan Evans (ECE) Theory of the Rest Fermion

by

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Abstract

The ECE equation of the rest fermion is derived from the tetrad postulate and ECE Lemma in terms of 2 x 2 tetrads which are related to the Pauli matrices. It is shown that the rest fermion can be described with 2 x 2 matrices, and there is no need for the 4 x 4 matrices introduced by Dirac. The antiparticle is generated from the particle without the use of the Dirac sea by reversing helicity, for example be reversing the sign of the third Pauli matrix. The ECE equation of the rest fermion is therefore preferred by Ockham's Razor to the Dirac equation. It is shown that the Dirac equation is obtained from the ECE equation as a special case.

Keywords: Einstein Cartan Evans (ECE) unified field theory, equation of the rest fermion, Pauli matrices, Dirac equation.

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7.1 Introduction

In the Einstein Cartan Evans (ECE) unified field theory [1] - [10] the wave equations of physics are obtained straightforwardly from the fundamental tetrad postulate of Cartan geometry. The tetrad postulate is re-arranged to give a wave equation in which the eigenoperator is the d'Alembertian of Minkowski spacetime. The eigenfunction is the tetrad and the eigenvalues are scalar curvatures R. This wave equation is known as the ECE Lemma, and is a purely geometrical equation. It is made into an equation of physics by using the postulate:

$$R = -kT \tag{7.1}$$

where T has the units of mass density (kilograms per cubic metre), and where k has the units of metres per kilogram. The units of R are inverse square metres. It is convenient to chose k to be the Einstein constant, which is related to the Newton gravitational constant G:

$$k = \frac{8\pi G}{c^2} = 1.86595 \text{ x } 10^{-26} \text{ m kgm}^{-1} \tag{7.2}$$

In general the curvature is defined as:

$$R = q_a^\lambda \partial^\mu (\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda) \tag{7.3}$$

where q^a_λ is the Cartan tetrad, $\Gamma^\nu_{\mu\lambda}$ is the connection of Riemann geometry and $\omega^a_{\mu b}$ is the spin connection of Cartan geometry. Therefore in ECE theory the wave equations of physics derive from:

$$(\Box + kT) q^a_\mu = 0 \tag{7.4}$$

and are deterministic and geometrical. The wave equations of physics in ECE theory are equations of general relativity and are rigorously objective for this reason. The Copenhagen indeterminacy is rejected in ECE theory because there is nothing "absolutely unknowable" about geometry, or about the hypothesis [1]. There is no reason to think that natural philosophy is an acausal, non-Baconian, subject as in the Copenhagen claims.

In Section 2, the wave equation (7.4) is considered in the limit:

$$R = \kappa^2 = (\frac{mc}{\hbar})^2 \tag{7.5}$$

where the curvature R is directly proportional to mass m. In this limit the ECE equation of the rest fermion is developed by using the Pauli matrices as basis elements. It is shown that the four Pauli matrices may be considered as tetrads in the limit (7.5), where the fermion field becomes isolated from all other fields. In the presence of gravitation the curvature R is no longer a constant, but becomes dependent on the properties of spacetime through equation (7.3). By using the first Pauli matrix using simple algebra it is shown that the equation of the rest fermion may be developed entirely in terms of 2 x 2 matrices. There is no need for 4 x 4 matrices as introduced by Dirac. The ECE equation of the rest fermion is therefore preferred by Ockham's Razor to the Dirac equation of the rest fermion. The Dirac equation may be derived from the ECE equation by re-arranging the 2 x 2 eigenfunction matrices into column four vectors, giving the Dirac spinor elements of the rest particle. By factorizing the d'Alembertian operator with the 4 x 4 Dirac matrices, the Dirac equation is recovered straightforwardly from the simpler and more powerful ECE equation. However, the 2 x 2 matrices contain all the information needed to describe the rest fermion. This is a fundamental discovery in mathematics and physics.

In Section 3 the antiparticle is generated from the particle by reversing helicity, for example by reversing the sign of the third Pauli matrix, keeping everything else the same. The existence of negative energy is rejected as unphysical on the classical level, and the concept of the Dirac sea is not needed to generate the antiparticle. When there is no momentum, particle and antiparticle are the same rest particle. This ECE theory is therefore preferred to the Dirac sea theory by Ockham's Razor.

