Euler Bernoulli resonance in a spherically symmetric spacetime: application to counter gravitation.

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Abstract

Euler Bernoulli resonance is developed as a subject of general relativity in a spherically symmetric spacetime. The equations of motion of dynamics and electrodynamics are obtained from the metric, and the hamiltonian defined precisely, showing that spacetime has both gravitational and electromagnetic energy. Details are given on the correct approach to the Newtonian limit, the Orbital Theorem generalized and a check carried on the Maclaurin series used for the description of the spacetime in successive approximations. The vector potential of electrodynamics is defined from the metric, which is defined in terms of the Cartan tetrads. The fundamental meaning of the tetrad is developed as a transformation matrix, and simple examples given of resonance equations of practical use in counter gravitational circuits.

Keywords: Einstein Cartan Evans (ECE theory), Euler Bernoulli resonance, equations of motion, dynamics, electrodynamics, metric, Cartan tetrad, rotation matrix, counter gravitation.

1. Introduction

In this series of 153 papers to date of the Einstein Cartan Evans (ECE) unified field theory, it has been shown that gravitational and electromagnetic energy is available from the hamiltonian of the metric of spacetime. By definition, the hamiltonian is conserved. In general relativity, the hamiltonian is defined as being half the rest energy of a particle of mass m. The rest energy is the familiar mc^2, where c is the vacuum speed of light, and so the hamiltonian is conserved because for a given mass m the rest energy does not change with the dynamics, i.e. it is said to be "conserved". Note carefully that general relativity is different fundamentally in philosophy from classical physics, in which the hamiltonian is the sum of the kinetic energy (T) and potential energy (V). General relativity deals with kinetic energy, and does not consider the concept of potential energy. The latter is replaced by geometry. The hamiltonian H in general relativity is pure kinetic energy, and so is the same as the lagrangian by definition. Gravitational and electromagnetic energy may be transferred from the hamiltonian H of spacetime in the general theory of relativity, and used in devices of great potential importance. It has been shown [1-10] in earlier papers that one practical implementation of counter gravitation is an electromagnetic device whose effect on the electromagnetic field is maximized by Euler Bernoulli resonance. The latter comes from force equations in dynamics which conserve the classical total energy [11] as is well known. In Section 2 the conditions under which this resonance amplification of counter gravitation occurs are defined in a spherically symmetric spacetime defined by the metric [12]. Equations of dynamics and electrodynamics are derived from the metric in Section 3, which gives all details and discusses the correct approach to the Newtonian limit in dynamics. The Orbital Theorem of UFT 111 of this series [1-10] is generalized, and a check carried out on the validity of the Maclaurin series used for successive approximations to the metric of spherical spacetime. These successive approximations contain new physics. The electromagnetic vector potential is derived from the metric and shown to contain irrotational and divergenceless components defined by the metric. These components derive the Helmholtz Theorem from the metric. The latter is defined in terms of the Cartan tetrads [1-10], which are shown to be defined in terms of transformation matrices of one coordinate system to another. Field and wave equations of ECE are defined by the tetrads, so a general method is devised to obtain them from the metric. Finally in Section 4, simple examples of Euler Bernoulli resonance structures are discussed in dynamics and circuit theory. This theory leads to circuit designs that maximize at resonance the effect of electrodynamics on gravitation, producing counter gravitational devices on board an aircraft or spacecraft.

2. Spherical spacetime and Euler Bernoulli resonance

In the non-relativistic, linear, classical limit in dynamics, Euler Bernoulli resonance originates [11] in the driven oscillator equation:

$$m\ddot{r} + 2\beta\dot{r} + \mathrm{k}\,r = F_0 \cos(\omega \mathrm{t}) \qquad (1)$$

where r is the displacement, m the mass, β the friction coefficient, k is the constant of

Hooke´s law and where the right hand side is a time dependent driving force. If the friction term is omitted the simplest resonant structure is:

$$m\ddot{r} + \mathrm{k}\, r = F_0 \cos(\omega \mathrm{t}) \tag{2}$$

which can be rewritten as:

$$m\ddot{r} + \omega_0^2\, r = A \cos(\omega \mathrm{t}) \tag{3}$$

where the characteristic frequency is defined by:

$$\omega_0^2 = \frac{\mathrm{k}}{m} \; , \quad A = \frac{F_0}{m} \quad . \tag{4}$$

