Derivation of relativity and the Sagnac Effect from the rotation of the Minkowski and other metrics of the ECE Orbital Theorem: the effect of rotation on spectra.

by
M .W. Evans,
British Civil List
(www.aias.us)

Abstract

It has been shown in Paper 111 of this series (UFT 111 on www.aias.us) that the metrics of physics are derived from the orbital theorem of spherically symmetric spacetime. The simplest solution of the orbital theorem is the Minkowski metric. It is shown that rotation of the Minkowski metric is sufficient to produce all the features of special relativity. An example is the Sagnac effect, which is the rotation of the lightlike Minkowski metric. The Sagnac effect is derived straightforwardly and developed to show that mechanical rotation in general affects spectra of all kinds. The main features of special relativity are all derived from a rotating Minkowski metric.

Keywords: ECE theory, rotating Minkowski metric, Sagnac effect, special relativity.

1. Introduction

The orbital theorem of Paper 111 of this series (UFT 111 on www.aias.us) is the basis for all metrics in physics, the simplest solution of the theorem being the Minkowski metric [1-12]. It is shown in this paper that rotation of the Minkowski metric produces all the well known features of special relativity, and also produces the Sagnac effect as in UFT 145 (www.aias.us). In Section 2 it is shown that the Sagnac effect can be deduced very simply by rotating the Minkowski metric about the Z axis in the null geodesic condition appropriate to a photon traversing the circumference of a circle. Rotating another solution of the orbital theorem produces the effect of gravitation on the Sagnac effect (UFT 145). It is shown that the Sagnac effect may be derived from the Cartan torsion, and that the effect is a birefringence produced in Minkowski spacetime by rotation of the frame of reference. In ECE theory (UFT 45 and 46 on www.aias.us) the Sagnac effect is also a frame rotation in a plane, which is the same as rotating the Minkowski metric in a plane about the Z axis for a null geodesic. In Section 3 it is shown that rotation of the Minkowski metric produces the gamma factor of Lorentz, which is the basis of all special relativity. The gamma factor is used to define the relativistic momentum, relativistic kinetic energy and the Einstein energy equation of special relativity. From the latter the Dirac equation in wave format is obtained using the operator relations of quantum mechanics. Factorization of the Minkowski metric into the Dirac gamma matrices produces the first order format of the Dirac equation. In UFT 129 and 130 (www.aias.us) it was shown that the Dirac equation can also be developed in first order format with 2 x 2 matrices, this simplifying its structure. The Dirac equation therefore originates in the rotation of Minkowski spacetime, and as is well known, produces the g factor of the electron and the Thomas factor 1 / 2. Self consistently, the Thomas precession is also the rotation of Minkowski spacetime.

2. Sagnac Effect

As shown in the preceding paper (UFT 145) the Minkowski metric can be derived from the orbital theorem of UFT 111 and in cylindrical polar coordinates is:

$$ds^2 = c^2\, dt^2 - d\,r^2 - r^2\, d\varphi^2 - dZ^2 \tag{1}$$

where

$$X = r\cos\varphi \quad , \quad Y = r\sin\varphi \quad . \tag{2}$$

Now rotate the metric in the XY plane for the null geodesic. This condition is:

$$ds^2 = d\,r^2 = dZ^2 = 0 \quad . \tag{3}$$

The rotation takes place at an angular frequency Ω and is defined by the Born coordinate rotation of the infinitesimal of angle as follows:

$$d\varphi' = d\varphi + \Omega\, dt \quad . \tag{4}$$

From these equations:

$$c^2\, dt^2 = r^2\, (d\varphi + \Omega\, dt\,) \tag{5}$$

so the infinitesimal of time is defined by:

$$dt = \pm\, \frac{r}{c}\, (d\varphi + \Omega\, dt\,) \quad . \tag{6}$$

Therefore the Sagnac effect is:

$$dt = \frac{d\varphi}{\omega \pm \Omega} \tag{7}$$

where the angular frequency ω is defined by

$$\omega = \frac{c}{r} \quad . \tag{8}$$

From Eq. (7) the time taken to traverse 360º or 2π radians around a circle is:

$$t = \frac{2\pi}{\omega \pm \Omega} \quad . \tag{9}$$

The rotational angular frequency Ω of the metric is that of the Sagnac platform. Therefore as in UFT 45 and 46 (www.aias.us) the rotation of the platform is the rotation of the frame of reference itself. In ECE theory this is a concept of general relativity and unified field theory. The latter manifests itself as in UFT 145 for example, as an effect of gravitation on light traversing the perimeter of the Sagnac platform. The Sagnac effect demonstrates precisely that physics is a unified field theory, and in ECE the effect of gravitation on light is deduced from the orbital theorem of UFT 111.

