Chapter 12

The Antisymmetry Law of Cartan Geometry: Applications to Electromagnetism and Gravitation

by

M. W. Evans[1], D. W. Lindstrom[2], H. Eckardt[3]

Alpha Institute for Advanced Study (AIAS)
(www.aias.us, www.atomicprecision.com)

Abstract

A novel antisymmetry law of Cartan geometry is developed from the fundamental antisymmetry of the commutator of covariant derivatives acting on any tensor in any spacetime. The law is illustrated with respect to new fundamental antisymmetries of the curvature and torsion tensors and curvature and torsion forms. These laws are expressed in vector format and developed for use with the Einstein Cartan Evans (ECE) theory of electrodynamics. The ECE electrodynamical laws are summarized for ease of reference. Their Hodge dual structures are developed and also summarized, and the fundamental properties of the ECE

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[1]e-mail: emyrone@aol.com
[2]e-mail: dwlindstrom@gmail.com
[3]e-mail: horsteck@aol.com

potential added to the ECE engineering model. The antisymmetry constraints are developed by Lindstrom and Eckardt in Section 4 for use with computer simulation of new energy and counter gravitational devices. Tesla resonance is recognized to be the various spin connection resonances of the ECE engineering model.

Keywords: Commutator antisymmetry, antisymmetry law of Cartan geometry and ECE theory, Hodge duality, ECE electromagnetic potential, computer simulation with ECE theory and antisymmetry constraints.

12.1 Introduction

In recent papers of this series [1] - [10] on the Einstein Cartan Evans (ECE) field theory novel antisymmetry laws have been developed from the well known antisymmetry of the commutator of covariant derivatives acting on any tensor in any spacetime of any dimension [11] . These laws are straightforward to understand but are powerful constraints on electrodynamics and gravitation. They show that theories of gravitation are incorrect fundamentally if they neglect spacetime torsion, and theories of electromagnetism are incorrect if they are based on U(1) gauge symmetry. They introduce a fundamentally new antisymmetry law into Cartan geometry itself, and this is developed in Section 2 in differential form, tensor and vector notations. The vector format of this law is used to summarize the ECE laws of electrodynamics which are the basis of the ECE engineering model [1] - [10]. The latter is the only theory of electrodynamics capable of describing Tesla resonance [12], a useful source of electric power. In Section 3 the Hodge dual structures of the ECE field theory are summarized and reviewed, and the properties of the ECE electromagnetic potential summarized for use with the engineering model. In Section 4, the Lindstrom constraint of Paper 133 is developed to produce a completely defined or well posed problem for use with computer simulation of devices taking electric power from spacetime through Tesla resonance, and for computer simulation of devices that produce counter gravitation.

12.2 Geometrical Antisymmetry Laws and Application to Physics

Consider the action of the commutator of covariant derivatives on the vector V^ρ in any spacetime of any dimension:

$$[D_\mu, D_\nu]V^\rho = D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho). \tag{12.1}$$

This equation is identically antisymmetric:

$$D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho) := -(D_\nu(D_\mu V^\rho) - D_\mu(D_\nu V^\rho)) \tag{12.2}$$

i.e.

$$D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho) := D_\mu(D_\nu V^\rho) - D_\nu(D_\mu V^\rho) \tag{12.3}$$

Q.E.D. Its only possible solutions are:

$$D_\mu(D_\nu V^\rho) = -D_\nu(D_\mu V^\rho) \tag{12.4}$$

because of antisymmetry in μ and ν . From fundamentals [1] - [11]:

$$D_\mu(D_\nu V^\rho) = \partial_\mu(\partial_\nu V^\rho) + (\partial_\mu \Gamma^\rho_{\nu\sigma}) V^\sigma + \Gamma^\rho_{\nu\sigma} \partial_\mu V^\sigma - \Gamma^\lambda_{\mu\nu} \partial_\lambda V^\rho - \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\lambda\sigma} V^\sigma$$
$$+ \Gamma^\rho_{\mu\sigma} \partial_\nu V^\sigma + \Gamma^\rho_{\mu\sigma} \Gamma^\sigma_{\nu\lambda} V^\lambda$$
$$\tag{12.5}$$

Therefore when we consider $D_\nu(D_\mu V^\rho)$, every term on the right hand side of Eq. (12.5) must change sign when:

$$\mu \to \nu, \quad \nu \to \mu. \tag{12.6}$$

In the limit of Minkowski spacetime:

$$D_\mu(D_\nu V^\rho) \to \partial_\mu(\partial_\nu V^\rho) \tag{12.7}$$

in which case:

$$\partial_\mu(\partial_\nu V^\rho) = -\partial_\nu(\partial_\mu V^\rho). \tag{12.8}$$

However, in Minkowski spacetime, by coordinate orthogonality:

$$\partial_\mu(\partial_\nu V^\rho) = \partial_\nu(\partial_\mu V^\rho). \tag{12.9}$$

Therefore:

$$\partial_\mu(\partial_\nu V^\rho) = 0. \tag{12.10}$$

For example, consider the position vector in two dimensions

$$\mathbf{V} = \mathbf{r} = X\mathbf{i} + Y\mathbf{j}. \tag{12.11}$$

In this case:

$$\frac{\partial \mathbf{V}}{\partial X} = \mathbf{i}, \ \frac{\partial \mathbf{V}}{\partial Y} = \mathbf{j} \tag{12.12}$$

and:

$$\frac{\partial}{\partial Y}\left(\frac{\partial \mathbf{V}}{\partial X}\right) = \frac{\partial}{\partial X}\left(\frac{\partial \mathbf{V}}{\partial Y}\right) = 0, \tag{12.13}$$

Q.E.D. Regrouping the algebra of Eq. (12.5): [1] - [11]:

$$[D_\mu, D_\nu]V^\rho = (\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \ \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\mu} - \ \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}) \ V^\sigma - (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})D_\lambda V^\rho$$
$$= R^\rho_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho$$

$$(12.14)$$

where $R^\rho{}_{\sigma\mu\nu}$ is the curvature tensor of any spacetime in any dimension, and where $T^\lambda{}_{\mu\nu}$ is its torsion tensor. Therefore:

$$\partial_\mu \Gamma^\rho_{\nu\sigma} = -\partial_\nu \Gamma^\rho_{\mu\sigma}, \tag{12.15}$$

$$\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} = -\Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}, \tag{12.16}$$

