Equations of motion from the Minkowski metric.

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Abstract

A kinetic equation of motion is derived from the Minkowski metric in terms of constants of motion. It is shown that all known orbits can be described when the free Minkowski metric is constrained by relations between infinitesimals. The free metric corresponds to the limit when there is no inverse square law force of attraction present, and the free metric gives rise to Kepler's second law, valid for all orbits. When the inverse square law is introduced, relations appear between infinitesimals of the metric, the metric is constrained by experimental data. All known orbits can be described by a constrained Minkowski metric. It is shown that the precessing elliptical orbits of planets and the precessing elliptical orbital of the electron in a hydrogen atom can be obtained from a new type of metric which adds an inverse square attraction term to the Minkowski metric.

Keywords: ECE metric theory, equations of motion from the Minkowski metric.

1. Introduction

In earlier papers of this series [1-10] the Minkowski and gravitational metrics have been shown to be solutions of the ECE Orbital Theorem of UFT 111 (www.aias.us) of this series of 149 papers to date. It has been shown that the Einstein field equation is incorrect (UFT 139) because of its use of a symmetric connection. The latter must take the antisymmetry of the commutator by definition. Contemporary scholarship has also shown that Schwarzschild did not derive the gravitational metric incorrectly named after him. The Minkowski and gravitational metrics must be derived from a correct theory, the simplest of which is the ECE Orbital Theorem. Contemporary astronomical observation shows that the orbits of stars in whirlpool galaxies cannot be described even qualitatively by the gravitational metric. However, in UFT 148 of this series it was shown that all orbits can be based on the simpler Minkowski metric provided that its infinitesimals are constrained by data from orbital observation. By correctly incorporating torsion into the basic geometry of relativity it has been shown [1-10] that all the metrics derived [11] from the Einstein field equation are incorrect mathematically, so it is no surprise that they cannot describe the totality of cosmological data now known. The use of dark matter is rejected in the rigorously relativistic ECE theory as unscientific, or at best wholly empirical or ad hoc.

In Section 2 the kinetic equation of motion of the free Minkowski metric is derived using the action and Lagrange equation to define three constants of motion of the free Minkowski metric: the relativistic energy E, the relativistic momentum p, and the relativistic angular momentum L. This method produces an equation of motion which is shown to be a well defined limit of the equation of motion derived from the gravitational metric. In Section 3 the effect of introducing for example an inverse square law of attraction is shown to be equivalent to constraining the free Minkowski metric with well defined mathematical relations between the infinitesimals of the metric. These relations are derived from orbital data of all kinds in the same way that the inverse square law must in the last analysis be derived from data. The free metric has no such constraints, and gives an equation of motion equivalent to Kepler's second law, a purely geometrical law applicable to any orbit.

In Section 3 the free Minkowski metric is constrained with an inverse square law of attraction, and the resulting metric is shown to produce a precessing elliptical orbit without use of the gravitational metric at all. We refer to the gravitational metric as the one which is incorrectly referred to in the twentieth century literature [2] as the "Schwarzschild metric" although he did not derive it in his only two papers of 1916 [2]. He derived a different metric which had no singularity.

2. Equation of motion from the free Minkowski metric

The Minkowski metric in cylindrical polar coordinates [12] is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\boldsymbol{r} \cdot d\boldsymbol{r} \tag{1}$$

where

$$\mathrm{d}\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r} = \mathrm{d}\, r^2 + r^2\, \mathrm{d}\varphi^2 + \mathrm{d}Z^2 \qquad (2)$$
$$= \mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2 \quad .$$

The total linear velocity is defined as:

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} \quad . \qquad (3)$$

Therefore:

$$c^2 \mathrm{d}\tau^2 = c^2\, \mathrm{d}t^2 - v^2\, \mathrm{d}t^2 \qquad (4)$$

so

$$\mathrm{d}\tau^2 = \left(1 - \frac{v^2}{c^2} \right) \mathrm{d}t^2 \quad . \qquad (5)$$

