Metrics for gravitation and electromagnetism in spherical and cylindrical spacetime.

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Abstract

Spherical and cylindrical symmetry of spacetime is used to develop metrics for gravitation and electromagnetism, and the interaction of gravitation and electromagnetism. The metrics are used to define the lagrangian and hamiltonian, equations of motion and orbital equations. In an early approximation, the electrodynamical metric is shown to reduce correctly to the minimal prescription, relativistic Hamilton Jacobi equation and Dirac equation for an electron interaction with a four potential in the minimal prescription. Some computations are given of the effect of successive terms in the approximation of the metric for spherically symmetric spacetime. This development is based on a basic concept of ECE theory, that equations of dynamics and electrodynamics have the same format.

Keywords: ECE theory, metric, gravitation, electrodynamics, interaction of gravitation and electromagnetism, relativistic Hamilton Jacobi equation, Dirac equation.

1. Introduction

In this paper , the 152nd in a series of papers [1–10] developing Einstein Cartan Evans (ECE) theory, a new metrical analysis of gravitation and electrodynamics is developed in order to give a coherent description of these fundamental fields from the basic structure of spacetime, and in order to show straightforwardly that there is electrodynamical energy in spacetime. This energy can be deduced directly from the metric by constructing the lagrangian and hamiltonian, following well known methods for gravitation. It is well known and well accepted [11, 12] that the lagrangian and hamiltonian of gravitation can be obtained directly from the metric, so it follows that the hamiltonian and lagrangian of electromagnetism can also be obtained directly from the metric. Both gravitation and electromagnetism are manifestations of the metric that represents spherical spacetime or spacetime of some chosen symmetry such as cylindrical symmetry. It has long been accepted that the hamiltonian of gravitation is due to the metric, and so the hamiltonian of electromagnetism and the unified field are also obtained directly from the same metric in ECE unified field theory. The hamiltonian is conserved, as is well known, so the theory conserves total energy as a constant of motion. The theory also conserves linear and angular momenta as the other constants of motion. Therefore electromagnetic power can be obtained from spacetime while conserving energy and momentum.

In Section 2, the assumption of spherically symmetric spacetime is used to deduce the general format of the metric for gravitation and electromagnetism and the unified field. Successive approximations of the general metric are used to deduce the hamiltonian, lagrangian, equations of motion and orbital equations. Some considerations are given to a metric in cylindrically symmetric spacetime. In Section 3 the minimal prescription is recovered correctly from the electrodynamical metric and used to deduce the relativistic Hamilton Jacobi equation and the Dirac equation of quantum field theory from the metric in a first approximation. It follows that the Dirac equation is a first approximation to a more accurate equation hitherto unknown. The problem of interaction of electromagnetism and gravitation can also be addressed using this metrical method. In Section 4 some numerical results are given of the effect of adding successive terms to the approximation of the metric for spherically symmetric spacetime.

2. Metrics from spherical and cylindrical spacetime.

For spherically symmetric spacetime, we start with the metric [1–10]:

$$\mathrm{d}s^2 = c^2\mathrm{d}\tau^2 = e^{-r_0/r}\, c^2\mathrm{d}t^2 - e^{r_0/r}\mathrm{d}r^2 - r^2\,\mathrm{d}\varphi^2 \tag{1}$$

in cylindrical polar coordinates in the XY plane. Even more general metrics of spherical spacetime may be used, but Eq. (1) is tractable analytically. It is a solution of the Orbital Theorem of spherically symmetric spacetime developed in UFT 111 (www.aias.us of the National Library of Wales and British National Archives www.webarchive.org.uk). For gravitation in Eq. (1):

$$r_0 = \frac{2MG}{c^2} \tag{2}$$

where M is the mass of an attracting object, G is Newton´s constant and c is the vacuum speed of light. For electromagnetism in Eq. (1):

$$r_0 = 2\,\frac{e_1}{m}\,\frac{e_2}{4\pi c^2 \epsilon_0} \tag{3}$$

where e_1 is the charge of an attracted object, e_2 is the charge of the attracting object, and ϵ_0 is the vacuum permittivity in S.I. units. For the unified field:

$$r_0 = 2\,\left(\frac{MG}{c^2} + \frac{1}{m}\,\frac{e_1 e_2}{4\pi c^2 \epsilon_0}\right) \tag{4}$$

For all three types of field the hamiltonian H is conserved and is the invariant defined as half the rest energy [11,12] as is well known in general relativity:

$$H = \mathcal{L} = T = \frac{1}{2}\, mc^2 = \frac{m}{2}\left(e^{-r_0/r} c^2 \left(\frac{dt}{d\tau}\right)^2 - e^{r_0/r}\left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\varphi}{d\tau}\right)^2\right) \;. \tag{5}$$

In Eq. (5) $\mathcal{L}$ is the lagrangian and T is the kinetic energy. As is well known, there is no potential energy in general relativity. So the classical ideas of attraction, potential energy, effective potential energy, centrifugal repulsion and so forth are all subsumed into, and given by, the metric. However, these classical ideas are so widely taught and are so familiar that in general relativity reference is still made to an “effective potential”. Newton did not know what caused “attraction”, but he knew how it worked in the context of his own era. In general relativity, the concept is replaced by properties of the metric. In ECE the same rules apply to electromagnetism and the unified field. In the standard model, electrodynamics is self-inconsistently a concept imposed on flat spacetime, the so called U(1) sector symmetry. ECE theory has shown [1–10] that the standard model is deeply flawed in many respects. UFT 150 for example shows that Einstein´s famous (or infamous) calculation of light deflection is wildy wrong. It is no longer possible to accept uncritically any aspect of the standard model of physics.