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}. \tag{12.17}$$

If it is asserted for the sake of argument that:

$$\Gamma^\lambda_{\mu\nu} \neq -\Gamma^\lambda_{\nu\mu} \tag{12.18}$$

then it follows that $\Gamma^\lambda_{\mu\nu}$ must have a symmetric component:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} \tag{12.19}$$

because any asymmetric matrix with lower indices μ and ν is by definition the sum of a symmetric part (S) and antisymmetric part (A):

$$\begin{aligned}
\Gamma^\lambda_{\mu\nu} &= \Gamma^\lambda_{\mu\nu}(S) + \Gamma^\lambda_{\mu\nu}(A) \\
&= \frac{1}{2}(\Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\nu\mu}) - \frac{1}{2}(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}).
\end{aligned} \tag{12.20}$$

The connection is not a tensor because it does not transform as a tensor under the general coordinate transformation [1] - [11], but its lower two indices define a matrix for each $\mu\nu$. However, if:

$$\mu = \nu \tag{12.21}$$

then:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} = 0 \tag{12.22}$$

so Eq. (12.18) is not true, Q.E.D. Therefore:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu}. \tag{12.23}$$

Eqs. (12.15) and (12.16) are proven in the same way and are also directly the result of the antisymmetry of the commutator:

$$[D_\mu, D_\nu]V^\rho = -[D_\nu, D_\mu]V^\rho. \tag{12.24}$$

There is no symmetric part to the commutator, which means that if the indices μ and ν are the same, the commutator vanishes, and so do ALL the terms on the right hand side of Eqs. (12.5) and (12.14).

The standard model [11] assumes incorrectly that only certain sums or differences of terms are antisymmetric, i.e. it assumes:

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu}, \; T^\lambda{}_{\mu\nu} = -T^\lambda{}_{\nu\mu} \tag{12.25}$$

where $R^\rho{}_{\sigma\mu\nu}$ is the sum of four terms and $T^\lambda{}_{\mu\nu}$ is the difference of two terms. The standard model compounds these errors by assuming that:

$$\Gamma^\lambda_{\mu\nu} =? \Gamma^\lambda_{\nu\mu}. \tag{12.26}$$

This is a gross error because μ is assumed to be the same as ν, in which case the commutator vanishes, and all terms on the right hand side of Eq. (12.14) vanish.

The correct antisymmetry of the identically non-zero torsion tensor is:

$$T^\lambda{}_{\mu\nu} = -T^\lambda{}_{\nu\mu} :\neq 0 \tag{12.27}$$

in which the connection is identically antisymmetric. The correct antisymmetries of the curvature tensor are:

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu}, \tag{12.28}$$

$$\partial_\mu \Gamma^\rho_{\nu\sigma} = -\partial_\nu \Gamma^\rho_{\mu\sigma}, \tag{12.29}$$

$$\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} = -\Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{12.30}$$

Translating to vector notation:

$$\boldsymbol{\nabla} \times \mathbf{R}^\rho_{\sigma 1} = \mathbf{0}, \quad \frac{\partial \mathbf{R}^\rho_{\sigma 2}}{\partial t} = 0 \tag{12.31}$$

where

$$\begin{aligned}\mathbf{R}^\rho_{\sigma 1} &= R^\rho_{\sigma 01}\mathbf{i} + R^\rho_{\sigma 02}\mathbf{j} + R^\rho_{\sigma 03}\mathbf{k}, \\ \mathbf{R}^\rho_{\sigma 2} &= R^\rho_{\sigma 23}\mathbf{i} + R^\rho_{\sigma 31}\mathbf{j} + R^\rho_{\sigma 12}\mathbf{k}.\end{aligned} \tag{12.32}$$

Therefore there exists the novel identity of Riemann geometry:

$$\boldsymbol{\nabla} \times \mathbf{R}^\rho_{\sigma 1} + \frac{1}{c}\frac{\partial \mathbf{R}^\rho_{\sigma 2}}{\partial t} := 0. \tag{12.33}$$

The vector $\mathbf{R}^\rho_{\sigma 1}$ is irrotational and the vector $\mathbf{R}^\rho_{\sigma 2}$ is independent of time. Denote:

$$R^\rho_{\sigma\mu\nu} := A^\rho_{\sigma\mu\nu} + B^\rho_{\sigma\mu\nu} \tag{12.34}$$

where:

$$A^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma}, \tag{12.35}$$

$$B^\rho_{\sigma\mu\nu} = \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{12.36}$$

Then for each ρ and σ :

$$A_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu \tag{12.37}$$

$$B_{\mu\nu} = \Gamma_{\mu\lambda}\Gamma^\lambda_\nu - \Gamma_{\nu\lambda}\Gamma^\lambda_\mu \tag{12.38}$$

The orbital antisymmetries are, for each ρ and σ :

$$A_{0i} = \partial_0 \Gamma_i - \partial_i \Gamma_0 \tag{12.39}$$

$$B_{0i} = \Gamma_{0\lambda} \Gamma_i^\lambda - \Gamma_{i\lambda} \Gamma_0^\lambda \tag{12.40}$$

Define the connection four vectors for each ρ and σ :

$$\begin{aligned}
\Gamma_\mu &= (\Gamma_0, -\mathbf{\Gamma}), \\
\Gamma_{\mu\lambda} &= (\Gamma_{0\lambda}, -\mathbf{\Gamma}_\lambda), \\
\Gamma_\mu^\lambda &= (\Gamma_0^\lambda, -\mathbf{\Gamma}^\lambda)
\end{aligned} \tag{12.41}$$

and the following vectors for each ρ and σ :

$$\begin{aligned}
\mathbf{A}_1 &= A_{01}\mathbf{i} + A_{02}\mathbf{j} + A_{03}\mathbf{k}, \\
\mathbf{A}_2 &= A_{23}\mathbf{i} + A_{31}\mathbf{j} + A_{12}\mathbf{k}.
\end{aligned} \tag{12.42}$$

Then:

$$\mathbf{A}_1 = -\mathbf{\nabla}\Gamma_0 - \frac{1}{c}\frac{\partial \mathbf{\Gamma}}{\partial t}, \tag{12.43}$$

$$\mathbf{A}_2 = \mathbf{\nabla} \times \mathbf{\Gamma}. \tag{12.44}$$

The antisymmetry law means that:

$$\mathbf{\nabla}\Gamma_0 = \frac{1}{c}\frac{\partial \mathbf{\Gamma}}{\partial t}. \tag{12.45}$$

Therefore:

$$\mathbf{\nabla} \times \mathbf{A}_1 = \frac{1}{c}\frac{\partial \mathbf{A}_2}{\partial t} = 0. \tag{12.46}$$