The infinitesimal of proper time is:

$$\mathrm{d}\tau = \left(1 - \frac{v^2}{c^2} \right) \mathrm{d}t^2 \qquad (6)$$

and

$$\mathrm{d}t = \gamma\, \mathrm{d}\tau \qquad (7)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-½} \quad . \qquad (8)$$

If we restrict attention to the XY plane, then:

$$\mathrm{d}Z = 0 \quad . \qquad (9)$$

The rest energy is defined as:

$$E_0 = mc^2 = m\left(\frac{\partial S}{\partial \tau} \right)^2 = m\, g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \qquad (10)$$

where S is the action [12]. Therefore:

$$E_0 = mc^2 \left(\frac{dt}{d\tau}\right)^2 - m\left(\frac{dr}{d\tau}\right)^2 - mr^2\left(\frac{d\varphi}{d\tau}\right)^2 \quad . \qquad (11)$$

The Lagrange equation for this system is:

$$\frac{d}{d\tau}\left(\frac{\partial E_0}{\partial \dot{x}^{\mu}}\right) = \frac{\partial E_0}{\partial x^{\mu}} = 0 \tag{12}$$

so

$$\frac{d}{d\tau}\left(mr^2 \frac{d\varphi}{d\tau}\right) = 0 \ , \tag{13}$$

$$\frac{d}{d\tau}\left(m \frac{dr}{d\tau}\right) = 0 \ , \tag{14}$$

$$\frac{d}{d\tau}\left(mc^2 \frac{dt}{d\tau}\right) = 0 \ . \tag{15}$$

From Eqs. (13) to (15) the constants of motion of the system are:

$$E = mc^2 \frac{dt}{d\tau} = \gamma mc^2 \quad , \tag{16}$$

$$L = mr^2 \frac{d\varphi}{d\tau} = \gamma mr^2 \frac{d\varphi}{dt} \quad , \tag{17}$$

$$p_l = m \frac{dr}{d\tau} = \gamma m \frac{dr}{dt} \quad , \tag{18}$$

where E is relativistic energy, p_l is the central component of the relativistic momentum and L is relativistic angular momentum. Therefore the equation of motion is:

$$(\gamma^2 - 1)\, mc^2 = \frac{p_l{}^2}{m} + \frac{L^2}{mr^2} \tag{19}$$

which is the Einstein energy equation:

$$E^2 = c^2\, p^2 + m^2\, c^4 \tag{20}$$

where

$$p^2 = \frac{1}{c^2}\left(E^2 - m^2\, c^4\right) = \gamma^2 m^2\, v^2$$

$$= p_l{}^2 + \frac{L^2}{r^2} \tag{21}$$

is the total relativistic momentum defined by [12]:

$$\boldsymbol{p} = \gamma m\, \boldsymbol{v} \quad . \tag{22}$$

Therefore E is the total relativistic energy defined [12] by:

$$E = \gamma mc^2 \quad . \tag{23}$$

Here T is the relativistic kinetic energy defined by

$$T = (\gamma - 1)\, mc^2 \tag{24}$$

and E_0 is the rest energy. The equation of motion may be written as:

$$(\gamma + 1)\, T = \frac{p^2}{m} \quad . \tag{25}$$

Note carefully that the total relativistic momentum is defined as the sum of a linear relativistic momentum and angular term.

Eq. (25) is equivalent to the description:

$$m\, v^2 = m\, \left(\left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\varphi}{dt}\right)^2\right) \tag{26}$$

i.e.

$$d\boldsymbol{r} \,.\, d\boldsymbol{r} = c^2\, (dt^2 - d\tau^2) = v^2\, dt^2 \quad . \tag{27}$$