The Euler Lagrange equation of motion:

$$\frac{d}{d\tau}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) = \frac{\partial \mathcal{L}}{\partial x^\mu} \tag{6}$$

when applied to the lagrangian defined in Eq.(5) produces the following three constants of motion, the total energy E, the linear momentum p and the angular momentum L. In general relativity [11, 12] the hamiltonian is defined as half the rest energy, so H and E are defined differently. However, both are conserved. We follow these traditional definitions in this paper, but generalize the theory of relativity much further than hitherto. The constants of motion follow from Eq. (6) and are:

$$E = mc^2\, e^{-r_0/\,r}\, \frac{dt}{d\tau} \quad , \tag{7}$$

$$p \ = me^{r_0/\,r}\, \frac{dr}{d\tau} \quad , \tag{8}$$

$$L \ = mr^2\, \frac{d\varphi}{d\tau} \quad . \tag{9}$$

So using these definitions:

$$H = \frac{1}{2}\, mc^2 = \frac{1}{2}\left(e^{r_0/\,r}\ \frac{E^2}{mc^2} - e^{-r_0/\,r}\, \frac{p^2}{m} - \frac{L^2}{mr^2} \right) \quad . \tag{10}$$

It is helpful at the outset to check Eq. (10) by recovering well known results from it. In the limit:

$$\frac{r_0}{r} \longrightarrow 0 \tag{11}$$

the hamiltonian becomes:

$$H = \frac{1}{2}\, mc^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - \frac{p^2}{m} - \frac{L^2}{mr^2} \right) \quad . \tag{12}$$

If there is no angular momentum in the system:

$$L = 0 \tag{13}$$

then:

$$H = \frac{1}{2}\, mc^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - \frac{p^2}{m} \right) \tag{14}$$

which is the Einstein energy equation of special relativity:

$$E^2 = c^2 p^2 + m^2 c^4 \quad . \tag{15}$$

As is well known, this equation quantizes to the Klein Gordon equation, corrected to the Dirac equation [13, 14]. In papers of this series [1–10] the Dirac equation has been derived from the tetrad postulate of geometry and simplified to an equation in 2 x 2 matrices, something that for many years was thought to be impossible. So the 4 x 4 Dirac matrices were used. Again, these 4 x 4 matrices are so widely taught and so familiar that we use them in this paper for the sake of reference only.

In the Minkowski notation:

$$p^{\mu} = \left(\frac{E}{c} , \boldsymbol{p} \right) \tag{16}$$

$$p_{\mu} = \left(\frac{E}{c} , -\boldsymbol{p} \right) \tag{17}$$

the hamiltonian of Eq. (14) is the well known invariant [1–10]:

$$H = \frac{1}{2m} p^{\mu} p_{\mu} \quad . \tag{18}$$

Using the methods of recent UFT papers (www.aias.us), the general orbital equation from the metric (1) is:

$$\frac{d\varphi}{dr} = \frac{1}{r^2} \left(\frac{1}{b^2} - e^{-r_0/r} \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-½} \quad . \tag{19}$$

So for example the light deflection due to gravitation is (UFT 150):

$$\Delta \varphi = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left(\frac{1}{b^2} - e^{-r_0/r} \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-½} dr \quad . \tag{20}$$

Now use the well known Maclaurin series:

$$e^{-r_0/r} = 1 - \frac{r_0}{r} + \frac{1}{2!} \left(\frac{r_0}{r} \right)^2 - \frac{1}{3!} \left(\frac{r_0}{r} \right)^3 + \ldots \tag{21}$$

The first level of approximation is:

$$e^{-r_0/r} \longrightarrow 1 \tag{22}$$

giving the Minkowski metric. The second level of approximation is:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} \tag{23}$$

giving the gravitational metric with Eq. (2), and its equivalent for electromagnetism with Eq. (3). Its equivalent for the unified field is given by Eq. (4). These are all solutions of the Orbital Theorem of UFT 111. In ECE theory the Einstein field equation is discarded because it is easily shown to be incorrect due to neglect of spacetime torsion [1 – 10], or equivalently, the incorrect use of a symmetric connection. The one to one correspondence between the commutator and connection (UFT 139) means that any connection in Cartan geometry must be antisymmetric. Approximation (23) in gravitation is able to describe the relativistic Keplerian orbits in the solar system, but is completely unable to describe whirlpool galaxies. In the obsolete standard model approximation (23) is incorrectly known as the Schwarzschild metric, even though it is easily seen by the simplest kind of scholarship (a literature search) that Schwarzschild did not derive it [1–10]. This type of incongruity shows that the standard model of physics is irrational in some key aspects and that the standard model has degenerated into dogmatism.