The difference in time for light traversing the spinning platform clockwise and anticlockwise is:

$$\Delta t = 2\pi \left(\frac{1}{\omega - \Omega} - \frac{1}{\omega + \Omega}\right) \tag{10}$$

$$= 2\pi\, r \left(\frac{1}{v - c} - \frac{1}{v + c}\right)$$

where we have used:

$$\omega = \frac{c}{r} \quad , \quad \Omega = \frac{v}{r} \quad . \tag{11}$$

Here $\boldsymbol{v}$ is the tangential linear velocity at the rim of a circular platform of radius $\boldsymbol{r}$ rotating with an angular velocity Ω in radians per second. Eq. (10) has been discussed for about a century, because the speed of light in one direction is $c - v$, and in the opposite direction $c + v$. In special relativity the speed c cannot be exceeded. However, special relativity is defined as one frame moving with respect to another linearly. In this context the Minkowski frame is static and unchanged. The Sagnac effect is derived as just shown by ROTATION of the Minkowski frame itself, and as such is not definable as special relativity. The latter always means a static Minkowski frame. The Maxwell Heaviside (MH) equations of special relativity are defined in a static Minkowski frame, and so MH cannot describe the Sagnac effect as is well known. The ECE theory describes the Sagnac effect as in UFT 45 and 46 as a frame rotation in generally covariant unified field theory [1-10]. The Sagnac effect has therefore been derived in a fully relativistic context, giving the simple result of Eq.(10). The latter can be developed straightforwardly as follows in terms of the area Ar of the Sagnac platform. For a circular platform:

$$Ar = \pi r^2 \tag{12}$$

where r is the radius of the platform with circumference $2\pi r$. From Eq. (10):

$$\Delta t = \frac{4\Omega Ar}{(c - v)(c + v)} \quad . \tag{13}$$

For non relativistic velocity v :

$$v << c \tag{14}$$

and Eq. (13) is approximated by:

$$\Delta t \simeq \frac{4\Omega Ar}{c^2} \quad . \tag{15}$$

The Sagnac effect is proportional to the product of the area Ar enclosed by the light beam and the angular velocity of the spinning platform, Ω , in radians per second. Sagnac interferometers are very sensitive to rotational motion, as is well known in gyro technology, and have a phenomenal frequency resolution of up to one part in 10^{25} .

The phase difference in radians due to Δt of Eq. (10) is:

$$\Delta\varphi = \omega\, \Delta t \tag{16}$$

$$\Delta\varphi = 2\pi r \left(\frac{\omega}{(c - v)} - \frac{\omega}{(c + v)} \right)$$

where:

$$\Delta\varphi \simeq \frac{4\omega\Omega Ar}{c^2} \quad . \tag{17}$$

From Eq. (16) the following wave numbers can be defined classically:

$$\kappa_1 = \frac{\omega}{(c - v)} \;, \;\; \kappa_2 = \frac{\omega}{(c + v)} \tag{18}$$

so that:

$$\Delta\varphi = 2\pi r\,(\kappa_1 - \kappa_2) \;\; . \tag{19}$$

From Eq. (13):

$$\Delta\varphi = 4\left(\frac{\Omega}{\omega}\right)\kappa_1\kappa_2\, Ar \tag{20}$$

so the phase change $\Delta\varphi$ is:

$$\Delta\varphi = 2\pi r\,(\kappa_1 - \kappa_2) = 4\left(\frac{\Omega}{\omega}\right)\kappa_1\kappa_2\, Ar \;\; . \tag{21}$$