Similarly:

$$\mathbf{\nabla} \times \mathbf{B}_1 = \frac{1}{c}\frac{\partial \mathbf{B}_2}{\partial t} = 0 \tag{12.47}$$

where, for each ρ and σ :

$$\begin{aligned}
\mathbf{B}_1 &= B_{01}\mathbf{i} + B_{02}\mathbf{j} + B_{03}\mathbf{k}, \\
\mathbf{B}_2 &= B_{23}\mathbf{i} + B_{31}\mathbf{j} + B_{12}\mathbf{k}
\end{aligned} \tag{12.48}$$

and where for each ρ and σ :

$$\begin{aligned}
\mathbf{B}_1 &= -\Gamma_{0\lambda}\mathbf{\Gamma}^\lambda + \mathbf{\Gamma}_\lambda \Gamma_0^\lambda, \\
\mathbf{B}_2 &= \mathbf{\Gamma}_\lambda \times \mathbf{\Gamma}^\lambda
\end{aligned} \tag{12.49}$$

Restoring the ρ and σ indices we recover Eqs. (12.31) and (12.33). These are the fundamental equations of Riemann geometry in vector format, with the novel antisymmetry constraints of previous work included.

Similarly, geometrical antisymmetry is fundamentally important to Cartan geometry, notably to the first Cartan Maurer structure equation:

$$T^a = d \wedge q^a + \omega_b^a \wedge q^b \tag{12.50}$$

and the second Cartan Maurer structure equation:

$$R_b^a = d \wedge \omega_b^a + \omega_c^a \wedge \omega_b^c \tag{12.51}$$

in standard [11] differential form notation. Here T^a is the torsion form, d ^ is the exterior derivative, q^a is the tetrad form, ω_b^a is the spin connection form and R_b^a is the curvature form. Considering the torsion, Eq. (12.50) in tensor notation is:

$$T_{\mu\nu}^a = \partial_\mu q_\nu^a - \partial_\nu q_\mu^a + \omega_{\mu b}^a q_\nu^b - \omega_{\nu b}^a q_\mu^b \tag{12.52}$$

where by definition [11]:

$$\omega_{\mu\nu}^a = \omega_{\mu b}^a q_\nu^b \tag{12.53}$$

To translate Eq. (12.52) to vector notation, the torsion is analysed in terms of its orbital component:

$$T_{0i}^a = \partial_0 q_i^a - \partial_i q_0^a + \omega_{0b}^a q_i^b - \omega_{ib}^a q_0^b, \tag{12.54}$$

$$i = 1, 2, 3,$$

and its spin component:

$$T_{ij}^a = \partial_i q_j^a - \partial_j q_i^a + \omega_{ib}^a q_j^b - \omega_{jb}^a q_i^b, \tag{12.55}$$

$$j = 1, 2, 3.$$

Define the vectors:

$$\mathbf{T}^a(\text{orb.}) = T_{01}^a \mathbf{i} + T_{02}^a \mathbf{j} + T_{03}^a \mathbf{k} \tag{12.56}$$

$$\mathbf{T}^a(\text{sp.}) = T_{23}^a \mathbf{i} + T_{31}^a \mathbf{j} + T_{12}^a \mathbf{k} \tag{12.57}$$

The torsion is defined in terms of the tetrad and spin connection, which are both four-vectors as follows:

$$q_\mu^a = (q_0^a, -\mathbf{q}^a), \tag{12.58}$$

$$\omega_{\mu b}^a = (\omega_{0b}^a, \ -\omega_b^a) \tag{12.59}$$

in a four dimensional spacetime. The four derivative is defined with a sign change as follows:

$$\partial_\mu = (\frac{1}{c}\frac{\partial}{\partial t}, \boldsymbol{\nabla}). \tag{12.60}$$

It follows that:

$$\mathbf{T}^a(\text{orb.}) = -\frac{1}{c}\frac{\partial \mathbf{q}^a}{\partial t} - \boldsymbol{\nabla} q_0^a - \omega_{0b}^a \mathbf{q}^b + \boldsymbol{\omega}_b^a q_0^b \tag{12.61}$$

and:

$$\mathbf{T}^a(\text{sp.}) = \boldsymbol{\nabla} \times \mathbf{q}^a - \boldsymbol{\omega}_b^a \times \mathbf{q}^b \tag{12.62}$$

which is the first Cartan Maurer structure equation in terms of vectors.

In Paper 133 of the ECE series (www.aias.us) it was shown that the fundamental tetrad postulate of Cartan geometry [1] - [11] may be expressed as:

$$\Gamma_{\mu\nu}^a = \partial_\mu q_\mu^a + \omega_{\mu\nu}^a. \tag{12.63}$$

The fundamental antisymmetry (12.23) therefore implies that:

$$\partial_\mu q_\nu^a + \omega_{\mu\nu}^a = -(\partial_\nu q_\mu^a + \omega_{\nu\mu}^a) \tag{12.64}$$

i.e.:

$$\partial_\mu q_\nu^a + \partial_\nu q_\mu^a + \omega_{\mu b}^a q_\nu^b + \omega_{\nu b}^a q_\mu^b = 0 \tag{12.65}$$

which is a novel and fundamental constraint on the first Cartan Maurer structure equation:

$$T_{\mu\nu}^a = \partial_\mu q_\nu^a - \partial_\nu q_\mu^a + \omega_{\mu b}^a q_\nu^b - \omega_{\nu b}^a q_\mu^b. \tag{12.66}$$

The Cartan Bianchi identity [1] - [11] in differential form notation is:

$$d \wedge T^a := j^a \tag{12.67}$$

where the current j^a is defined as:

$$j^a = R_b^a \wedge q^b - \omega_b^a \wedge T^b. \tag{12.68}$$

In tensor notation, Eq. (12.69) is:

$$\partial_\mu T_{\nu\rho}^a + \partial_\rho T_{\mu\nu}^a + \partial_\nu T_{\rho\mu}^a = j_{\mu\nu\rho}^a + j_{\rho\mu\nu}^a + j_{\nu\rho\mu}^a \tag{12.69}$$

where:

$$j_{\mu\nu\rho}^a = R_{\mu\nu\rho}^a - \omega_{\mu b}^a T_{\nu\rho}^b \tag{12.70}$$

and so on. In vector notation, Eq. (12.69) is expressed as two equations:

$$\boldsymbol{\nabla} \cdot \mathbf{T}^a(\text{sp.}) = j_0^a \tag{12.71}$$

and

$$\boldsymbol{\nabla} \times \mathbf{T}^a(\text{orb.}) + \frac{1}{c}\frac{\partial \mathbf{T}^a(\text{sp.})}{\partial t} = \mathbf{j}^a \tag{12.72}$$

where the time-like part of the current is:

$$j_0^a = -(j_{123}^a + j_{312}^a + j_{213}^a) \tag{12.73}$$

and where the space-like part is:

$$\mathbf{j}^a = j_X^a \mathbf{i} + j_Y^a \mathbf{j} + j_Z^a \mathbf{k} \tag{12.74}$$

where

$$j_X^a = -(j_{012}^a + j_{201}^a + j_{120}^a) \tag{12.75}$$

and so on. For each a :

$$
\begin{aligned}
T_{\mu\nu} &= \begin{pmatrix} 0 & T_{01} & T_{02} & T_{03} \\ T_{10} & 0 & T_{12} & T_{13} \\ T_{20} & T_{21} & 0 & T_{23} \\ T_{30} & T_{31} & T_{32} & 0 \end{pmatrix} \\
&= \begin{pmatrix} 0 & T_X(orb.) & T_Y(orb.) & T_Z(orb.) \\ -T_X(orb.) & 0 & -T_Z(sp.) & T_Y(sp.) \\ -T_Y(orb.) & T_Z(sp.) & 0 & -T_X(sp.) \\ -T_Z(orb.) & -T_Y(sp.) & T_X(sp.) & 0 \end{pmatrix}.
\end{aligned}
\tag{12.76}
$$

The homogeneous field equations of ECE electrodynamics, and of the ECE engineering model, are based directly on this geometry [1] - [11]. It is also known from recent work that the field equations must be constrained by antisymmetry (Eq. (12.65) in tensor notation). The basic ECE hypothesis is:

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu} \tag{12.77}$$

where $cA^{(0)}$ has the units of volts and in ECE theory is a basic property of the vacuum observable in the radiative corrections and also in Tesla resonance. In general, the homogeneous field equations are:

$$\partial_\mu \widetilde{F}^{\mu\nu} = \widetilde{J}^\nu / \epsilon_0 \tag{12.78}$$

where $\widetilde{F}^{\mu\nu}$ is the electromagnetic field tensor, and where $\widetilde{J}^\nu$ is the homogeneous or magnetic four current density. There is no geometrical reason why $\widetilde{J}^\nu$ should be zero in general. From experimental data in the laboratory, it is claimed that:

$$\widetilde{J}^\nu = 0. \tag{12.79}$$

Accepting this claim for the sake of argument, it follows that the homogeneous field equations in vector notation are:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = 0 \tag{12.80}$$

and:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = 0. \tag{12.81}$$

For each a , the field tensor $\widetilde{F}^{\mu\nu}$ is:

$$\widetilde{F}^{\mu\nu} = \begin{pmatrix} 0 & -cB_X & -cB_Y & -cB_Z \\ cB_X & 0 & E_Z & -E_Y \\ cB_Y & -E_Z & 0 & E_X \\ cB_Z & E_Y & -E_X & 0 \end{pmatrix} \tag{12.82}$$

and is the Hodge dual of

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_X & -E_Y & -E_Z \\ E_X & 0 & -cB_Z & cB_Y \\ E_Y & cB_Z & 0 & -cB_X \\ E_Z & -cB_Y & cB_Y & 0 \end{pmatrix} \tag{12.83}$$

The Hodge duality between these tensors is defined (see section 3) as:

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \tag{12.84}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric four-dimensional unit tensor defined by:

$$\begin{aligned}
\varepsilon^{0123} &= -\varepsilon^{1230} &= \varepsilon^{2301} = -\varepsilon^{3012} && = 1 \\
\varepsilon^{1023} &= -\varepsilon^{2130} &= \varepsilon^{3201} = -\varepsilon^{0312} && = -1 \\
\varepsilon^{1032} &= -\varepsilon^{2103} &= \varepsilon^{3210} = -\varepsilon^{0321} && = 1 \\
\varepsilon^{1302} &= -\varepsilon^{2013} &= \varepsilon^{3120} = -\varepsilon^{0231} && = -1 \\
&\text{etc.}
\end{aligned} \tag{12.85}$$

12.3   Hodge Duality, Inhomogeneous Field Eqaution and Electromagnetic Potential

The Hodge duality (12.84) means that for each a , elements of the field tensor and its Hodge dual are related as follows:

$$\widetilde{F}^{01} = F^{23}; \ \widetilde{F}^{02} = F^{31}; \ \widetilde{F}^{03} = F^{12} \tag{12.86}$$

$$\widetilde{F}^{12} = F^{30}; \ \widetilde{F}^{31} = F^{20}; \ \widetilde{F}^{23} = F^{10}. \tag{12.87}$$

It is seen that this is a re-arrangement of a four dimensional antisymmetric tensor to give another four dimensional antisymmetric tensor. The indices in Eq. (12.86) are in cyclic permutation:

$$0123, 0231, 0312 \tag{12.88}$$

and also those in Eq. (12.87):

$$1230, 3120, 2310 \tag{12.89}$$

Eqs. (12.86) and (12.87) mean that there are two ways of writing an antisymmetric tensor in four dimensions. The basic field tensors of the ECE engineering model are therefore related by Eqs. (12.86) and (12.87) and are defined by Eqs. (12.82) and (12.83).

Consider the fundamental commutator structure of Riemann geometry:

$$[D_\mu, D_\nu]V = R_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V. \tag{12.90}$$

Raising indices term by term gives:

$$[D^\mu, D^\nu]V = R^{\rho\sigma\mu\nu}V^\sigma - T^{\lambda\mu\nu}D_\lambda V. \tag{12.91}$$

The Hodge duals of the terms appearing in this equation are defined as follows:

$$[D_\mu, D_\nu]_{\text{HD}}V^\rho = \frac{1}{2}\|g|^{1/2}\epsilon_{\mu\nu\alpha\beta}[D^\alpha, D^\beta]V^\rho \tag{12.92}$$

$$\widetilde{R}^\rho_{\sigma\mu\nu} = [D_\mu, D_\nu]_{\text{HD}}V^\rho \tag{12.93}$$

$$\widetilde{T}^\lambda_{\mu\nu} = \frac{1}{2}\|g|^{1/2}\epsilon_{\mu\nu\alpha\beta}T^{\lambda\alpha\beta} \tag{12.94}$$

where $\|g|^{1/2}$ is the square root of the modulus or positive value of the determinant of the metric [1] - [11]. This is a weighting factor used to define the Hodge dual in the general four dimensional spacetime. In Eq. (12.91) it cancels out however. By definition [1] - [11], the antisymmetric tensor in Eqs. (12.92) to (12.94) is the Minkowski spacetime tensor.