The next level of approximation is:

$$e^{-r_0/r} = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 \tag{24}$$

and for gravitation this type of metric has been shown in the UFT papers to be able to describe the inward spiralling and precessing ellipse of a binary pulsar without the use of "gravitational radiation", so called. The metrics that produce "Hawking radiation", so called, are obviously incorrect due to neglect of torsion, and this incorrectness has been demonstrated in great detail [1–10] using computer algebra.

By direct, well known, observation, the orbit of stars in a whirlpool galaxy is approximated by a spiral format [1–10] such as the logarithmic spiral:

$$\frac{d\varphi}{dr} = \xi r \tag{25}$$

and so for a whirlpool galaxy:

$$e^{-r_0/r} = \left(\frac{1}{b^2} - \frac{\xi^2}{r^2}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)^{-1} \quad . \tag{26}$$

A metric of type (1) is able to describe all known orbits in given limits, and without the use of fictitious and unscientific "dark matter" as in the standard model. Some computations using this metric are given in Section 4. In previous ECE papers on the spiral galaxies, they were explained using the very simple idea of constant spacetime angular momentum. The metric (1) is consistent with this idea as follows. The spiral shape of a galaxy may be explained even in the Minkowski approximation (22) using:

$$\frac{E^2}{mc^2} - mc^2 = m\left(\frac{dr}{d\tau}\right)^2 + \frac{L^2}{mr^2} \tag{27}$$

which gives the orbital equation of Minkowski spacetime:

$$\frac{d\varphi}{dr} = \frac{1}{r}\left(\left(\frac{v}{\omega r}\right)^2 - 1\right)^{-1/2} \tag{28}$$

where the velocity is defined by:

$$v^2 = \left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\varphi}{d\tau}\right)^2 \tag{29}$$

and the angular velocity by:

$$\omega = \frac{d\varphi}{d\tau} \quad . \tag{30}$$

In a spiral galaxy it is observed that as:

$$r \longrightarrow \infty \tag{31}$$

the velocity becomes constant. This result is explained straightforwardly from the Minkowski orbit as follows. From UFT 151 (www.aias.us) it is known that the Minkowski orbit is described by:

$$\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{b^2}\left(1 - \frac{1}{\gamma^2}\right) = \left(\frac{v}{c}\right)^2 \frac{1}{b^2} = \left(\frac{v}{r^2\omega}\right)^2 \tag{32}$$

where

$$\left.\begin{aligned} b = c\,\frac{L}{E} = r^2\,\frac{\omega}{c} \quad &, \qquad \omega = \frac{d\varphi}{d\tau} \quad , \\ \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad &, \qquad d\tau = \left(1 - \frac{v^2}{c^2}\right) dt \quad . \end{aligned}\right\} \tag{33}$$

Therefore

$$\frac{v}{r^2\omega} = \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{-1/2} \tag{34}$$

is a constant of motion of the Minkowski orbit. The angular momentum:

$$L = mr^2\omega = mr^2\,\frac{d\varphi}{d\tau} = \gamma\, mr^2\,\frac{d\varphi}{dt} \tag{35}$$

is also constant in the limit:

$$r \longrightarrow \infty \tag{36}$$

so the velocity v is constant. Thus, in Eq. (28):

$$\frac{v}{r\omega} = \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{-1/2} r \quad . \tag{37}$$

Define:

$$B := \left(\frac{1}{b^2} - \frac{1}{a^2}\right)^{-1/2} \tag{38}$$

then the orbit is:

$$\frac{d\varphi}{dr} = \frac{1}{r}\,(B^2 r^2 - 1)^{-1/2} \tag{39}$$

i.e.

$$\frac{d\varphi}{dr} \xrightarrow[r \to \infty]{} \frac{1}{B r^2} \tag{40}$$

This is the equation of the hyperbolic spiral, which was used in Fig. (6.7) of the fifth volume of ref. [1] to match the observed pattern of stars on a whirlpool galaxy. So metric (1) explains the main characteristics of a whirlpool galaxy in the limit:

$$\frac{r_0}{r} \longrightarrow 0$$

the angular momentum being a constant of motion of the metric.

In the opposite limit:

$$e^{-r_0/r} \longrightarrow 0 \quad , \qquad \frac{r_0}{r} \longrightarrow \infty \quad , \tag{42}$$

there is a very heavy mass M at the centre of the galaxy and the distance r is such that:

$$r_0 >> r \quad . \tag{43}$$

In this limit the orbit in Eq. (19) becomes:

$$\frac{d\varphi}{dr} \longrightarrow \frac{b}{r^2} \tag{44}$$

which is again the hyperbolic spiral.

Having tested the metric (1) in this way for all known orbits of gravitational theory, it may be applied to electrodynamics (Eq. (3)), and the unified field (Eq. (4)).