This is an example of the ECE phase introduced in UFT 6 and 9 (www.aias.us):

$$\Delta\varphi = \oint \boldsymbol{\kappa} \,.\, \mathrm{d}\boldsymbol{r} = \int \kappa^2 \mathrm{d}\, Ar \quad . \tag{22}$$

The ECE phase is the source of all phase phenomena in physics, such as the Berry phase, the topological phases, the Aharonov Bohm and Wu Yang phase, the Tomita Chiao phase change and so forth. The Sagnac effect can be thought of as the Tomita Chiao helical fibre, in which plane polarized light is rotated simply by traversing the helical fibre. A very high resolution fibre optic gyro can be constructed by winding an optical fibre many times on a drum. There is no explanation for this gyro in MH theory. So large areas of optics cannot be described by MH theory, they need ECE theory for basic description.

It has been shown that rotation of the Minkowski metric produces the effect:

$$\omega_r = \omega \pm \Omega \quad , \quad \kappa_r = \frac{\omega}{c \pm v} \quad . \tag{23}$$

If the velocity v_r is defined as in conventional optics [12] by:

$$\kappa_r = \frac{\omega_r}{v_r} = (\mu \in)^{1/2}\omega_r \tag{24}$$

where μ and $\in$ are permeability and permittivity, then the refractive index is, formally:

$$n = \frac{c}{v_r} \quad . \tag{25}$$

Therefore the Sagnac effect can be thought of in terms of two refractive indices:

$$n_{+} = \left(\frac{\omega}{(\omega + \Omega)}\right)\left(\frac{c}{(c+v)}\right) = \left(\frac{c}{(c+v)}\right)^{2} \tag{26}$$

and

$$n_{-} = \left(\frac{\omega}{(\omega - \Omega)}\right)\left(\frac{c}{(c-v)}\right) = \left(\frac{c}{(c-v)}\right)^{2} \tag{27}$$

giving the birefringence:

$$\Delta n = n_{+} - n_{-} = 4\, v\, c \left(\frac{c}{c^{2} - v^{2}}\right)^{2} \quad . \tag{28}$$

The Sagnac effect can also be defined as the mechanically induced rotation of the electromagnetic phase:

$$e^{\,i(\omega \pm \Omega)\mathrm{t}} = e^{\,i\omega \mathrm{t}}\; e^{\,\pm i\Omega \mathrm{t}} \tag{29}$$

rotation induced by the rotation generator [12]:

$$\phi = e^{\,\pm i\Omega \mathrm{t}} \tag{30}$$

and this concept has the important consequence that mechanical rotation affects the phase of electromagnetic radiation under any circumstance. Therefore spectra of all kinds are affected by mechanical rotation of all kinds in any region of the electromagnetic spectrum. In the notes to Paper 146 posted on www.aias.us and accompanying this paper, an instrument is designed to pick up this effect, being a combination of a Sagnac and Michelson Fourier transform interferometer.

This rotation generator description of the Sagnac effect can be generalized in ECE theory as the phase

$$\gamma^{a} := \kappa \int T^{a}_{\mu\nu}\, \mathrm{d}\sigma^{\mu\nu} \tag{31}$$

where κ has the dimensions of wave number and where $T^{a}_{\mu\nu}$ is the Cartan torsion

$$T^{a}_{\mu\nu} = \partial_{\mu} q^{a}_{\nu} - \partial_{\nu} q^{a}_{\mu} + \omega^{a}_{\mu\nu} - \omega^{a}_{\nu\mu} \quad . \tag{32}$$

Here $\mathrm{d}\sigma^{\mu\nu}$ is the area infinitesimal in four dimensions, q^{a}_{μ} is the Cartan tetrad, and $\omega^{a}_{\mu\nu}$ is the Cartan spin connection. Consider the unit diagonal tetrad:

$$q^{a}_{\mu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

then the torsion is defined by a mixed index wave number tensor:

$$\kappa_{\mu\nu}^{a} = T_{\mu\nu}^{a} = 2\ \omega_{\mu\nu}^{a} \quad . \tag{34}$$