Using:

$$\frac{d\varphi}{dt} = \frac{d\varphi}{dr}\frac{dr}{dt} \tag{28}$$

Eq. (26) becomes:

$$m\, v^2 = m\, \left(\frac{dr}{dt}\right)^2 \left(1 + r^2 \left(\frac{d\varphi}{dr}\right)^2\right) \tag{29}$$

which is an orbital equation:

$$\frac{d\varphi}{dr} = \frac{\dot{r}}{r}\, (v^2 - \dot{r}^2)^{-1/2} \tag{30}$$

The above equation is a kinetic energy equation from what we term "the free Minkowski metric". In its non relativistic limit:

$$(\gamma^2 - 1)\, m\, c^2 \longrightarrow \left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - 1\right) m\, c^2 \tag{31}$$

$$\sim \quad m\, v^2$$

so the non relativistic limit of Eq. (19) is

$$T = \frac{1}{2}\, m\, v^2 = \frac{p^2}{2\, m} \quad . \tag{32}$$

In this limit:

$$L \longrightarrow mr^2 \frac{d\varphi}{dt} \tag{33}$$

$$p \longrightarrow m \frac{dr}{dt} \tag{34}$$

In the conventional non-relativistic treatment of orbits [12] the second term on the right hand side of Eq. (19) is called "the centrifugal potential energy". However, it is not potential energy at all, it is part of the kinetic energy.

3. The constrained Minkowski metric.

All metrically based equations of motion are kinetic in nature because the Lagrangian is defined in relativity as the well known:

$$\mathcal{L} = T = \frac{1}{2}\, m\, c^2 = \frac{1}{2}\, m\, g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \tag{35}$$

and is defined as being the kinetic energy T. Therefore:

$$\mathcal{L} = H = T \tag{36}$$

where H is the hamiltonian. The free Minkowski metric developed in Section 2 is the metric in which there is no functional relation between the infinitesimals dr and dφ . The effect of introducing what is known in Newtonian dynamics as "force of attraction" is to introduce a relation between dr and dφ in a Minkowski metric. This is the principle of orbits introduced in UFT 148 (www.aias.us). For example, if the orbit is observed in astronomy to be the precessing ellipse [12]:

$$r = \frac{\alpha}{(\,1 + \epsilon \cos y\varphi)} \tag{37}$$

the constrained Minkowski metric that describes the orbit is derived by differentiating Eq. (37) to produce:

$$\frac{dr}{d\varphi} = \left(\frac{y\epsilon}{\alpha} \sin(y\varphi) \right) r^2 \quad . \tag{38}$$

So there appears an additional constraint, a relation between the infinitesimals dr and dφ. In the free Minkowski metric of Section 2, this constraint is absent. Here ϵ is the eccentricity, and α and y are observed parameters. It was shown by Sommerfeld (see accompanying notes) that a precessing ellipse can be obtained from a hamiltonian which when adopted for gravitation becomes:

$$H = (\gamma - 1)\, m\, c^2 - \frac{mMG}{r} \tag{39}$$

in which a mass m is attracted to a mass M a distance r away, and in which G is the Newton constant. It is shown as follows that all the experimental features of precessing elliptical orbits and orbitals can be obtained from the following metric:

$$ds^2 = c^2 dt^2 \left(1 - \frac{r_0}{r} \right) - d\boldsymbol{r} \cdot d\boldsymbol{r} \tag{40}$$

where

$$r_0 = \frac{2mG}{r} \quad . \tag{41}$$

This is the Minkowski metric with the time infinitesimal changed by:

$$dt^2 \longrightarrow \left(1 - \frac{2MG}{c^2 r} \right) dt^2 \tag{42}$$

as observed in the gravitational red shift. Note carefully that the metric (40) is not the usual metric (mis-called "the Schwarzschild metric"):

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r} \right) - \left(1 - \frac{r_0}{r} \right)^{-1} dr^2 - r^2 d\varphi^2 \tag{43}$$

associated with gravitation. The metric (43) is incorrectly attributed to Schwarzschild and is obtained from the incorrect [1-10] Einstein field equation.