Consider for example the H atom, in which one electron of charge:

$$e_1 = |\, e \,| = e \tag{45}$$

orbits one proton of charge:

$$e_2 = e \quad . \tag{46}$$

In this example:

$$r_1 = \frac{e^2}{2m\pi c^2 \epsilon_0} = 5.636 \text{ x } 10^{-15} \text{ m} \quad , \quad r_2 = \frac{2MG}{c^2} = 2.262 \text{ x } 10^{-84} \text{ m} \quad . \tag{47}$$

Electromagnetism and gravitation can be described in terms of the radii:

$$r_1 >> r_2 \quad . \tag{48}$$

Their combined effect is described by:

$$r_0 = r_1 + r_2 \quad . \tag{49}$$

For the H atom the interaction of the electron and proton is dominated entirely by the electrodynamical interaction, usually incorporated in the Schroedinger equation through the Coulomb law [14]. However, on the opposite cosmological scale r_1 is influenced in general by r_2 in a unified field theory. It is seen that r_1 is of the order of the proton radius, whose best estimate is:

$$r \text{ (proton)} = (0.8 – 0.86) \text{ x } 10^{-15} \text{ m.} \tag{50}$$

Therefore r_2 is well inside the proton radius. This is a thought experiment where no quantum effects are considered as yet, a thought experiment designed to estimate the relative importance of electrostatic and gravitational interaction in the H atom. From the metric (1), the equation of motion of the electron proton system on the classical level is:

$$m \left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - e^{-r_0/r} \left(mc^2 + \frac{L^2}{mr^2}\right) \tag{51}$$

which in the approximation (23) becomes

$$m \left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right) \left(mc^2 + \frac{L^2}{mr^2}\right) \tag{52}$$

i.e. :

$$\frac{E^2}{mc^2} - mc^2 = m \left(\frac{dr}{d\tau}\right)^2 - \frac{r_0}{r} mc^2 - \frac{r_0}{r} \frac{L^2}{mr^2} + \frac{L^2}{mr^2} \quad . \tag{53}$$

Adopting the traditional nomenclature [11, 12] the “effective potential” is:

$$V = \frac{-(r_1 + r_2)}{r} \left(mc^2 + \frac{L^2}{mr^2}\right) + \frac{L^2}{mr^2} \tag{54}$$

and the “inverse square attraction” is:

$$V(\text{inv.sq.}) = \frac{-(r_1 + r_2)}{r} mc^2 = - \frac{e_1 e_2}{4\pi r \epsilon_0} - \frac{mMG}{r} \tag{55}$$

which is a linear combination of the attractive Coulomb and Newton potential energies . The metric gives these laws plus centrifugal and relativistic corrections. This is a relativistic classical model consisting of one charged mass orbiting another. It is seen that on this level of approximation there is no "cross term", i.e. no contribution to the potential energy of attraction from the combined effect of gravitation and electrostatics.

In a more accurate approximation:

$$\exp\left(\frac{-(r_1+r_2)}{r}\right) \sim 1 - \frac{(r_1+r_2)}{r} + \frac{1}{2}\left(\frac{r_1+r_2}{r}\right)^2 \tag{56}$$

electromagnetism influences gravitation. The effective potential is changed, and its inverse square part is:

$$V(\text{inv.sq.}) = -\frac{e_1 e_2}{4\pi r \epsilon_0} - \frac{mMG}{r} + \frac{mc^2}{4}\left(\frac{r_1+r_2}{r}\right)^2 \quad . \tag{57}$$

The cross term is:

$$V\,(\text{cross term}) = \frac{mc^2}{2}\,\frac{r_1\, r_2}{r^2} = \left(\frac{mG}{r^2}\right)\left(\frac{e_1 e_2}{2\pi c^2 \epsilon_0}\right) \tag{58}$$

and is positive valued or repulsive, opposing the gravitational attraction.

This simple calculation shows that electrodynamics may be used to lessen the pull of gravitation, producing an array of new industries. In the units:

$$e_1 = e_2 = r = m = M = 1 \tag{59}$$

the Coulombic attraction is $-(4\pi\epsilon_0)^{-1}$, and the cross term is $G/(2\pi c^2\epsilon_0)$ of the order of 10^{-27} smaller. The engineering problem, quite obviously, is to maximize the influence of electromagnetism on gravitation and several previous ECE papers (www.aias.us) have addressed this problem in terms of spin connection resonance. This calculation is meant to show only that such an influence may exist in nature, using metric (1) as a new approach. The opposite influence of gravitation on electromagnetism is known from light deflection by gravitation, and is again described by metric (1) in terms of photon mass (UFT 150).