Using the well known complex circular basis [1-10]:

$$a = (1), (2), (3) \tag{35}$$

consider the special case of propagation along the Z axis, so:

$$T_{12}^{(3)} = \kappa_{12}^{(3)} \tag{36}$$

which is the tensor equivalent of the scalar valued vector component:

$$T_{3}^{(3)} = \kappa_{3}^{(3)} \tag{37}$$

where the wave number vector is:

$$\boldsymbol{\kappa} = \boldsymbol{\kappa}^{(3)} = \kappa_{3}^{(3)}\mathbf{k} \quad . \tag{38}$$

This vector is part of the four-wave number:

$$\kappa_{\mu}^{(3)} = (\kappa_{0}^{(3)}, -\boldsymbol{\kappa}^{(3)}) \quad . \tag{39}$$

Using the ECE phase theorem (22), the phase (31) is therefore

$$\gamma^{a} := \kappa \int \kappa_{\mu\nu}^{a}\ d\sigma^{\mu\nu} = \oint \kappa_{\mu}^{a}\ dx^{\mu} \quad . \tag{40}$$

In the Sagnac effect, Eq. (40) shows that:

$$\Delta \gamma^{(3)} = \omega\ \Delta t = 2\pi \left(\frac{\omega}{(\omega - \Omega)} - \frac{\omega}{(\omega + \Omega)} \right) \quad . \tag{41}$$

Finally in this section recall that in UFT 145 the effect of gravitation on the Sagnac effect was shown to change:

$$\frac{d\varphi}{dt} = \omega \pm \Omega \tag{42}$$

to

$$\frac{d\varphi}{dt} = x\omega \pm \Omega \tag{43}$$

where:

$$x = (1 - \frac{2MG}{c^2 R})^{½} \quad . \tag{44}$$

Here M is the mass of a gravitating object that acts on the photon of mass m traversing the circumference of the Sagnac platform, G is Newton's constant, and R is the distance between m and M. This effect can be used as follows to measure the gravitomagnetic field on the surface of the Earth (UFT 117 and 119) within the context of ECE theory:

$$\boldsymbol{\Omega}_g = -\frac{1}{c^2}\, \boldsymbol{v} \times \boldsymbol{g} = \frac{mG}{c^2 R^3}\, \boldsymbol{L} \quad . \tag{45}$$

Here $\boldsymbol{L}$ is the mean angular momentum of the Earth, considered in a first approximation as a sphere with angular velocity ω_E at the equator:

$$L = \frac{2}{5} m R^2 \omega_E \quad . \tag{46}$$

Therefore:

$$\Omega_g = \frac{\omega_E}{5} \left(\frac{2mG}{c^2 R}\right) \tag{47}$$

and the Sagnac effect is modified by the gravitomagnetic field to:

$$\frac{d\varphi}{dt} = \left(1 - \frac{5\Omega_g}{\omega_E}\right)^{½} \omega \pm \Omega \quad . \tag{48}$$

The frequency ω is therefore shifted to:

$$\omega \longrightarrow \left(1 - \frac{5\Omega_g}{\omega_E}\right)^{½} \omega \tag{49}$$

$$\Delta\omega \simeq \frac{5}{2} \frac{\Omega_g \omega}{\omega_E} \quad . \tag{50}$$

Using the measured quantities:

$$\left.\begin{aligned} \omega_E &= 7.29 \times 10^{-5} \text{ rad s}^{-1} \\ m &= 5.98 \times 10^{24} \text{ kgm} \\ R &= 6.37 \times 10^{6} \text{ m} \end{aligned}\right\} \tag{51}$$

the gravitomagnetic angular frequency at the Earth' equator on the surface is:

$$\Omega_g = 2.03 \times 10^{-14} \text{ rad s}^{-1} \quad . \tag{52}$$

One year is 3.156×10^7 seconds, and one radian is 2.0626×10^5 seconds of arc (arcseconds), so

$$\Omega_g = 0.13 \text{ arcseconds per year} \quad . \tag{53}$$

The frequency resolution of large area Sagnac interferometers is such that this may be measurable in such an instrument located on the surface at the equator, or indeed anywhere on the surface.