Therefore:

$$[D_\mu, D_\nu]_{\text{HD}}V^\rho = \widetilde{R}^\rho_{\sigma\mu\nu}V^\sigma - \widetilde{T}^\lambda_{\mu\nu}D_\lambda V^\rho \tag{12.95}$$

results from Eq. (12.90). For example, if we consider:

$$[D_2, D_3]V^\rho = R^\rho{}_{\sigma23}V^\sigma - T^\lambda{}_{23}D_\lambda V^\rho \tag{12.96}$$

then:

$$[D_0, D_1]_{\text{HD}}V^\rho = \widetilde{R}^\rho_{\sigma01}V^\sigma - \widetilde{T}^\lambda_{01}D_\lambda V^\rho \tag{12.97}$$

and the commutator has been rearranged. Its indices have been changed from 2,3 to 0,1 using an antisymmetric unit tensor. Therefore:

$$\begin{aligned}
[D_0, D_1]_{\text{HD}} &= [D_2, D_3] \\
[D_0, D_2]_{\text{HD}} &= [D_3, D_1] \\
[D_0, D_3]_{\text{HD}} &= [D_1, D_2] \\
[D_1, D_2]_{\text{HD}} &= [D_3, D_0] \\
[D_3, D_1]_{\text{HD}} &= [D_2, D_0] \\
[D_2, D_3]_{\text{HD}} &= [D_1, D_0]
\end{aligned} \tag{12.98}$$

and Eq. (12.95) is an example of Eq. (12.90). This means that the tensors $\widetilde{R}^{\rho}_{\sigma\mu\nu}$ and $\widetilde{T}^{\lambda}_{\mu\nu}$ are related to each other in the same way as the tensors $R^{\rho}_{\sigma\mu\nu}$ and $T^{\lambda}_{\mu\nu}$.

The way that $R^{\rho}_{\sigma\mu\nu}$ and $T^{\lambda}_{\mu\nu}$ are related to each other is given by the Cartan Bianchi identity:

$$D \wedge T^a := R^a_b \wedge q^b \tag{12.99}$$

so $\widetilde{R}^{\rho}_{\sigma\mu\nu}$ and $\widetilde{T}^{\lambda}_{\mu\nu}$ are related to each other by the identity:

$$D \wedge \widetilde{T}^a := \widetilde{R}^a_b \wedge q^b \tag{12.100}$$

which is the Cartan Evans identity. In tensor notation Eq. (12.99) becomes the homogeneous field equation of ECE theory, and Eq. (12.100) becomes the inhomogeneous field equation. These are respectively:

$$D_\mu \widetilde{T}^{a\mu\nu} := \widetilde{R}^a{}_{\mu}{}^{\mu\nu} \tag{12.101}$$

and

$$D_\mu T^{a\mu\nu} := R^a{}_{\mu}{}^{\mu\nu}. \tag{12.102}$$

For each a in these equations:

$$\widetilde{T}^{01} = T^{23}; \ \widetilde{T}^{02} = T^{31}; \ \widetilde{T}^{03} = T^{12}$$
$$\widetilde{T}^{12} = T^{30}; \ \widetilde{T}^{31} = T^{20}; \ \widetilde{T}^{23} = T^{10} \tag{12.103}$$

and for each a and b:

$$\widetilde{R}^{01} = R^{23}; \ \widetilde{R}^{02} = R^{31}; \ \widetilde{R}^{03} = R^{12}$$
$$\widetilde{R}^{12} = R^{30}; \ \widetilde{R}^{31} = R^{20}; \ \widetilde{R}^{23} = R^{10} \tag{12.104}$$

In tensor notation, and using the fundamental ECE hypothesis (12.77), these equations become the homogeneous field equation of ECE electrodynamics:

$$\partial_\mu \widetilde{F}^{a\mu\nu} = \widetilde{J}^{a\nu}/\varepsilon_0 \tag{12.105}$$

and the inhomogeneous field equation:

$$\partial_\mu F^{a\mu\nu} = J^{a\nu}/\varepsilon_0. \tag{12.106}$$

If the experimental claim for the absence of magnetic four current density is accepted, then:

$$\partial_\mu \widetilde{F}^{a\mu\nu} = 0 \tag{12.107}$$

and

$$\partial_\mu F^{a\mu\nu} = J^a/\varepsilon_0. \tag{12.108}$$

In vector notation:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = 0$$

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = 0$$

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \rho^a / \varepsilon_0 \qquad (12.109)$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c} \frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}^a$$

Note carefully that the vector notation subsumes the existence of the metric. The latter is not known in general because the equations are not written in a Minkowski spacetime. They are written in a general spacetime. The metric is used to raise and lower indices [1] - [11] as usual, so care has to be taken to use a consistent scheme of calculation throughout. See the accompanying notes for paper 134 for more details.

As in previous work the electric and magnetic fields are related to the potential four vector and spin connection four vector, giving the following results:

$$\mathbf{E}^a = -c\boldsymbol{\nabla} A_0^a - \frac{\partial \mathbf{A}^a}{\partial t} - c\,\omega_{0b}^a \mathbf{A}^b + cA_0^b \boldsymbol{\omega}_b^a \qquad (12.110)$$

and

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}_b^a \times \mathbf{A}^b. \qquad (12.111)$$

The potential four vector of ECE theory is a vector valued one-form, i.e. a mixed index rank two tensor which is a one-form for each a :

$$A_\mu^a = (A_0^a, -\mathbf{A}^a) \qquad (12.112)$$

and the spin connection is a tensor valued one-form which is a one-form for each a and b:

$$\omega_{\mu b}^a = (\,\omega_{0b}^a, -\boldsymbol{\omega}_b^a). \qquad (12.113)$$