The electrodynamic metric defined by Eqs. (1) and (23) produces the equation of motion:

$$H = \frac{1}{2}mc^2 = \frac{1}{2}\left(\left(1 - \frac{r_0}{r}\right)\frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right)^{-1}\frac{p^2}{m} - \frac{L^2}{mr^2}\right) \tag{60}$$

in which the term:

$$V = -\frac{e_1 e_2}{4\pi r \epsilon_0}\left(1 + \left(\frac{L}{mc}\right)^2 \frac{1}{r^2}\right) + \frac{L^2}{2mr^2} \tag{61}$$

is known traditionally in gravitational general relativity as "the effective potential energy", even though there is no concept of potential energy in general relativity. Adopting this traditional nomenclature it is seen that the Coulomb potential is corrected to:

$$V\,(\text{Coulomb}) = -\frac{e_1 e_2}{4\pi r \epsilon_0}\left(1 + \left(\frac{L}{mc}\right)^2 \frac{1}{r^2}\right) \tag{62}$$

in which $L/(mc)$ is a constant. If there is no angular momentum in the system:

$$L = 0 \tag{63}$$

then there is no general relativistic correction to the Coulomb law. It is well known that the Coulomb law is the most accurate in physics, so any correction to it must be observed in a well designed experiment. In the H atom [14] however, there is quantized angular momentum present in the effective potential of the Schroedinger equation of the H atom:

$$V\,(\text{H}) = -\frac{e^2}{4\pi r \epsilon_0} + \frac{l(l+1)\hbar^2}{2\mu r^2} \tag{64}$$

and so this is corrected in general relativity to:

$$V\,(\text{H}) = -\frac{e^2}{4\pi r \epsilon_0}\left(1 + l(l+1)\left(\frac{\hbar}{mc}\right)^2 \frac{1}{r^2}\right)_{\mu\sim m} + \frac{l(l+1)\hbar^2}{2mr^2} \tag{65}$$

in which the squared Compton wavelength is:

$$\left(\frac{\hbar}{mc}\right)^2 = 1.4912 \times 10^{-25}\ \text{m}^2 \quad . \tag{66}$$

The relativistic correction will have an effect on the atomic spectrum of H and this is a problem that can be addressed by adopting quantum chemistry packages.

Finally in this section some consideration is given to cylindrical spacetime in order to demonstrate that the theory of this paper is applicable to any symmetry of spacetime and that the theory can be developed in many directions. A possible cylindrically symmetric metric is:

$$ds^2 = c^2 d\tau^2 = e^{-Z_0/Z} c^2 dt^2 - dr^2 - r^2 d\varphi^2 - e^{Z_0/Z}\, dZ^2 \tag{67}$$

whose hamiltonian and lagrangian are defined by the kinetic energy (half rest energy):

$$\mathcal{L} = H = T = \frac{1}{2} m c^2 = \frac{1}{2} m\left(e^{-Z_0/Z} c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\varphi}{d\tau}\right)^2\right) - e^{Z_0/Z}\left(\frac{dZ}{d\tau}\right)^2) \ . \tag{68}$$

From this definition the following equation of motion is obtained:

$$m \frac{d\alpha^2}{d\tau^2} = \frac{E^2}{mc^2} - e^{-Z_0/Z} (m c^2 + \frac{L^2}{mr^2}) \tag{69}$$

where

$$d\alpha^2 = dZ^2 + e^{-Z_0/Z} dr^2 \quad . \tag{70}$$

The orbital equation is:

$$\frac{d\varphi}{d\alpha} = \frac{1}{r^2} \left(\frac{1}{b^2} - e^{-Z_0/Z} \left(\frac{1}{a^2} + \frac{1}{Z^2}\right)\right)^{-½} \tag{71}$$

and the light deflection due to gravitation in this cylindrical spacetime is:

$$\Delta \varphi = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left(\frac{1}{b^2} - e^{-Z_0/Z} \left(\frac{1}{a^2} + \frac{1}{Z^2}\right)\right)^{-½} d\alpha \quad . \tag{72}$$

In order to make this expression analytically tractable, it is assumed that r is proportional to Z:

$$r = \beta Z \tag{73}$$

so the light deflection due to gravitation can be computed as follows in terms of parameters:

$$\Delta \varphi = 2 \int_{R_0}^{\infty} \frac{1}{Z^2} \left(\frac{1}{b^2} - e^{-Z_0/Z} \left(\frac{1}{a^2} + \frac{1}{Z^2}\right)\right)^{-½} (1 + \beta e^{-Z_0/Z})^{½} dZ \tag{74}$$

3. Metric, minimal prescription and Dirac equation.

Consider the effect of the term r_0/r on Eq (14). The hamiltonian is changed to:

$$H = \frac{1}{2} m c^2 = \frac{1}{2} \left(\left(\exp\left(\frac{r_0}{2r}\right) \frac{E}{c}\right)^2 - \left(\left(\exp\left(\frac{-r_0}{2r}\right) p\right)^2\right) \quad . \tag{75}$$