The solution of this equation is:

$$r\,(t) = \left(\frac{A}{\omega_0^2 - \omega^2} \right) \cos(\omega \mathrm{t}) \tag{5}$$

and amplitude and kinetic energy resonance [11] occur at:

$$\omega = \omega_0 \quad . \tag{6}$$

This theory can be translated [11] into circuit theory as discussed in Section 4. Eq. (3) is a mechanism that supplies a natural system with energy from an external source at a rate equal to that absorbed. Energy is transferred and total energy conserved. The driving term in general relativity is spacetime described by a metric which defines the hamiltonian or conserved total energy. Energy is transferred from spacetime to the system (a circuit design for example, or atom or molecule). The transfer process may be observed in the radiative corrections for example (UFT 85 of www.aias.us). The spacetime energy is transferred to the unforced oscillator:

$$\ddot{r} + \omega_0^2\, r = 0 \tag{7}$$

which is any kind of naturally occurring system such as a circuit. In Eq. (7) the unforced oscillator is the spring of Hooke´s law used for simplicity of argument, but it could be an atom or molecule as is well known [11]. The key engineering problem is to design a circuit to transfer spacetime energy to the circuit. This is a non-trivial problem which has been thought about for over a century, but the fact that a metric defines the hamiltonian is irrefutable.

The metric of the spherical spacetime is:

$$\mathrm{d}s^2 = c^2 \mathrm{d}\tau^2 = e^{-r_0/r}\, c^2 \mathrm{d}t^2 - \mathrm{d}\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r} \tag{8}$$

where H is the hamiltonian, $\mathcal{L}$ the lagrangian, and T the kinetic energy. These are defined as half the rest energy:

$$H = \mathcal{L} = T = \frac{1}{2}\, mc^2 = \frac{m}{2}\left(e^{-r_0/r} c^2 \left(\frac{dt}{d\tau}\right)^2 - e^{r_0/r}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\varphi}{d\tau}\right)^2\right) \tag{9}$$

where m is the mass of a test particle and c the vacuum speed of light. Eq. (9) comes from the metric of spherical spacetime in cylindrical polar coordinates:

$$ds^2 = c^2 d\tau^2 = e^{-r_0/r}\, c^2 dt^2 - e^{r_0/r} dr^2 - r^2\, d\varphi^2 \tag{10}$$

where $d\tau$ is the infinitesimal of proper time (the time measured in a frame placed on the moving particle). The infinitesimal of time measured in the laboratory frame is dt and in the cylindrical polar system in the XY plane:

$$d\boldsymbol{r} \,.\, d\boldsymbol{r} = e^{r_0/r} dr^2 + r^2\, d\varphi^2 \quad . \tag{11}$$

In these equations r_0 is a characteristic distance to be defined. As in previous papers [1-11] the total energy E and angular momentum L are constants of motion:

$$E = mc^2\, e^{-r_0/r}\, \frac{dt}{d\tau} \quad , \quad L = mr^2 \frac{d\varphi}{d\tau} \tag{12}$$

so the equation of motion is:

$$\frac{1}{2}\, m \left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - e^{-r_0/r}\left(mc^2 + \frac{L^2}{mr^2}\right)\right) \quad . \tag{13}$$

Now make the approximation:

$$e^{-r_0/r} \sim 1 - \frac{r_0}{r} \tag{14}$$

to derive the relativistic Kepler orbits and their equivalents in electrodynamics. For gravitation [1-11]:

$$r_0 = \frac{2MG}{c^2} \tag{15}$$

where M is a mass that attracts m , and G is Newton´s constant. Although this system is defined in Eq. (9) to be purely kinetic, it is customary to use the classical concept of potential energy for convenience only, so that the “potential energy” term:

$$V = -\frac{1}{2}\, mc^2 \frac{r_0}{r} = -\frac{mMG}{r} \tag{16}$$

defines the Newtonian inverse square force of attraction between m and M:

$$F = -\frac{dV}{dr} = -\frac{mMG}{r^2} \quad . \tag{17}$$

The Coulomb attraction between charge e_1 on mass m and charge e_2 on mass M is obtained by defining:

$$r_0 = \frac{2e_1 e_2}{4m\pi c^2 \epsilon_0} \tag{18}$$

where ϵ_0 is the S.I. vacuum permittivity. The Coulombic potential energy is:

$$V = -\frac{1}{2} mc^2 \frac{r_0}{r} = -\frac{e_1 e_2}{4\pi c^2 \epsilon_0} \tag{19}$$

and the inverse square law of attraction between e_1 and e_2 is:

$$F = -\frac{dV}{dr} = -\frac{e_1 e_2}{4\pi r^2 \epsilon_0} \tag{20}$$

The Newton and Coulomb laws have been derived from the metric. These force laws both have the format:

$$F = -\frac{dV}{dr} = -\frac{1}{2} mc^2 \frac{r_0}{r^2} \tag{21}$$

and show that spacetime itself is responsible for the phenomena of attraction of two masses and two charges. The potential energy for attraction is due to spacetime itself. This is of course an idea of general relativity. So an oscillatory driving force may also be obtained from spacetime as follows:

$$F = -\frac{1}{2} mc^2 \frac{r_0}{r^2} = F_0 \cos(\omega \mathrm{t}) \quad . \tag{22}$$