For each a therefore, Φ^a is the time-like and scalar valued potential in volts, and for each a, $\mathbf{A}^a$ is the space-like and vector valued potential. By definition:

$$\Phi^a = cA_0^a \qquad (12.114)$$

and A_0^a is scalar-valued for each a. Quantities such as A_i^a , $i = 1, 2, 3$ are components of the space-like three-vector part of the four-vector A_μ^a for each a. If the complex circular basis is used then by definition the following components vanish:

$$A_Z^{(1)} = A_Z^{(2)} = A_X^{(3)} = A_Y^{(3)} = 0. \qquad (12.115)$$

By definition, the electric and magnetic fields are space-like three-vectors, in which:

$$a = (1), (2), (3). \qquad (12.116)$$

Similarly the vector potential is a space-like three-vector, taking the a indices defined in Eq. (12.116). Therefore:

$$\mathbf{E}^{(0)} = \mathbf{B}^{(0)} = \mathbf{A}^{(0)} = 0 \tag{12.117}$$

and:

$$E_i^{(0)} = B_i^{(0)} = A_i^{(0)} = 0 \tag{12.118}$$

where:

$$A_\mu^{(0)} = (\frac{\Phi^{(0)}}{c}, 0). \tag{12.119}$$

The magnetic field of the ECE engineering model is therefore:

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}_b^a \times \mathbf{A}^b \tag{12.120}$$

$$a, b = (1), (2), (3)$$

and the electric field of the ECE engineering model is therefore:

$$\mathbf{E}^a = -\nabla\Phi^a - \frac{\partial \mathbf{A}^a}{\partial t} - c\,\omega_{0b}^a \mathbf{A}^b + cA_0^b \boldsymbol{\omega}_b^a, \tag{12.121}$$

$$a = (1), (2), (3); \ \ b = (0), (1), (2), (3).$$

In the definition of the electric field the components A_0^a appear. They are time-like and scalar-valued for all a. In summary:

$$A_\mu^a = (\frac{\Phi^a}{c}, \mathbf{A}^a), \ \ a = (1), (2), (3), \tag{12.122}$$

$$A_\mu^{(0)} = (\frac{\Phi^{(0)}}{c}, 0).$$

Therefore $\Phi^{(0)}$ is the scalar potential of a scalar wave and $\Phi^{(i)}$ are scalar potentials for waves of polarizations:

$$(i) = (1), (2), (3). \tag{12.123}$$

In the next section the antisymmetry laws will be applied to this engineering model in order to define a well posed problem for the computer simulation of devices.

12.4 Electromagnetic Equations Suitable for Numerical Analysis

In the previous section, general equations in a vector format were presented for the electromagnetic portion of the ECE theory. A completely specified set of

equations for the electromagnet portion of the ECE theory is available is one restricts the model to that of a single polarization. The equations can be written in a compact elegant format if one restricts the solution to that specified by the Lindstrom constraint [5]. In this case, the complete set of equations for a single polarization is

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \tag{12.124}$$

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{12.125}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho/\varepsilon_0, \tag{12.126}$$

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \tag{12.127}$$

with the field intensities defined by

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} + \boldsymbol{\omega}\Phi, \tag{12.128}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}. \tag{12.129}$$

The electric component of the antisymmetry equation for a single polarization is

$$\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} - \boldsymbol{\omega}\Phi = 0 \tag{12.130}$$

and the magnetic antisymmetry relation restricted by the Lindstrom constraint is

$$\boldsymbol{\nabla} \times \mathbf{A} = -\boldsymbol{\omega} \times \mathbf{A}. \tag{12.131}$$

If we apply the antisymmetry equations (12.130) and (12.131) to the field intensities $\mathbf{E}$ and $\mathbf{B}$ we see two independent definitions for $\mathbf{E}$ and a single definition for $\mathbf{B}$, namely

$$\mathbf{E} = -2\frac{\partial \mathbf{A}}{\partial t} - 2\omega_0 \mathbf{A} \tag{12.132}$$

or

$$\mathbf{E} = -2\boldsymbol{\nabla}\Phi + 2\boldsymbol{\omega}\Phi \tag{12.133}$$

and

$$\mathbf{B} = 2\boldsymbol{\nabla} \times \mathbf{A}. \tag{12.134}$$

$\mathbf{B}$ is obviously compatible with Gauss' Law, Eq. (12.124).

Applying the two alternative equations (12.132) and (12.133) for $\mathbf{E}$, and (12.134) for $\mathbf{B}$, to Faraday's Law, Eq. (12.125) gives for both cases:

$$\boldsymbol{\nabla} \times (\boldsymbol{\omega}\Phi + \frac{\partial \mathbf{A}}{\partial t}) = 0, \tag{12.135}$$

$$\boldsymbol{\nabla} \times (\omega_0 \mathbf{A}) = 0. \tag{12.136}$$

Note that if we take the curl of Eq.(12.130) and apply Eq. (12.136) we get Eq. (12.135) meaning that Eq. (12.135) contains no new information that is not already given by the electric component of the antisymmetry equations.

Now we derive three alternative formulations for the field equations in potential and spin connection formulation. Using Eqs. (12.133) and (12.134) and inserting them into Faraday's Law (12.125), Coulomb's Law (12.126) and the Ampere-Maxwell Law (12.127) we obtain

$$\boldsymbol{\nabla} \times (\boldsymbol{\omega}\Phi) - \frac{\partial}{\partial t}(\boldsymbol{\omega} \times \mathbf{A}) = 0 \tag{12.137}$$

$$-\nabla^2 \Phi + \boldsymbol{\nabla} \cdot (\boldsymbol{\omega}\Phi) = \frac{1}{2\varepsilon_0}\rho \tag{12.138}$$

$$-\boldsymbol{\nabla} \times (\boldsymbol{\omega} \times \mathbf{A}) - \frac{1}{c^2}\frac{\partial}{\partial t}(\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\Phi) = \frac{1}{2}\mu_0 \mathbf{J} \tag{12.139}$$

Eq. (12.138) is the well-known form of the resonant Coulomb Law. Eqs. (129-131) represent a set of seven equations for seven unknowns $\boldsymbol{\omega}, \mathbf{A}, \boldsymbol{\Phi}$, but according to Appendix A, the Coulomb and Ampere-Maxwell Law are not independent from one another. This can also be seen by the following: Take the divergence of Eq. (12.139):

$$\frac{1}{c^2}\frac{\partial}{\partial t}(-\nabla^2 \Phi + \boldsymbol{\nabla} \cdot (\boldsymbol{\omega}\Phi)) = \frac{1}{2}\mu_0 \boldsymbol{\nabla} \cdot \mathbf{J}. \tag{12.140}$$

Time integration of this equation gives

$$-\nabla^2 \Phi + \boldsymbol{\nabla} \cdot (\boldsymbol{\omega}\Phi) = \frac{1}{2\varepsilon_0}\rho \tag{12.141}$$

with

$$\rho = \int \boldsymbol{\nabla} \cdot \mathbf{J} dt. \tag{12.142}$$

So there is a connection between current and charge density (continuity equation) which must be respected to obtain linear dependence of Eqs. (12.138) and (12.139). If both quantities are chosen independently as normally is done for modeling real systems, all equations are linearly independent. Please note that this consideration cannot be transferred to the Gauss and Faraday Laws since there are no density terms at the right-hand side.