3. Development of relativity from rotation of the Minkowski metric.

Eq. (5) may be written as:

$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) dt^2 = \frac{1}{\omega^2}\left(d\varphi^2 + 2\,\Omega\, d\varphi\, dt\right) \tag{54}$$

from which the infinitesimal of proper time may be defined as:

$$d\tau = \frac{dt}{\gamma} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt \tag{55}$$

and the relativistic angular velocity as:

$$\omega' = \omega \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad . \tag{56}$$

The Thomas precession is then the phase difference:

$$\alpha = \omega'\tau - \omega t = (\gamma - 1)\, \omega t \tag{57}$$

as discussed in UFT 145 (www.aias.us). Eq. (57) is equivalent to the relativistic angle formula for the Thomas precession (its simplest description):

$$\theta' = \gamma\, \theta \quad . \tag{58}$$

The well known:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{59}$$

is that of the Lorentz boost in special relativity. The proper time is a Lorentz invariant and is defined [13] as:

$$\tau = \frac{t}{\gamma} = (1 - \frac{v^2}{c^2})^{½} t \quad . \tag{60}$$

For a particle with mass, the proper time is the time in the frame of reference in which the particle is instantaneously at rest, but the photon, being theoretically massless, has no rest frame, so its proper time from Eq. (55) is:

$$d\tau = 0 \quad . \tag{61}$$

The well known β factor of special relativity is:

$$\beta = \frac{v}{c} \quad . \tag{62}$$

These factors are usually derived through the Lorentz boost [13, 14] as is well known, but can also be derived from rotation of the Minkowski frame. The factors γ and β are related by:

$$\gamma = (1 - \beta^2)^{-½} \quad . \tag{63}$$

These factors are therefore also features of the rotation of the Minkowski frame and of the Thomas precession. The latter was derived in 1927 as the Thomas angular velocity [12]:

$$\boldsymbol{\omega}_T = \left(\frac{\gamma^2}{1+\gamma}\right) \frac{\boldsymbol{a} \times \boldsymbol{v}}{c^2} \tag{64}$$

due to the cross product of acceleration $\boldsymbol{a}$ and velocity $\boldsymbol{v}$. Eqs. (57) and (64) seem without further analysis to bear no relation to each other, but are both features of rotation of the Minkowski metric. Also, the well known formula [12]:

$$c^2 t'^2 - (X'^2 + Y^2 + Z'^2) = c^2 t^2 - (X^2 + Y^2 + Z^2) \tag{65}$$

from which the Lorentz boost is derived can be thought of as the invariant of the rotation of the four vector:

$$x^\mu = (ct, X, Y, Z) \tag{66}$$

in the Minkowski spacetime. This is seen from the fact that the equivalent of Eq. (65) in three dimensions is the well known invariant for rotation in three dimensions

$$X'^2 + Y'^2 + Z'^2 = X^2 + Y^2 + Z^2 \quad . \tag{67}$$

So the Lorentz boost is an algebraic consequence of the rotation of x^μ in Minkowski spacetime, and Eq. (4) is similar: rotation of the Minkowski metric. It follows that all the equations of what is known as "special relativity" are equations that derive from a rotation of the Minkowski metric. The Sagnac effect is the special case:

$$ds^2 = dr^2 = dZ^2 = 0 \tag{68}$$

of a null geodesic in two dimensions, X and Y.

We may proceed to construct what is known as "special relativity" by defining the relativistic momentum [13]:

$$\boldsymbol{p} = \gamma\, m\, \boldsymbol{v} = \gamma\, m \frac{d\boldsymbol{r}}{dt} = m \frac{d\boldsymbol{r}}{d\tau} \tag{69}$$

using the proper time, which is derived from the rotating metric of Eq.(4). A more precise and self consistent definition of "special relativity" follows, because rotation of the metric also produces the result of Eq.(60) and there is no paradox or conflict with the idea that c cannot be exceeded. Using well known methods, written out in full in the notes accompanying this paper (notes to UFT 146 on www.aias.us), the relativistic kinetic energy:

$$T = mc^2(\gamma - 1) \tag{70}$$

can be derived from the relativistic momentum (69). The Thomas precession is the phase shift:

$$\alpha = (\gamma - 1)\, \theta \tag{71}$$

where:

$$\theta = \omega t \tag{72}$$

so the relativistic kinetic energy is:

$$T = mc^2 \frac{\alpha}{\theta} \tag{73}$$

where the rest energy is:

$$E_0 = mc^2 \quad . \tag{74}$$

The Einstein energy equation:

$$E^2 = p^2c^2 + {E_0}^2 \tag{75}$$

is a direct consequence [13] of Eq. (69), and in Eq. (75), E is the total energy defined by:

$$E = T + mc^2 = mc^2 \left(1 + \frac{\alpha}{\theta} \right) \quad . \tag{76}$$

So both E and T are defined by the phase α of the Thomas precession (71), and the Thomas precession is the rotation of the Minkowski metric.