In this case:

$$r_0 = -\left(\frac{2r^2}{mc^2}\right) F_0 \cos(\omega \mathrm{t}) \tag{23}$$

and the metric being used is:

$$\mathrm{d}s^2 = c^2 \mathrm{d}\tau^2 = c^2 \left(1 - \frac{r_0}{r}\right) \mathrm{d}t^2 - \left(1 - \frac{r_0}{r}\right)^{-1} \mathrm{d}r^2 - r^2 \mathrm{d}\,\varphi^2 \quad . \tag{24}$$

Writing:

$$\frac{dV_0}{dr} = -F_0 \cos(\omega \mathrm{t}) \tag{25}$$

then:

$$r_0 = \left(\frac{2r^2}{mc^2}\right)\frac{dV_0}{dr} \quad . \tag{26}$$

The potential energy V_0 produced by spacetime defines a driving force that drives any naturally occurring unforced oscillator into resonance. At resonance the kinetic energy and amplitude are maximized. As in Section 4, the driving force in circuit theory is an electromotive force defined by the metric. This is the metric based definition of electric energy from spacetime. It has been accepted for nearly a century that gravitational energy may be obtained from the metric.

3. Hamiltonian, equations of motion and Newtonian limit.

The hamiltonian in general relativity is defined by:

$$H = \frac{1}{2}\, m \left(\frac{ds}{d\tau}\right)^2 \tag{27}$$

so $H/\,m$ is pure geometry. The total and conserved energy H is pure geometry for a given m. There is no potential energy in general relativity, so the lagrangian is the same as the hamiltonian. The Euler Lagrange equation is used [1-10] to define constants of motion, which are quantities which are unchanged, or conserved, as the dynamics evolve. The constants of motion are the total energy E:

$$E = mc^2 e^{-r_0/\,r}\left(\frac{dt}{d\tau}\right)^2 \tag{28}$$

the total linear momentum p :

$$p = m e^{r_0/\,r}\,\frac{dr}{d\tau} \tag{29}$$

and the total angular momentum L:

$$L = mr^2\left(\frac{d\varphi}{d\tau}\right)^2 \quad . \tag{30}$$

Multiply both sides of Eq. (9) by $e^{-r_0/\,r}$:

$$\frac{1}{2}\, mc^2 e^{-r_0/\,r} = \frac{1}{2}\, m\left(e^{-r_0/\,r}\, c\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - e^{-r_0/\,r}\, r^2\left(\frac{d\varphi}{d\tau}\right)^2\right) \tag{31}$$

re-arrange terms:

$$\frac{1}{2}\, m\, (\frac{dr}{d\tau})^2 = \frac{1}{2}\, m\, (e^{-r_0/\,r}\, c\, (\frac{dt}{d\tau})^2 \,-\, c^2\, e^{-r_0/\,r} \,-e^{-r_0/\,r}\, r^2\, (\frac{d\varphi}{d\tau})^2) \tag{32}$$

and the equation of motion for spherical spacetime is:

$$\frac{1}{2}\, m\, (\frac{dr}{d\tau})^2 = \frac{1}{2}\, (\frac{E^2}{mc^2} - e^{-r_0/\,r}\, (\, mc^2 + \frac{L^2}{mr^2}))\quad . \tag{33}$$

The orbital equation is found by eliminating the proper time as follows:

$$\frac{dr}{d\tau} = \frac{d\varphi}{d\tau}\frac{dr}{d\varphi} = (\frac{L^2}{mr^2}\,)\,\frac{dr}{d\varphi} \tag{34}$$

to obtain:

$$(\frac{dr}{d\varphi})^2 = r^4\, (\frac{1}{b^2} - \, e^{-r_0/\,r}\, (\frac{1}{a^2} + \frac{1}{r^2}\,)) \tag{35}$$

where the constants of motion a and b have the units of length and are defined by:

$$a = \frac{L}{mc} \quad , \quad b = \frac{cL}{E} \quad . \tag{36}$$

The orbital equation is obtained by inverting Eq. (35) and taking the square root:

$$\frac{d\varphi}{dr} = \frac{1}{r^2}\, (\frac{1}{b^2} - \, e^{-r_0/\,r}\, (\frac{1}{a^2} + \frac{1}{r^2}\,))^{-½} \quad . \tag{37}$$

Light deflection by the Sun was calculated correctly for the first time in UFT 150 on www.aias.us and is given by:

$$\Delta\, \varphi \;=\; 2\int_{R_0}^{\infty} \frac{1}{r^2}\, (\frac{1}{b^2} - (\,1 - \frac{r_0}{r}\,)\,(\frac{1}{a^2} + \frac{1}{r^2}\,))^{-½}\, dr \tag{38}$$

where R_0 is the distance of closest approach of a photon of mass m to the Sun of mass M. According to Section 2, these considerations apply also to charge interaction.