We derive a second version of the equation set by starting with Eqs. (12.132) and (12.134). Faraday's Law (12.125) then reads

$$\boldsymbol{\nabla} \times (-2\frac{\partial \mathbf{A}}{\partial t} - -2\omega_0 \mathbf{A}) + 2\frac{\partial}{\partial t}(\boldsymbol{\nabla} \times \mathbf{A}) = 0 \tag{12.143}$$

which can be simplified to

$$\boldsymbol{\nabla} \times (\omega_0 \mathbf{A}) = 0 \tag{12.144}$$

and is identical to Eq. (12.136). The Coulomb Law and Ampere-Maxwell take the form

$$\boldsymbol{\nabla}.\frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\nabla} \cdot (\omega_0 \mathbf{A}) = \frac{1}{2\varepsilon_0}\rho, \tag{12.145}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{A} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{1}{c^2}\frac{\partial}{\partial t}(\omega_0 \mathbf{A}) = \frac{1}{2}\mu_0 \mathbf{J}. \tag{12.146}$$

Eq. (12.145) is compatible with (12.144) and tells that $\omega 0 \mathbf{A}$ represents a pure source field. Eqs. (12.145) and (12.146) represent four equations for four variables $\omega 0$, $\mathbf{A}$. As discussed before, these equations are independent if the charge and current density are chosen in an unrelated way. This form of the electromagnetic field equations is most simple and can be compared with other known equations of physics. Eq. (12.146) is a wave equation in three dimensions with transversal and longitudinal solutions. This goes beyond Maxwellian electrodynamics. Eq. (12.145) is a non-linear diffusion equation. The non-linearity is caused by the spin connection, indicating that there is a flow of potential present in addition to standard theory. This could be considered to represent interaction with a surrounding vacuum (or space time) which is the source of energy in case of resonance effects.

Now we derive the third version of the equation set. Although not necessary, Eq. (12.136) means that we can write

$$\omega_0 \mathbf{A} = -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\psi) \tag{12.147}$$

where the time derivative has been introduced for elegance only. It is shown in the Appendix A, that Coulomb's Law (12.126) and the Maxwell–Ampere Eq. (12.119) reduce to three independent equations. If we substitute (12.132) and (12.134) into (12.126) and (12.127), we get

$$\boldsymbol{\nabla} \cdot \frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\nabla} \cdot (\omega_0 \mathbf{A}) = -\frac{1}{2\varepsilon_0}\rho, \tag{12.148}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{A} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{1}{c^2}\frac{\partial}{\partial t}(\omega_0 \mathbf{A}) = \frac{1}{2}\mu_0 \mathbf{J}. \tag{12.149}$$

Using the vector identity

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{A} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \tag{12.150}$$

time-integrating Eq. (12.148), and substituting the expression for $\boldsymbol{\nabla} \cdot \mathbf{A}$ into Eq. (12.149) we have immediately

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\mathbf{A} + \int \omega_0 \mathbf{A}dt) = \frac{1}{2}\mu_0 \mathbf{J} + \frac{1}{2}\int \frac{\boldsymbol{\nabla}\rho}{\varepsilon_0}dt \tag{12.151}$$

Using Eq. (12.147), this can be written more elegantly as

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\mathbf{A} - \boldsymbol{\nabla}\Phi) = \frac{1}{2}\mu_0 \mathbf{J} + \frac{1}{2\varepsilon_0}\int \boldsymbol{\nabla}\rho \, dt. \tag{12.152}$$

By using Eq. (12.132) we find

$$\int \mathbf{E}\,dt = -2\mathbf{A} - 2\int \omega_0 \mathbf{A}\,dt = -2\mathbf{A} + 2\boldsymbol{\nabla}\psi \qquad (12.153)$$

which term appears in (12.152). Alternatively, (12.153) is according to (12.133):

$$\int \mathbf{E}\,dt = -2\int \boldsymbol{\nabla}\Phi\,dt + 2\int \boldsymbol{\omega}\Phi\,dt. \qquad (12.154)$$

Substituting this alternative form of (12.153) into (12.152), we obtain

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\int \boldsymbol{\nabla}\Phi dt - \int \boldsymbol{\omega}\Phi\,dt) = \frac{1}{2}\mu_0 \mathbf{J} + \frac{1}{2\varepsilon_0}\int \boldsymbol{\nabla}\rho\,dt \qquad (12.155)$$

or after taking the time derivative:

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\Phi) = \frac{1}{2}\mu_0\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{2\varepsilon_0}\boldsymbol{\nabla}\rho. \qquad (12.156)$$

In total, Equations (12.147), (12.152) and (12.156) represent nine equations in nine unknowns:

$$\omega_0 \mathbf{A} = -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\psi) \qquad (12.157)$$

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\mathbf{A} - \boldsymbol{\nabla}\psi) = \frac{1}{2}\mu_0 \mathbf{J} + \frac{1}{2\varepsilon_0}?\boldsymbol{\nabla}\rho dt \qquad (12.158)$$

$$(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2})(\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\Phi) = \frac{1}{2}\mu_0\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{2\varepsilon_0}\boldsymbol{\nabla}\rho \qquad (12.159)$$

The equations are entirely independent, and so represent a balanced set.

It is interesting to note how singularities can arise in the solution scenario. For example, if one takes the cross product of the electric portion of the anti-symmetry equation (12.128) with $\mathbf{A}$, one gets

$$\boldsymbol{\nabla}\Phi \times \mathbf{A} - \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{A} - \omega_0 \mathbf{A} \times \mathbf{A} - -\Phi\boldsymbol{\omega} \times \mathbf{A} = 0. \qquad (12.160)$$

Assuming that the time derivative of $\mathbf{A}$ is parallel to $\mathbf{A}$, this simplifies to

$$\boldsymbol{\nabla}\Phi \times \mathbf{A} = \Phi\boldsymbol{\omega} \times \mathbf{A}. \qquad (12.161)$$

Eq. (12.131) can finally be used to remove $\boldsymbol{\omega} \times \mathbf{A}$:

$$\boldsymbol{\nabla} \times \mathbf{A} = -\frac{1}{\Phi}\boldsymbol{\nabla}\Phi \times \mathbf{A}. \qquad (12.162)$$

Singularities occur whenever Φ is zero and $\boldsymbol{\nabla}\Phi$ and $\mathbf{A}$ are not. Coupled with the obvious driven resonances in (12.158) and (12.159), a rich supply of non-linear solutions becomes available.