In the approximation:

$$\exp\left(\frac{r_0}{2r}\right) = 1 + \frac{r_0}{2r} \quad , \tag{76}$$

$$\exp\left(\frac{-r_0}{2r}\right) = 1 - \frac{r_0}{2r} \tag{77}$$

the change in the hamiltonian can be represented by:

$$E \longrightarrow E + (\frac{r_0}{2r}) E \ , \tag{78}$$

$$p \longrightarrow p - (\frac{r_0}{2r}) p \ . \tag{79}$$

This is the well known minimal prescription:

$$p^\mu \longrightarrow p^\mu - e A^\mu \tag{80}$$

where the four potential of electromagnetism is:

$$A^\mu = (\frac{\varphi}{c}, \boldsymbol{A}) \ . \tag{81}$$

It follows that:

$$\varphi = - \frac{e_2}{4\pi r \epsilon_0} (\frac{E}{m c^2}) \tag{82}$$

$$A = \frac{e_2}{4\pi r c \epsilon_0} (\frac{p}{mc}) \ . \tag{83}$$

For a particle at rest:

$$\varphi = - \frac{e_2}{4\pi r \epsilon_0} \ , \tag{84}$$

$$A = 0 \tag{85}$$

and self consistently, the problem is one of electrostatics, with the Coulomb potential (83) and no vector potential. With the definitions (81,82) the metric gives the well known relativistic Hamilton Jacobi equation and the correct Dirac equation [13] of the electron in an electromagnetic four potential:

$$(\gamma^\mu (p_\mu - e A_\mu) - mc) \psi = 0 \tag{86}$$

The Dirac equation gives the g factor of the electron, the Thomas precession, the correct fine and hyperfine structure in atomic spectra, ESR, NMR MRI, RFR and antiparticles. Therefore these well known results are all obtained from metric (1) in the approximation (23). This suggests that the Dirac equation itself is an approximation of a hitherto unknown and more generally valid equation derivable from the unapproximated metric (1). In previous work in ECE theory the Dirac equation concept has been simplified using 2 x 2 matrices [1–10] and the Dirac equation derived directly from geometry.

4. Numerical studies of the effect of using approximations of metric (1) on light deflection.

The exponential form of the metric factor (Eq. (1)) has been shown to be derivable straightforwardly. Other well known metrics come out to be approximations of a series expansion of this factor up to a certain degree. The Maclaurin expansion (21) with $r_0 = 1$ has been plotted to third order in Fig. 1. For the second order the asymptote of $r \to 0$ is wrong. From third order onwards the difference to the exact value is minimal for r>1. There is practically no deviation from the exact values for r >10 for all degrees of approximation.
The angle of light deflection for the sun has been calculated according to paper 150:

$$\Delta\varphi = 2\int_0^{1/R_0}\left(\frac{1}{{r_0}^2} - \exp(-r_0 u)\left(\frac{1}{{R_0}^2} + u^2\right)\right) du \tag{87}$$

where R_0 is the Sun´s radius and r_0 is the so-called Schwarzschild radius. The integral gives the well-known value of order 10^{-6} already in the 0^{th} order, see Fig. 2.

In the integration procedure for light deflection we operate in the range $r \gg r_0$ since R_0 is by 5 orders of magnitude larger than r_0. Therefore the light deflection is no good experiment to determine the true physical form of the metric factor, see Fig. 1. This result is corroborated by the finding in paper 150 that the u dependence of the integrand has practically no effect on the result. The latter is obtained by the constant part $1/{r_0}^2$ in very good approximation.

To see any effects of the approximation order of Eq. (21), the integration radius R_0 has to be changed drastically. The photon mass is proportional to r_0/R_0 according to Eq. (60) of paper 150. Therefore a varying R_0 leads to a variation in photon mass which is only meaningful in narrow limits because the photon mass is a rest mass. Nevertheless we have varied R_0 to study the effect. This is only visible when R_0 is in the order of the Schwarzschild radius. This may be the case for neutron stars (or other very compact stars). From Fig. 2 it can be seen that the approximation order is only relevant if R_0 comes near to r_0. Order 0 means Minkowski metric, order 4 stands for the exact exponential function. It comes out quite clearly that for the sun the Minkowski metric works excellent while for neutron stars it is significantly wrong. A particular result is that for $R_0 < r_0$ the integrand is negative and therefore no light deflection is possible. This leads to a new interpretation of the Schwarzschild radius, one could even speak of a "black hole" in this case because no light is reflected, but this does nothing to say about the light coming from the center directly. The maximum deflection angle is about 5.2 rad, less than a complete circle which would be 2π. All this is valid of course only if the extrapolated assumptions are valid.

In a whirlpool galaxy the stars are arranged in a logarithmic spiral

$$r(\varphi) = r_0 \exp(\xi\varphi) \tag{88}$$

where ξ is defined from the metric by the equation

$$e^{-r_0/2r} = \frac{\frac{1}{b^2} - \frac{\xi^2}{r^2}}{\frac{1}{a^2} + \frac{1}{r^2}} \tag{89}$$

(paper 151, Eqs. (31-34)). Eq. (89) can be solved for $\xi(r)$, resulting in

$$\xi = \frac{1}{ab}\, e^{-r_0/2r}\sqrt{a^2 r^2 e^{r_0/r} - b^2 r^2 - a^2 b^2}. \tag{90}$$

For a true spiral, ξ should be constant. If this is possible, can be seen from Fig. 3 where $\xi(r)$ has been plotted for $b = r_0 = 1$ and various a values. Obviously ξ is constant over a wide range of r for $a = 0.8$.