The approximation (14) is usually used for the relativistic Kepler problem, and results in:

$$\frac{1}{2}\, (\frac{E^2}{mc^2} - mc^2) = \frac{1}{2}\, m\, (\frac{dr}{d\tau})^2 + V \quad . \tag{39}$$

In this theory it is customary to use the "effective potential", despite the fact that only the kinetic energy T is defined (see Eq. (9)). So the effective potential energy in joules is:

$$V = -\frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \tag{40}$$

and "the force" in newtons is:

$$F = -\frac{dV}{dr} \quad . \tag{41}$$

These classical ideas are used so often that the words "potential energy" and "force" are still used in general relativity, but all is now geometry and the metric. In ECE unified field theory this is true not only for gravitation but all fields, i.e. the unified field. Using the classical language the total force between m and M is:

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 \tag{42}$$

where

$$F_1 = |\boldsymbol{F}_1| = -mMG/r^2 \tag{43}$$

$$F_2 = |\boldsymbol{F}_2| = L^2/(mr^3) \tag{44}$$

$$F_3 = |\boldsymbol{F}_3| = -3MGL^2/(mc^2r^4) \quad . \tag{45}$$

In ECE theory these forces also occur between two charges e_1 and e_2 . The force F_1 is negative and attractive and is the inverse square force of attraction. The force F_2 is positive and repulsive, and is the centrifugal force. The force F_3 is negative and attractive, and produces the precession of elliptical orbits. Only F_1 and F_2 are present in Newtonian orbital theory, which produces static ellipses and the three Kepler laws [11].

The metric being used to produce all this well known information in dynamics is:

$$c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - d\boldsymbol{r} \cdot d\boldsymbol{r} \tag{46}$$

where by definition:

$$d\boldsymbol{r} \cdot d\boldsymbol{r} = \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\varphi^2 \quad . \tag{47}$$

The linear velocity v of the particle m is defined by

$$v^2 dt^2 = d\boldsymbol{r} \cdot d\boldsymbol{r} \quad . \tag{48}$$

In a frame in which m is at rest (i.e. in a frame of reference placed on the particle):

$$v = 0 \tag{49}$$

because the particle does not move in its own frame of reference. So:

$$\frac{d\tau}{dt} = (1 - \frac{r_0}{r})^{½} \qquad \text{if} \qquad v = 0 \quad . \tag{50}$$

Otherwise, by definition:

$$d\tau^2 = (1 - \frac{r_0}{r})^{½}\ dt^2 - (\frac{v}{c})^2\ dt^2 \tag{51}$$

and:

$$\frac{d\tau}{dt} = (1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2)^{½} \quad . \tag{52}$$

Therefore:

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{dr}{dt}\right)^2 \left(\frac{dt}{d\tau}\right)^2 = \left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2 \right)^{-1} \left(\frac{dr}{dt}\right)^2 \tag{53}$$

and the equation of motion (39) is therefore:

$$\frac{1}{2} (\frac{E^2}{mc^2} - mc^2) = \frac{m}{2} \left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2 \right)^{-1} \left(\frac{dr}{dt}\right)^2 + V \quad . \tag{54}$$

The conserved total energy E (not to be confused with H in general relativity) is:

$$E = mc^2\ (1 - \frac{r_0}{r})\ \frac{dt}{d\tau} = mc^2\ (1 - \frac{r_0}{r}) \left(1 - \frac{r_0}{r} - \frac{v^2}{c^2}\right)^{-½}, \tag{55}$$

the conserved total linear momentum is

$$p = m\ (1 - \frac{r_0}{r}) \left(1 - \frac{r_0}{r} - \frac{v^2}{c^2}\right)^{-½} \frac{dr}{dt} \tag{56}$$

and the conserved total angular momentum is:

$$L = m\ r^2 \left(1 - \frac{r_0}{r} - \frac{v^2}{c^2}\right)^{-½} \frac{d\varphi}{dt} \quad . \tag{57}$$