Finally we mention that the engineering model reduces to standard electro-magnetic theory if the constraint (12.130) is formulated in a more restricted form; see Appendix B. This explains the occurrence of the factor of 2 between ECE and standard theory.

12.5   Appendix A - Dependence of the Coulomb and Gauss Law on the Ampère-Maxwell Law

A proof is given here that Coulomb's Law and the Maxwell-Ampere Law reduce to three independent equations given that there is a conservation of charge. If we write Coulomb's Law with conservation of charge, and the Maxwell-Ampere Law in matrix form, we have

$$\begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} - \frac{1}{c^2} \begin{pmatrix} \frac{\partial}{\partial t} & 0 & 0 \\ 0 & \frac{\partial}{\partial t} & 0 \\ 0 & 0 & \frac{\partial}{\partial t} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \mu_0 \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}, \quad \text{(A-1)}$$

$$\begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \frac{1}{\epsilon_0} \int \boldsymbol{\nabla} \cdot \mathbf{J} \, dt. \quad \text{(A-2)}$$

If in Eq. (A-1), we take $\frac{\partial}{\partial x}$ of the first row, $\frac{\partial}{\partial y}$ of the second row, and $\frac{\partial}{\partial z}$ of the third row, we get

$$\begin{pmatrix} 0 & -\frac{\partial^2}{\partial x \partial z} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial y \partial z} & 0 & -\frac{\partial^2}{\partial y \partial x} \\ -\frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial x \partial z} & 0 \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} - \frac{1}{c^2} \begin{pmatrix} \frac{\partial^2}{\partial x \partial t} & 0 & 0 \\ 0 & \frac{\partial^2}{\partial y \partial t} & 0 \\ 0 & 0 & \frac{\partial^2}{\partial z \partial t} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$
$$\text{(A-3)}$$
$$= \mu_0 \begin{pmatrix} \frac{\partial}{\partial x} J_x \\ \frac{\partial}{\partial y} J_y \\ \frac{\partial}{\partial z} J_z \end{pmatrix}.$$

If we add to row 1 the sum of rows 2 and 3, this becomes

$$\begin{pmatrix} 0 & 0 & 0 \\ \frac{\partial^2}{\partial y \partial z} & 0 & -\frac{\partial^2}{\partial y \partial x} \\ -\frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial x \partial z} & 0 \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} - \frac{1}{c^2} \begin{pmatrix} \frac{\partial^2}{\partial x \partial t} & \frac{\partial^2}{\partial y \partial t} & \frac{\partial^2}{\partial z \partial t} \\ 0 & \frac{\partial^2}{\partial y \partial t} & 0 \\ 0 & 0 & \frac{\partial^2}{\partial z \partial t} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$
$$\text{(A-4)}$$
$$= \mu_0 \begin{pmatrix} \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y + \frac{\partial}{\partial z} J_z \\ \frac{\partial}{\partial y} J_y \\ \frac{\partial}{\partial z} J_z \end{pmatrix}.$$

We note that the equation given by row 1 is just Coulomb's equation (A-2). Thus the set of four equations has been reduced to three independent equations. That is to say, Coulomb's Law adds nothing to the Maxwell-Ampere equation given that conservation of charge applies.

A similar argument can be made for the pair of equations given by Gauss's Law and Faraday's Law, reducing the number of equations from four to three.

12.6 Appendix B - Derivation of Standard Electromagetic Theory from Specialized Antisymmetry Constraints

We show that the Lindstrom magnetic constraint plus a particular solution to the electric constraint reduces engineering model II to standard electromagnetic theory. In comparing ECE electromagnetic theory to standard electromagnetic theory, it has been noted that if the spin connection is reduced to zero in the ECE theory, that the definitions of electric and magnetic fields in terms of the electric and magnetic potentials reduce to that of traditional electromagnetic theory, i.e.

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\Phi, \tag{B-1}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{B-2}$$

These forms violate the antisymmetry conditions of ECE theory and in so doing generally invalidate standard electromagnetic theory.

Let us apply the following particular solutions to the antisymmetry equations ie.

$$\boldsymbol{\omega}\Phi = -\frac{\partial \mathbf{A}}{\partial t}, \tag{B-3}$$

$$\omega_0 \mathbf{A} = \boldsymbol{\nabla}\Phi, \tag{B-4}$$

$$\boldsymbol{\omega} \times \mathbf{A} = -\boldsymbol{\nabla} \times \mathbf{A}. \tag{B-5}$$

These satisfy the antisymmetry equations, which we will now apply to the ECE engineering model for a single polarization, called model II.

Using equations (B-3) through (B-5) the electric and magnetic field of the engineering model II become

$$\mathbf{E} = -2\frac{\partial \mathbf{A}}{\partial t} - 2\boldsymbol{\nabla}\Phi, \tag{B-6}$$

$$\mathbf{B} = 2\boldsymbol{\nabla} \times \mathbf{A}. \tag{B-7}$$

Traditional electromagnetic theory (Jackson) defines the electric and magnetic potential through the use of Gauss' and Faraday's Law, namely, since

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{B-8}$$

we can write

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{a}. \tag{B-9}$$

Comparing (B-9) to (B-7) we see

$$\mathbf{a} = 2\mathbf{A}. \tag{B-10}$$

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Substituting (B-9) into Faraday's equation

$$\boldsymbol{\nabla} \times E + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{B-11}$$

gives

$$\boldsymbol{\nabla} \times \mathbf{E} = -\boldsymbol{\nabla} \times \frac{\partial \mathbf{a}}{\partial t} \tag{B-12}$$

which has

$$\mathbf{E} = -\frac{\partial \mathbf{a}}{\partial t} - \boldsymbol{\nabla}\varphi \tag{B-13}$$

as the only solution. Note that we will use lower case symbols for the traditional electromagnetic theory.

Comparing (B-13) to (B-6) gives

$$\varphi = 2\Phi. \tag{B-14}$$

This shows that the engineering model II equations reduce to standard electromagnetic equations given the restrictions (B-3) through (B-5), which are a particular solution to the antisymmetry equations.

In this particular example, the fundamental comparison is not setting the spin connection to zero, but rather setting

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{a} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} = 2\boldsymbol{\nabla} \times \mathbf{A} \tag{B-15}$$

and imposing the particular solutions (B-3) through (B-5).

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