In the new metric (40) the free Minkowski metric is constrained by the presence of an additional term in dt^2. The new metric (40) can be written as:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\boldsymbol{r} \cdot d\boldsymbol{r} - \frac{2MG}{r} dt^2 \tag{44}$$

and its Lagrangian and Hamiltonian are:

$$\mathcal{L} = H = T = \frac{1}{2} m c^2 = \frac{1}{2} m c^2 \left(1 - \frac{r_0}{r} \right) \left(\frac{dt}{d\tau} \right)^2 - \frac{m}{2} \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\varphi}{d\tau} \right)^2 \right) \quad . \tag{45}$$

For purposes of comparison only, the mis-called Schwarzschild metric is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2\, d\varphi^2 - \frac{2MG}{r}\, dt^2 \tag{46}$$

and its Lagrangian and Hamiltonian are:

$$\mathcal{L} = H = T = \frac{1}{2}\, m\, c^2 = \frac{1}{2}\, m\, c^2 \left(1 - \frac{r_0}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - \frac{m}{2}\left(1 - \frac{r_0}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 - \frac{m}{2}\, r^2 \left(\frac{d\varphi}{d\tau}\right)^2 \ . \tag{47}$$

The Lagrange equation of the new metric is obtained from the Lagrangian (45) as:

$$\frac{d}{d\tau}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu}}\right) = \frac{\partial \mathcal{L}}{\partial x^{\mu}} = 0 \tag{48}$$

giving the three equations:

$$E = mc^2 \left(1 - \frac{r_0}{r}\right)\frac{dt}{d\tau} = \text{constant} \qquad , \tag{49}$$

$$p_l = m\frac{dr}{d\tau} = \text{constant} \qquad , \tag{50}$$

$$L = mr^2 \frac{d\varphi}{d\tau} = \text{constant} \qquad , \tag{51}$$

and the constants of motion of the new metric (44).

With these definitions we obtain a new equation of motion:

$$\frac{1}{2}\left(1 - \frac{r_0}{r}\right)\frac{{p_l}^2}{m} = \frac{E^2}{2mc^2} - \frac{1}{2}\left(1 - \frac{r_0}{r}\right)\left(mc^2 - \frac{L^2}{mr^2}\right) \quad . \tag{52}$$

The equation of motion from the gravitational metric (43) is:

$$\frac{1}{2}\frac{{p_l}^2}{m} = \frac{E^2}{2mc^2} - \frac{1}{2}\left(1 - \frac{r_0}{r}\right)\left(mc^2 - \frac{L^2}{mr^2}\right) \quad . \tag{53}$$

In both equations the right hand side is:

$$\text{RHS} = \frac{E^2}{2mc^2} - \frac{1}{2}\, mc^2 + \frac{mMG}{r} - \frac{L^2}{2mr^2} + \frac{MGL^2}{mc^2 r^3} \tag{54}$$

and gives a precessing elliptical orbit [12] as is well known. The method of obtaining the precessing ellipse in a text such as that by Marion and Thornton [12] is to define "an effective potential”:

$$U := -\frac{mMG}{r} - \frac{MGL^2}{mc^2r^3} + \frac{L^2}{2mr^2} \tag{55}$$

in the Newtonian limit. The inverse cubed term changes the Newtonian ellipse into a precessing ellipse. This is self-inconsistent because the original Lagrangian (36) is purely kinetic, there is no concept of “potential energy” or “force” in it. However, the end result, mathematically [12] is a precessing ellipse. In future work a more consistent method will be developed to derive the precessing ellipse, or any orbit, from a constrained Minkowski metric.