In the limit of special relativity the metric (47) approaches the Minkowski metric:

$$c^2 \mathrm{d}\tau^2 = c^2 \mathrm{d}t^2 - d\boldsymbol{r} \cdot d\boldsymbol{r} \tag{58}$$

so either:

$$r \longrightarrow \infty \tag{59}$$

or

$$r_0 \longrightarrow 0 \quad . \tag{60}$$

In the special relativistic limit:

$$E = \gamma mc^2 \quad , \qquad p = \gamma m \frac{dr}{dt} \quad , \qquad L = \gamma mr^2 \frac{d\varphi}{d\tau} \tag{61}$$

where:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad . \tag{62}$$

The angular velocity is defined by:

$$\omega = \frac{d\varphi}{d\tau} \quad . \tag{63}$$

So Eq. (39) becomes the special relativistic equation:

$$\frac{1}{2}(\gamma^2 - 1)\, mc^2 = -\frac{mMG}{r} + \frac{1}{2}\gamma^2 mv^2 \tag{64}$$

where the square of the total velocity in cylindrical polar coordinates is [11]:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 \quad . \tag{65}$$

In the limit:

$$r \longrightarrow \infty \tag{66}$$

Eq. (64) becomes:

$$\frac{1}{2}(\gamma^2 - 1)\, mc^2 = \frac{1}{2}\gamma^2 mv^2 = \frac{1}{2}\frac{p^2}{m} \tag{67}$$

which is the Einstein energy equation of special relativity for a free particle of mass m :

$$E^2 = p^2c^2 + m^2c^4 \quad . \tag{68}$$

The relativistic kinetic energy in special relativity is:

$$T = mc^2\,(\gamma - 1) \tag{69}$$

and if $v << c$:

$$\gamma \longrightarrow 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{70}$$

giving the classical kinetic energy:

$$T \longrightarrow \frac{1}{2}mv^2 \ . \tag{71}$$

In the limit:

$$\gamma \longrightarrow 1 \tag{72}$$

Eq. (67) becomes:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2 \qquad . \tag{73}$$

The equation of motion (39) may be written as:

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) = \frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 + V \tag{74}$$

where

$$V = -\frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \tag{75}$$

The Newtonian limit of Eqs. (74) and (75) is [11]:

$$E_N = \frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 + \frac{L^2}{2mr^2} - \frac{mMG}{r} \tag{76}$$

and in the classical dynamics E_N is identified with H. In general relativity, E is not the same as H as we have seen. In Eqs. (74) and (75):

$$\frac{E^2}{mc^2} = mc^2\left(1 - \frac{r_0}{r}\right)^2\left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2\right)^{-1}\frac{d\varphi}{dt} \tag{77}$$

$$L = mr^2\left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2\right)^{-1/2}\frac{d\varphi}{dt} \quad .$$
(78)

The way in which Eqs.(74) and (75) reduce to Eq.(76) is not usually explained in textbooks, it is taken for granted that they do. The method of reduction is non trivial and is as follows. On the left hand side of Eq. (74):

$$\left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2 \right) = (1 - \frac{r_0}{r}) \left(1 - \left(\frac{v}{c}\right)^2 \right) - \frac{r_0}{r} \left(\frac{v}{c}\right)^2 \quad . \tag{79}$$

Now assume:

$$\frac{r_0}{r} << 1 \quad , \quad \frac{v}{c} << 1 \quad , \tag{80}$$

so:

$$\left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2 \right) \sim (1 - \frac{r_0}{r}) \left(1 - \left(\frac{v}{c}\right)^2 \right) \tag{81}$$

and

$$\frac{E^2}{mc^2} \sim mc^2 \, (1 - \frac{r_0}{r}) \left(1 - \left(\frac{v}{c}\right)^2 \right)^{-1} \quad . \tag{82}$$

Now use:

$$\left(1 - \left(\frac{v}{c}\right)^2 \right)^{-1} = 1 + \left(\frac{v}{c}\right)^2 \tag{83}$$

so

$$\frac{E^2}{mc^2} \sim mc^2 \, (1 - \frac{r_0}{r}) \, (1 + \left(\frac{v}{c}\right)^2) \tag{84}$$

and

$$\frac{1}{2} \left(\frac{E^2}{mc^2} - mc^2 \right) \sim \frac{1}{2} \, mv^2 - \frac{1}{2} \, mc^2 \frac{r_0}{r} = \frac{1}{2} \, mv^2 - \frac{mMG}{r} \quad . \tag{85}$$

Q.E.D. There are therefore several non-trivial approximations. In the Newtonian limit, the right hand side of Eq. (74) is assumed to reduce to:

$$V \longrightarrow -\frac{mMG}{r} + \frac{L^2}{2mr^2} \tag{86}$$

where the angular momentum L reduces to:

$$L \longrightarrow mr^2 \frac{d\varphi}{dt} \quad . \tag{87}$$

So the right hand side of Eq.(74) becomes:

$$\text{RHS} \qquad \frac{1}{2}\, m\, (\frac{dr}{d\tau})^2 + \frac{1}{2}\; mr^2 \frac{d\varphi}{dt} - \frac{mMG}{r} \quad . \tag{88}$$