7.2 ECE Rest Fermion Equation With 2 x 2 Matrices

Consider the position vector $\boldsymbol{r}$ in three dimensions in the Cartesian representation, in which the basis elements are the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$:

$$\boldsymbol{r} = X\boldsymbol{i} + Y\boldsymbol{j} + Z\boldsymbol{k}. \tag{7.6}$$

Now choose the three Pauli matrices:

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{7.7}$$

as basis elements. The position vector in this representation is:

$$\boldsymbol{r} = X\boldsymbol{\sigma}^1 + Y\boldsymbol{\sigma}^2 + Z\boldsymbol{\sigma}^3. \tag{7.8}$$

The Cartesian unit vectors are related by the $O(3)$ symmetry cyclic equations:

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k} \text{ etc.} \tag{7.9}$$

and the Pauli matrices by the $SU(2)$ symmetry cyclic relations:

$$\left[\tfrac{\sigma^1}{2}, \quad \tfrac{\sigma^2}{2}\right] = i\frac{\sigma^3}{2} \text{ etc.} \tag{7.10}$$

with a factor $\frac{1}{2}$. The square of the position vector is a scalar:

$$r^2 = X^2\boldsymbol{i} \cdot \boldsymbol{i} + Y^2\boldsymbol{j} \cdot \boldsymbol{j} + Z^2\boldsymbol{\kappa} \cdot \boldsymbol{\kappa} \tag{7.11}$$

and in the Pauli representation is also a scalar:

$$r^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = X^2 \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^1 + Y^2 \boldsymbol{\sigma}^2 \cdot \boldsymbol{\sigma}^2 + Z^3 \boldsymbol{\sigma}^3 \cdot \boldsymbol{\sigma}^3. \tag{7.12}$$

On the left hand side appears the 2 x 2 unit matrix. The vectorial format of the Pauli matrices is related to the Cartesian unit vectors by:

$$\boldsymbol{\sigma}^1 = \sigma^1 \boldsymbol{i}, \tag{7.13}$$

$$\boldsymbol{\sigma}^2 = \sigma^2 \boldsymbol{j}, \tag{7.14}$$

$$\boldsymbol{\sigma}^3 = \sigma^3 \boldsymbol{k}, \tag{7.15}$$

and this is an example of Cartan geometry, in which the tetrad is defined as the mixed index rank two tensor that relates the vector V^a and V_μ:

$$V^a = q^a_\mu V^\mu \tag{7.16}$$

and in which the tetrad postulate is [1] - [10]:

$$D_\mu q^a_\nu = 0. \tag{7.17}$$

The complete vector field is:

$$DV = (D_\mu V^\nu) dx^\mu \otimes \partial_\nu = (D_\mu V^a) dx^\mu \otimes \hat{e}_a \tag{7.18}$$

and the basis elements in this equation are also related by the tetrad as follows:

$$\partial_\mu = q^a_\mu \hat{e}_a. \tag{7.19}$$

It follows therefore that the Pauli matrices are tetrads:

$$\sigma^1 = q^1_X, \tag{7.20}$$

$$\sigma^2 = q^2_Y, \tag{7.21}$$

$$\sigma^3 = q^3_Z. \tag{7.22}$$

The zero order Pauli matrix (the 2 x 2 unit matrix) is also a tetrad:

$$\sigma^0 = q^0_0. \tag{7.23}$$

Consider the wave equation:

$$(\Box + \kappa^2)\psi = 0 \tag{7.24}$$

where ψ is considered to have a time dependence but no distance dependence. Define the following four positive energy solutions of Eq. (7.24):

$$\begin{bmatrix} \psi^1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \phi^R_1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} e^{-i\phi}, \quad \phi = \frac{mc^2}{\hbar} t, \tag{7.25}$$

$$\begin{bmatrix} 0 & \psi^2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \phi_2^R \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} e^{-i\phi}, \tag{7.26}$$

$$\begin{bmatrix} 0 & 0 \\ \psi^3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \phi_1^L & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} e^{-i\phi}, \tag{7.27}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \psi^4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \phi_2^L \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} e^{-i\phi}, \tag{7.28}$$

where:

$$\psi = \begin{bmatrix} \psi^1 & \psi^2 \\ \psi^3 & \psi^4 \end{bmatrix} = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix}. \tag{7.29}$$