The last diagram is an example for the cylindrical metric described in Section 2 of this paper. The light deflection for such a metric example is given by Eq. (73). Rewritten to the coordinate u = 1/z it is

$$\Delta\varphi = 2\int_0^{1/R_0} \left(\frac{1}{b^2} - \mathrm{e}^{-z_0 u}\left(\frac{1}{a^2} + u^2\right)^{-1/2} (1 + \beta e^{-z_0 u})^{1/2}\right) du \quad . \tag{90}$$

The integrand of this equation is plotted in Fig. 4 with all constants equal to unity with exception of a . For a = 0.5 the integral diverges. For a =1 there is a singularity for u = 0 but the integral value exists. For a = 2 the integrand is regular and the integral exists. From these examples it can be seen that physically meaningful parameter combinations are at least possible for this cylindrical metric.

Finally we have checked the relativistic correction of the Schroedinger equation given in Eq. (65). It is an angular momentum correction to the Coulomb potential which is of order $1/c^2$. Therefore it is expected to be very small. We added the corresponding term to the solution program of the Hydrogen atom we used for papers 63 ff. The correction of the potential is largest near to the nucleus (r = 0). However, the only orbitals having a non-vanishing probability density at r = 0 are s orbitals for which is $l = 0$, i.e. these are not affected by the relativistic correction. The lowest possible orbitals being impacted are the 2p orbitals, but the effect on the energy levels is smaller than $10^{-5}\ Ryd$, this is below the achievable precision of the computer program. This result is in accordance with the fact that the effect is of order $1/c^2$ and therefore extremely small.

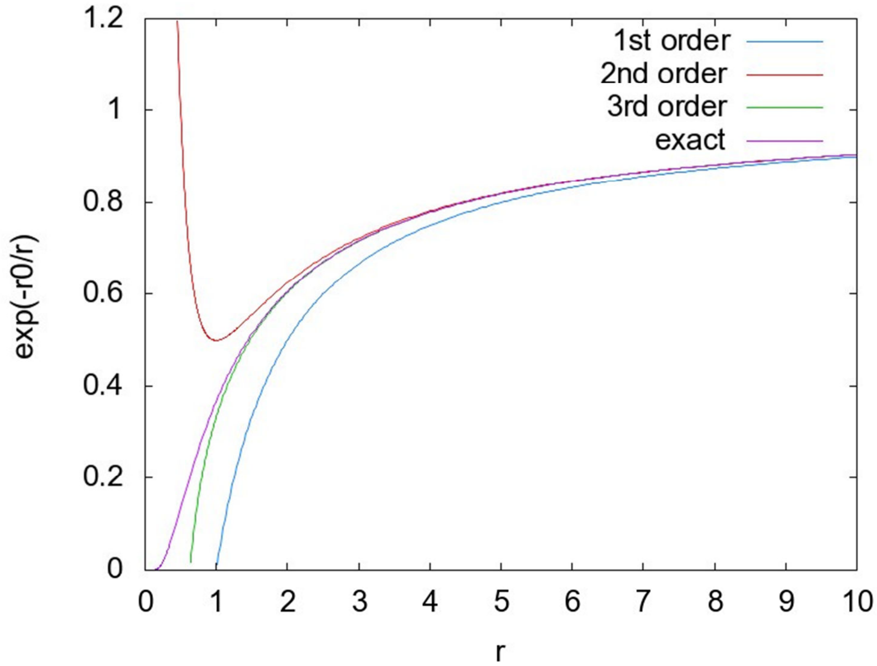

Fig. 1. Different degrees of approximation for the function $\exp(-r_0/r)$ with $r_0=1$.