This classical analysis can be extended to quantum mechanics by using the operator equivalence [11]:

$$p^{\mu} = i\,\hbar\,\partial^{\mu} \tag{77}$$

with Eq. (75). The details of this procedure are given in the accompanying notes to UFT 146 (www.aias.us). . The result is the Dirac equation in its wave format [1-11]:

$$(\square + \kappa^2)\,\psi = 0 \quad . \tag{78}$$

Here $\hbar$ is the reduced Planck constant, m is the mass of the electron, ψ is the Dirac spinor, and κ^{-1} is the Compton wavelength:

$$\frac{1}{\kappa} = \frac{\hbar}{mc} \quad . \tag{79}$$

In UFT 129 and 130, Eq. (78) is derived from the tetrad postulate in the context of ECE theory. By using the factorization of the Minkowski metric into the Dirac gamma matrices:

$$2g^{\mu\nu} = \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} \quad . \tag{80}$$

Eq. (78) reduces to the first order differential format of the Dirac equation. Again details are given in the accompanying notes to UFT 146 on www.aias.us. The factorization (80) automatically introduces the Pauli matrices:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_Y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{81}$$

and SU(2) representation space. So the Dirac equation is the rotation of a Minkowski spacetime after application of the operator equivalence (77) of quantum mechanics. In UFT 129 and 130 it was also shown that the Dirac equation can be written with 2 x 2 matrices without the need for the 4 x 4 gamma matrices of Dirac.

Finally, the accompanying notes to this paper (UFT 146 on www.aias.us) give all details of how the g factor of the electron and the Thomas 1 / 2 factor of spin orbit coupling are deduced from the Dirac equation, which can be thought of as a rotation of Pauli spinors in SU(2) representation space [1-11]. The latter type of rotation has therefore been traced in this paper to the rotation (4) of the Minkowski metric and the Minkowski metric itself has been shown in UFT 111 (www.aias.us) to be a very simple consequence of the ECE orbital theorem, the Frobenius theorem for spherically symmetric spacetimes. It appears that all other metrics of physics may also be deduced from the simple but powerful orbital theorem.

Acknowledgments

The British Government is thanked for a Civil List Pension, and many colleagues worldwide for interesting discussions.

References

[1] M. W. Evans et al., "Generally Covariant Unified Field Theory" (Abramis Academic, Suffolk, 2005 onwards), in seven volumes to date.

[2] M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (Abramis in press, 2010).

[3] M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002), in five volumes.

[4] M. W. Evans, "Modern Non-Linear Optics", (Second Edition, Wiley 2001), in three volumes.

[5] M. W. Evans and S. Kielich, ibid., (First edition, Wiley, 1992, 1993, 1997), in three volumes.

[6] M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

[7] M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).

[8] About 1,700 papers and books on the websites: www.aias.us (National Library of Wales and British Libraries permanent collection), www.atomicprecision.com and www.upitec.org.

[9] K. Pendergast, "The Life of Myron Evans" (Abramis / Arima, 2010, in press).

[10] ECE and precursor papers in a range of journals, 1992 to present, including "Foundations of Physics Letters", "Physica B", and conference journals.

[11] L .H. Ryder, "Quantum Field Theory" (Cambridge, 2nd ed., 1996).

[12] J. D. Jackson, "Classical Electrodynamics" (Wiley, 3rd. Ed., 1999).

[13] J. B. Marion and S. T. Thornton, "Classical Dynamics" (HBC, 3rd ed., 1988).

[14] P. W. Atkins, "Molecular Quantum Mechanics" (Oxford, 2nd (1983) and subsequent editions).