Here c is the speed of light in a vacuum and $\hbar$ is the reduced Planck constant. Simple matrix algebra, using the first Pauli matrix:

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{7.30}$$

shows that:

$$\begin{bmatrix} \phi_1^R & 0 \\ 0 & 0 \end{bmatrix} = \sigma^1 \begin{bmatrix} 0 & 0 \\ \phi_1^L & 0 \end{bmatrix}, \tag{7.31}$$

$$\begin{bmatrix} 0 & \phi_2^R \\ 0 & 0 \end{bmatrix} = \sigma^1 \begin{bmatrix} 0 & 0 \\ 0 & \phi_2^L \end{bmatrix}, \tag{7.32}$$

$$\begin{bmatrix} 0 & 0 \\ \phi_1^L & 0 \end{bmatrix} = \sigma^1 \begin{bmatrix} \phi_1^R & 0 \\ 0 & 0 \end{bmatrix}, \tag{7.33}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \phi_2^L \end{bmatrix} = \sigma^1 \begin{bmatrix} 0 & \phi_2^R \\ 0 & 0 \end{bmatrix}. \tag{7.34}$$

It follows that the ECE equation of the rest fermion is:

$$(i\sigma^1\partial_0 - \kappa)\psi = 0 \tag{7.35}$$

which may be rewritten as:

$$i\frac{\partial\psi}{\partial t} = \frac{mc^2}{\hbar}\sigma^1\psi \tag{7.36}$$

by multiplying both sides by σ^1 and using:

$$\sigma^1\sigma^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{7.37}$$

Eq. (7.35) may be derived straightforwardly from the ECE wave equation of the rest fermion:

$$(\partial^0\partial_0 + \kappa^2)\psi = 0 \tag{7.38}$$

as follows. Factorise the second time derivative:

$$\partial^0\partial_0 = (i\sigma^1\partial^0)(-i\sigma^1\partial_0) \tag{7.39}$$

and use Eq. (7.37) again to find that:

$$(i\sigma^1\partial^0 + \kappa)(i\sigma^1\partial_0 - \kappa)\psi = 0. \tag{7.40}$$

A solution of which is Eq. (7.35), QED.

The ECE equation of the rest fermion, Eq. (7.35), produces the following four equations inter-relating the four scalar valued components which appear in the equation:

$$i\frac{\partial\phi_1^R}{\partial t} = \frac{mc^2}{\hbar}\phi_1^L, \tag{7.41}$$

$$i\frac{\partial\phi_2^R}{\partial t} = \frac{mc^2}{\hbar}\phi_2^L, \tag{7.42}$$

$$i\frac{\partial\phi_1^L}{\partial t} = \frac{mc^2}{\hbar}\phi_1^R, \tag{7.43}$$

$$i\frac{\partial\phi_2^L}{\partial t} = \frac{mc^2}{\hbar}\phi_2^R. \tag{7.44}$$

These same equations are obtained from the Dirac equation of the rest fermion, but by using a more complicated structure with 4 x 4 matrices as is well known [11]. The Dirac equation of the rest fermion is:

$$i\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\frac{\partial\psi}{\partial t} = \frac{mc^2}{\hbar}\begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}\psi \tag{7.45}$$

where the spinor is a column four vector:

$$\psi = \begin{bmatrix}\phi^R\\ \phi^L\end{bmatrix} = \begin{bmatrix}\psi^1\\ \psi^2\\ \psi^3\\ \psi^4\end{bmatrix} = \begin{bmatrix}\phi_1^R\\ \phi_2^R\\ \phi_1^L\\ \phi_2^L\end{bmatrix}. \tag{7.46}$$

The four scalar valued components are those of the two Pauli spinors:

$$\phi^R = \begin{bmatrix}\phi_1^R\\ \phi_2^R\end{bmatrix}, \quad \phi^L = \begin{bmatrix}\phi_1^L\\ \phi_2^L\end{bmatrix}. \tag{7.47}$$