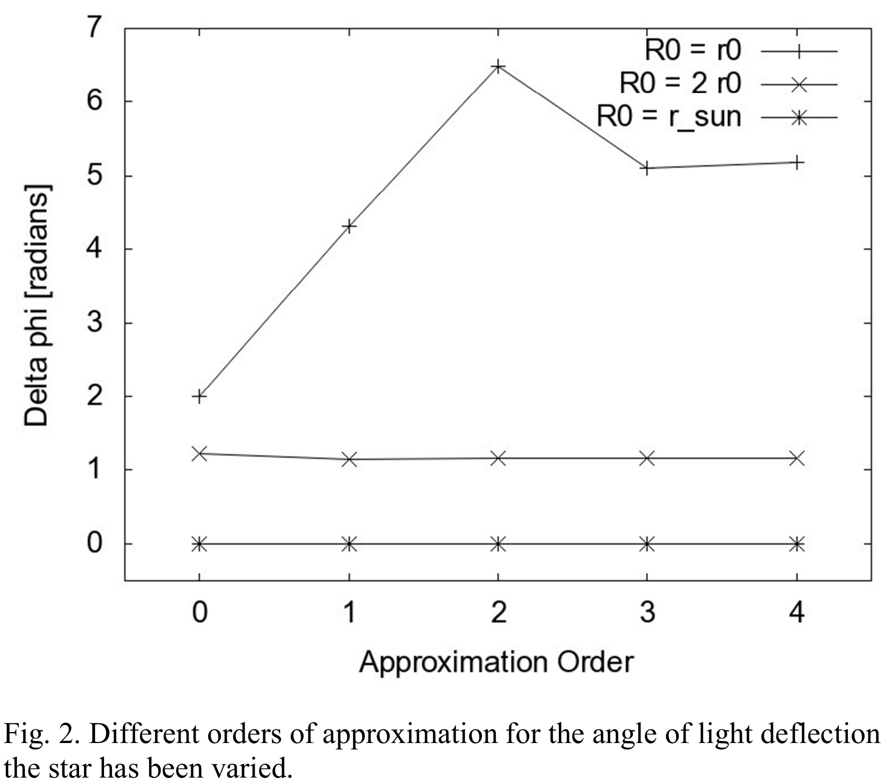

Fig. 2. Different orders of approximation for the angle of light deflection $\Delta\phi$. The radius R_0 of the star has been varied.

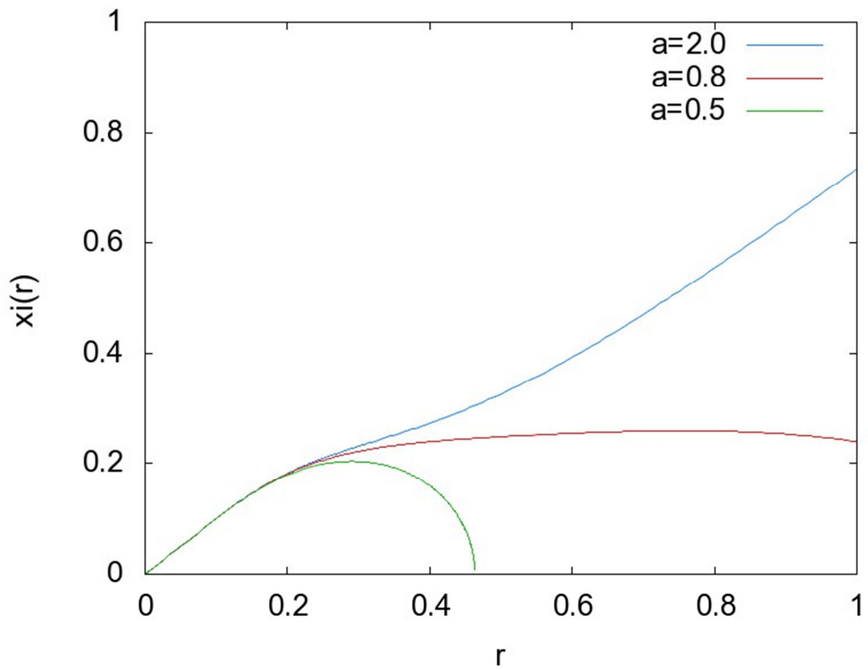

Fig. 3. Spiral parameter $\xi(r)$ for different parameters of a.

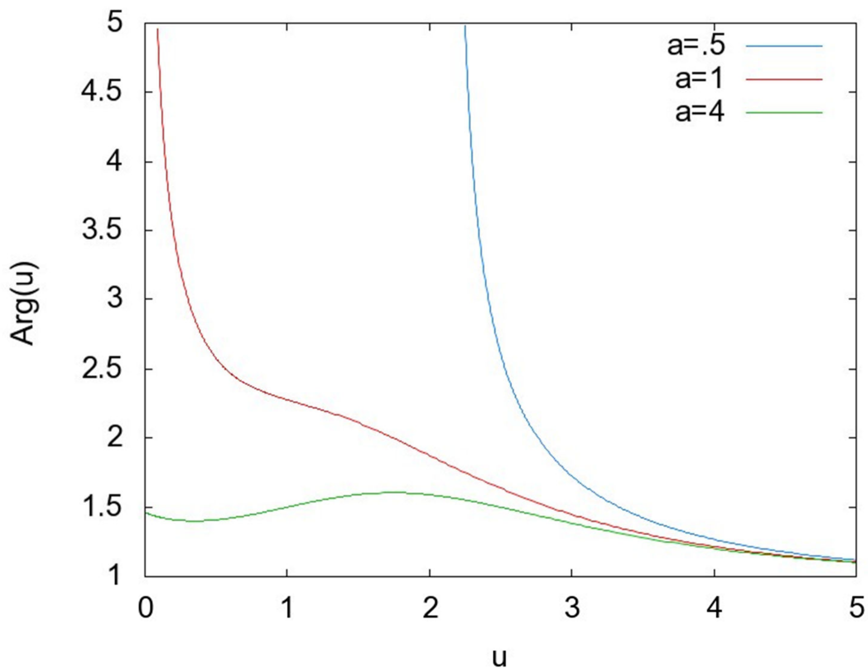

Fig. 4. Argument of integral of Eq. (73) for different a parameters.

Acknowledgments

The British Government is thanked for the award of a Civil List Pension to MWE, Alex Hill and colleagues for translations and typesetting, and David Burleigh for posting on www.aias.us and the National Archives site www.webarchive.org.uk . Colleagues worldwide are thanked for many interesting discussions.

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