Finally it is assumed that:

$$(\frac{dr}{d\tau})^2 \longrightarrow \left(1 - \frac{r_0}{r} - \left(\frac{v}{c}\right)^2\right)^{-1} (\frac{dr}{dt})^2 \longrightarrow (\frac{dr}{d\tau})^2 \tag{89}$$

Therefore:

$$\text{RHS} \longrightarrow \frac{1}{2}\, m\, ((\frac{dr}{d\tau})^2 + r^2 (\frac{d\varphi}{dt})^2\,) - \frac{mMG}{r} = \frac{1}{2}\, mv^2 - \frac{mMG}{r} \quad . \tag{90}$$

So Eq. (74) reduces to the Newtonian

$$E_N = \frac{1}{2}\, mv^2 - \frac{mMG}{r} = T + V \; . \tag{91}$$

Q.E.D. Similarly in ECE theory the Coulombic limit is obtained.

The above theory works well for relativistic Keplerian orbits [11], but for other types of orbit such as those in binary pulsars, the precessing ellipse also spirals inwards. In order to describe this orbit the general spherical spacetime is needed:

$$ds^2 = e^{-2\alpha}\, c^2 dt^2 - e^{2\beta} dr^2 - r^2\, d\varphi^2 \tag{92}$$

where [12], in general:

$$\alpha = \alpha\,(r\,,\,t) \quad , \quad \beta = \beta\,(r\,,\,t) \tag{93}$$

i.e. α and β are functions of r and t. The precessing elliptical orbits of the relativistic Kepler problem are given by:

$$e^{-2\alpha} = 1 - \frac{r_0}{r} \quad , \quad e^{2\beta} = \left(1 - \frac{r_0}{r}\right)^{-1} \quad , \tag{94}$$

but the additional inward spiralling of binary pulsars need:

$$e^{-2\alpha} = 1 - \frac{r_0}{r} - \frac{r_1}{r^2} \quad , \tag{95}$$

$$e^{2\beta} = \left(1 - \frac{r_0}{r} - \frac{r_1}{r^2}\right)^{-1} \; . \tag{96}$$

So it is reasonable to assume in general that:

$$e^{-2\alpha} = 1 - \frac{r_0}{r} - \frac{r_1}{r^2} - \ldots - \frac{r_n}{r^n} \; , \tag{97}$$

$$e^{2\beta} = \left(1 - \frac{r_0}{r} - \frac{r_1}{r^2} - \ldots - \frac{r_n}{r^n}\right)^{-1} . \tag{98}$$

This series can be built up by an extension of the Orbital Theorem of UFT 111 on www.aias.us: as follows. Extend the original Orbital Theorem:

$$mr = \frac{r}{n} = \int dr \tag{99}$$

to:

$$m_1 r = \int dr = r - \rho_1 \tag{100}$$

$$m_2 \frac{r^2}{2} = \int r dr = \frac{r^2}{2} - \frac{\rho_2}{2} \tag{101}$$

$$\vdots$$

$$m_n \frac{r^n}{n} = \int r^{n-1} dr = \frac{1}{n}(r^n - \rho_n) \tag{102}$$

where ρ_1 , ρ_2 , ... , ρ_n are constants of integration. So:

$$m = \frac{1}{n}(m_1 + m_2 + \ldots + m_n) = 1 - \frac{\rho_1}{nr} - \ldots - \frac{\rho_n}{nr^n} \tag{103}$$

where

$$m = 1 - \frac{r_0}{r} - \frac{r_1}{r^2} - \ldots - \frac{r_n}{r^n} \quad , \tag{104}$$

$$n = m^{-1} . \tag{105}$$

In chapter 15 of volume 6 of reference (1), r_1 was found by comparison with the orbit of binary pulsars.

The Maclaurin series expansion:

$$e^{-\alpha} = 1 - \alpha + \frac{\alpha^2}{2!} - \ldots \tag{106}$$

can be written up to the first two terms as:

$$\alpha(r) := \frac{r_0}{r} \tag{107}$$

so

$$\exp\left(-\frac{r_0}{r}\right) = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \ldots \tag{108}$$

but this function gives rise to a metric that gives an expanding precessing ellipse. The equation of motion for a metric of type (108) is

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) = \frac{1}{2}\, m \left(\frac{dr}{d\tau}\right)^2 + V \quad . \tag{109}$$

The effective potential is:

$$V = -\frac{mMG}{r} - \frac{MGL^2}{mc^2r^3} + \frac{L^2}{2mr^2}\left(1 + \left(\frac{2MG}{c^2r}\right)^2\right) + 2\, m\left(\frac{MG}{cr}\right)^2 \tag{110}$$

and contains hitherto unknown repulsive terms in addition to the centrifugal force. These terms force the orbit outwards by a small amount per revolution and may account for solar system anomalies observed by satellites recently.