The Lagrangian and Hamiltonian in Eq. (52) are both made up of kinetic energy only. This equation can be written as

$$\frac{1}{2}\, mc^2 \left(\left(1 - \frac{r_0}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - 1 \right) = \frac{m}{2}\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\varphi}{d\tau}\right)^2 \right) \tag{56}$$

and in the non relativistic limits defined by:

$$\frac{m}{2}\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\varphi}{d\tau}\right)^2 \right) \longrightarrow \frac{1}{2}\, mv^2 \tag{57}$$

$$\frac{1}{2}\, mc^2 \left(\left(1 - \frac{r_0}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - 1 \right) = \frac{1}{2}\, mc^2 \left(\gamma^2 - 1\right) - \frac{mMG\gamma}{r} \tag{58}$$

$$\longrightarrow \frac{1}{2}\, mv^2 - \frac{mMG}{r} \quad .$$

Eq. (56) becomes

$$\frac{1}{2}\, mv^2 - \frac{mMG}{r} \sim \frac{1}{2}\, mv^2 \quad . \tag{59}$$

This must be interpreted to mean that:

$$\frac{1}{2}\, mv^2 \longrightarrow \frac{1}{2}\, mv^2 - \frac{mMG}{r} \tag{60}$$

related to geodesics and $-m\, r_0 / r$ is regarded in relativity as KINETIC, not potential, energy. In relativity there is no concept of potential energy or force of attraction, orbits are metrics related to geodesic. The classical concept of “energy of attraction” is introduced into the Minkowski metric by making the following change in the time infinitesimal:

$$dt^2 \longrightarrow \left(1 - \frac{r_0}{r}\right) dt^2 \tag{61}$$

The Sommerfeld hamiltonian is obtained as the non-relativistic limit of the left hand side of Eq. (56), the limit:

$$H = \frac{1}{2} m\, c^2 \left(\gamma^2 - 1 \right) - \frac{mMG}{r}\, \gamma \tag{62}$$

$$\longrightarrow m\, c^2 \left(\gamma - 1 \right) - \frac{mMG}{r} \longrightarrow \frac{1}{2} m\, v^2 - \frac{mMG}{r}$$

and this Hamiltonian was shown by Sommerfeld (see notes accompanying this paper) to give a precessing orbital for the electron in the H atom in the old quantum theory. The Dirac equation of the hydrogen atom gives the same result.

The non relativistic limit of the right hand side of Eq. (56) is:

$$H = \frac{m}{2} \left(\left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\varphi}{d\tau}\right)^2 \right) \tag{63}$$

In this limit the usual result [12] is obtained for the kinetic energy of a planar orbit:

$$H = T = \frac{1}{2} m\, v^2 = \frac{m}{2} \left(\dot{r}^2 + r^2 \dot{\varphi}^2 \right) \tag{64}$$

The right hand sides of Eqs. (52) and (53) are the same, so the so called "effective potential" is the same in the new metric and the mis called Schwarzschild metric, so the same precessing ellipse is obtained with the method of Marion and Thornton [12]. Both metrics give the observed orbit accurately. However, the left hand sides of the two equations are not the same. In the new metric (44) the observation of the orbit means that the central component of the kinetic energy is observed to be:

$$T_l = \frac{1}{2} \left(1 - \frac{r_0}{r} \right) m \left(\frac{dr}{d\tau} \right)^2 \tag{65}$$

but in the mis-called Schwarzschild metric it is observed to be:

$$T_l = \frac{1}{2}\, m \left(\frac{dr}{d\tau} \right)^2 \quad . \tag{66}$$

Both the metric (43) and the metric (44) give the same precessing ellipse (37), so both are described by the constrained Minkowski metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - \left(1 + x^2 c^2 \right) r^2\, d\varphi^2 \quad , \tag{67}$$

$$x = \frac{y\epsilon}{\alpha} \sin(y\varphi) \quad .$$

In ECE theory the gravitational metric (43) is obtained as a possible solution of the Orbital Theorem of UFT 111, and NOT from the incorrect Einstein equation. Since the two apparently different metrics (43) and (44) give the same result for the precessing ellipse, it is concluded once more that the Einstein equation is incorrect, giving an infinite number of meaningless metrics [2]. The correct method is to use orbital data to produce a constrained Minkowski metric, and from that obtain tetrads, torsion and curvature in Cartan geometry and ECE theory [1-10].

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