Write the four positive valued solutions as follows:

$$\begin{bmatrix}\psi^1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\phi_1^R\\0\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\exp\left(-i\frac{mc^2}{\hbar}t\right), \tag{7.48}$$

$$\begin{bmatrix} 0 \\ \psi^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \phi_2^R \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \exp\,(-i\frac{mc^2}{\hbar}t), \tag{7.49}$$

$$\begin{bmatrix} 0 \\ 0 \\ \psi^3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \phi_1^L \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \exp\,(-i\frac{mc^2}{\hbar}t), \tag{7.50}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \psi^4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \phi_2^L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \exp\,(-i\frac{mc^2}{\hbar}t). \tag{7.51}$$

Note carefully that negative energy has been rejected as unphysical, so, contrary to the standard approach as used in a textbook such as ref. [11], the phases of all four solutions are the same and all four are positive energy solutions. In the approach of this paper, negative energy solutions do not occur on the classical and quantum levels, and there is in consequence no need for the Dirac sea. Finally in this Section note that Eqs. (7.45) to (7.51) produce the same results, Eqs. (7.41) to (7.44) as from the ECE equation of the rest fermion, but the latter equations has the clear advantage of simplicity, it dispels with the need for Dirac's 4 x 4 matrices, and the ECE theory also has the fundamental advantage of being a generally covariant unified field theory [1] - [10].

7.3 The Anti-Fermion

Define the anti-fermion as the fermion with opposite helicity and opposite electric charge. The helicity of the fermion is defined as:

$$h = \boldsymbol{\sigma} \cdot \boldsymbol{\rho} \tag{7.52}$$

and is sometimes described [11] as the projection of spin along a given direction of momentum. It follows that the rest fermion and rest anti-fermion are identical, because they have no momentum. In order for an anti-fermion to be distinct from the fermion the ECE Eq. (7.35) must be extended to a fermion with non-zero momentum. That will be the subject of the next paper in this series. The concept of negative energy is rejected on the classical level in ECE theory, so there is no problem with negative energy cascade [11] as in the standard literature. Dirac artificially introduced the negative energy problem on the quantum level, and was obliged thereby to postulate the non observable Dirac sea, in which the Pauli exclusion principle is used empirically to claim that the vacuum is filled with negative energy fermions paired off according to the Pauli exclusion principle. A vacancy in this unobservable entity, the Dirac sea, has energy $-|E|$. An electron with energy E is claimed to fill this vacancy, emitting energy $2E$. It us then claimed that the vacancy has charge $+e$ and positive energy. The vacancy is called the positron. Finally it is claimed that observable positrons in nature are "predicted" by the Dirac equation.

In ECE theory these claims are rejected. The positron in ECE theory is formed by reversing the sign of the chirality operator. In terms of Dirac matrices it is well known [11] that the chirality operator is the matrix:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{7.53}$$

where the Dirac matrices are defined in terms of the Minkowski metric:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \tag{7.54}$$

or in terms of the Pauli matrices:

$$\gamma^\mu = (\gamma^0, \gamma^i), \quad \gamma^0 = \begin{bmatrix} 0 & \sigma^0 \\ \sigma^0 & 0 \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{bmatrix}. \tag{7.55}$$

In ECE theory a simpler approach will be sought in paper 130 in order to eliminate the need for Dirac matrices. For example, the helicity of a fermion travelling in the Z axis, with momentum in the Z axis, is reversed by keeping the momentum direction the same and reversing the third Pauli matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{7.56}$$

The operation (7.56) is a parity operation, its equivalent in Cartesian coordinates is the reversal of the unit vector $\boldsymbol{k}$ to $-\boldsymbol{k}$. This reverses the chirality or handedness of the coordinate system, leaving everything else the same. The operation (7.56) in ECE theory produces the anti-fermion. In order to conserve CPT, the electric charge must be the opposite for the anti-fermion. Here C is the charge conjugation operator, T is the motion reversal operator, and P is the parity inversion operator. The direction of momentum (motion) is not changed, so CPT must be conserved by:

$$CPT = (-C)(-P)T. \tag{7.57}$$

ACKNOWLEDGMENTS
The British Government is thanked for the award of a Civil List pension in 2005 and armorial bearings in 2008 for services to Britain in science, and the staff of A.I.A.S. / T.G.A. for many interesting discussions.

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