The orbital equation from metric (108) is

$$\frac{d\varphi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \ldots\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} \tag{111}$$

from which corrections to light deflection may be calculated by extending the methods in UFT 150 on www.aias.us.

It is important to check the validity of the Maclaurin series:

$$e^{-r_0/r} = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \frac{1}{3!}\left(\frac{r_0}{r}\right)^3 \ldots \tag{112}$$

used in these metrical calculations. The Maclaurin series is defined by:

$$f(x) = f(0) + x\, f'(0) + \frac{x^2}{2!}\, f''(0) + \ldots \tag{113}$$

and is a special case of the Taylor series:

$$f(a + x) = f(a) + x\, f'(a) + \frac{x^2}{2!}\, f''(a) + \ldots \tag{114}$$

as is well known. The Maclaurin series can be used for combinations of elementary functions, for example:

$$\exp(\sin x) = 1 + x + \frac{x^2}{2!} - \frac{3x^4}{4!} - \ldots \tag{115}$$

is found from:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{4!} + \ldots \tag{116}$$

and

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \tag{117}$$

so

$$e^y = 1 + y \frac{y^2}{2!} + \frac{y^3}{3!} + \ldots \tag{118}$$
$$y = \sin x \quad .$$

Therefore:

$$e^y = 1 + \sin x + \frac{(\sin x)^2}{2!} + \ldots = 1 + (x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots) \quad . \tag{119}$$

Q.E.D. In order for this procedure to be valid the two power series must converge for a common interval of convergence [13]. The series:

$$e^{-\alpha} = 1 - \alpha + \frac{\alpha^2}{2!} - \frac{\alpha^3}{3!} + \ldots \tag{120}$$

converges for:

$$\alpha < 1 , \tag{121}$$

so if

$$\alpha = \frac{r_0}{r} \tag{122}$$

it converges if $r_0 <$ r. The Maclaurin series (120) is obtained from:

$$\exp(-(\alpha + a)) = f(a) + \alpha f'(a) + \frac{\alpha^2}{2!} f''(a) + \ldots \quad . \tag{123}$$

Here

$$f'(a) = \frac{d}{d\alpha} e^{-\alpha} \text{ (when } \alpha = a) \quad = - a\, e^{-a} \quad . \tag{124}$$

Eq. (122) can be regarded as a power series consisting of one term. The interval of convergence is:

$$0 < r_0 < r \tag{125}$$

i.e.

$$0 < \alpha < 1 \quad . \tag{126}$$

In this interval Eq. (120) and Eq. (122) both converge, so:

$$e^{-r_0/\,r} = 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \frac{1}{3!}\left(\frac{r_0}{r}\right)^3 + \ldots \tag{127}$$

Q.E.D.

In general, the Taylor series [13] is:

$$\mathrm{f}\,(x\,) = \mathrm{f}\,(a) + (x - a)\ \mathrm{f}'(a) + \frac{(x-a)^2}{2!}\ \mathrm{f}''(a) + \ldots \tag{128}$$

where

$$\mathrm{f}\,(x\,) = e^{-r_0/\,x} \tag{129}$$

$$\mathrm{f}'(x) = \frac{r_0}{x^2}\ e^{-r_0/\,x} \tag{130}$$

$$\mathrm{f}''(x) = \frac{r_0}{x^3}\left(\frac{r_0}{x} - 2\right)\ e^{-r_0/\,x} \quad . \tag{131}$$

So

$$e^{-r_0/\,r} = \ e^{-r_0/\,a}\ (\ 1 + \frac{r_0}{a^2}\ (r - a) + \frac{(r-a)^2}{2!}\ \frac{r_0}{a^3}\ e^{-r_0/\,x} + \ldots$$

$$= 1 - \frac{r_0}{r} + \frac{1}{2!}\left(\frac{r_0}{r}\right)^2 - \frac{1}{3!}\left(\frac{r_0}{r}\right)^3 + \ldots \tag{132}$$

In order to address problems in electrostatics and electrodynamics from a metrical approach, the scalar and vector potentials must be known. With reference to note 153(9) accompanying this paper (ref. [14] of this paper, National Library of Wales and British National Archives www.webarchive.org.uk, site www.aias.us, UFT 153) the linear velocity in cylindrical polar coordinates is [15]:

$$\boldsymbol{v} = \frac{dX}{dt}\,\boldsymbol{i} + \frac{dX}{dt}\,\boldsymbol{j} + \frac{dZ}{dt}\,\boldsymbol{k} = \frac{dr}{dt}\,\boldsymbol{e}_r + r\,\frac{d\varphi}{dt}\,\boldsymbol{e}_\varphi + \frac{dZ}{dt}\,\boldsymbol{k} \quad . \tag{133}$$

The minimal prescription [16] means that:

$$\boldsymbol{p} = m\,\boldsymbol{v} = e\,\boldsymbol{A} \quad , \qquad \boldsymbol{A} = \frac{m}{e}\,\boldsymbol{v} \quad . \tag{134}$$

Restrict consideration to the XY plane in the non relativistic limit of the metrical approach of this section to find that:

$$\boldsymbol{p} = m\,\boldsymbol{v} \quad , \quad p_r = m\frac{dr}{dt} \quad , \quad L = mr^2\frac{d\varphi}{dt} \tag{135}$$

where $\boldsymbol{p}$ is the total linear momentum, whose radial component is p_r, and where L is the angular momentum. The vector potential is therefore:

$$\boldsymbol{A} = \boldsymbol{A}_r + \boldsymbol{A}_\varphi \tag{136}$$

where

$$\boldsymbol{A}_r = \frac{m}{e}\frac{dr}{dt}\,\boldsymbol{e}_r \quad , \quad \boldsymbol{A}_\varphi = \frac{m}{e}\,r\frac{d\varphi}{dt}\,\boldsymbol{e}_\varphi \quad . \tag{137}$$

In Cartesian coordinates it has the irrotational component:

$$\boldsymbol{A}_r = \frac{m}{e}\left(\frac{\mathrm{X}\,\boldsymbol{i} + \mathrm{Y}\,\boldsymbol{j}}{(X^2+Y^2)^{1/2}}\right)\frac{dr}{dt} \tag{138}$$

and the divergenceless component:

$$\boldsymbol{A}_\varphi = \frac{m}{e}\left(-\,Y\,\boldsymbol{i} + X\,\boldsymbol{j}\right)\frac{d\varphi}{dt} \quad . \tag{139}$$

Therefore:

$$\nabla\cdot\boldsymbol{A}_\varphi = 0 \quad , \quad \nabla\times\boldsymbol{A}_r = \boldsymbol{0} \tag{140}$$

and so Eq. (136) is the Helmholtz Theorem [17], which has been obtained from the metric and minimal prescription in a straightforward way. Recently [1-10] the Helmholtz Theorem has been extended independently by Silver, by Moses, by Read and by Evans [18] to:

$$\boldsymbol{A} = \boldsymbol{A}^{(1)} + \boldsymbol{A}^{(2)} + \boldsymbol{A}^{(3)} \quad , \tag{141}$$

$$\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = \frac{1}{\sqrt{2}}\left(\boldsymbol{i} - i\,\boldsymbol{j}\right) \quad , \tag{142}$$

$$\boldsymbol{e}^{(1)} = \boldsymbol{k} \tag{143}$$

and this is the meaning of the index

$$a = (1)\,,\,(2)\,,\,(3) \tag{144}$$

in ECE theory. ECE theory and Cartan geometry profoundly enhance the fundamental structure of electrodynamics, giving for example the $\boldsymbol{B}^{(3)}$field [1-10] which has been routinely observable in the inverse Faraday effect for fifty years. For the $\boldsymbol{B}^{(3)}$field, the a index is (3) of the complex circular basis, and this observable and radiated magnetic flux density (in S.I. units of tesla) is given by fundamental geometry itself. Its significance is that electrodynamics becomes general relativity, as demanded by the philosophy of relativity and unified field theory.

The metrical method of this section is fundamental to general relativity and in note 153(10) [14] it is considered how tetrads may be obtained from elements of the metric. In Cartan geometry [1-10,12]. A general method of obtaining tetrads from metrics means that wave and field equations in ECE theory [1-10] are linked to metric based equations of motion. It is therefore fundamentally important to define clearly the meaning of the Cartan tetrad. It is a mixed index tensor [12]. If e^{a} denotes basis elements of the coordinate system labelled a and e^{μ} of the coordinate system labelled μ, then the tetrad tensor q^{a}_{μ} is defined by [12]

$$\mathrm{e}^{a} = q^{a}_{\mu}\, \mathrm{e}^{\mu} \ . \tag{145}$$

Let a denote the cylindrical polar coordinates and μ the Cartesian coordinates, and for simplicity of argument, confine attention to three space dimensions. Let the unit vectors of the cylindrical polar basis be defined by:

$$\boldsymbol{e}_r = (e^{(1)}, 0 , 0) \tag{146}$$

$$\boldsymbol{e}_\varphi = (0 , e^{(2)}, 0) \tag{147}$$

$$\boldsymbol{e}_Z = (0 , 0 , e^{(3)}) \quad . \tag{148}$$

In the Cartesian coordinate basis (see note 153(10)) these unit vectors are:

$$\boldsymbol{e}_r = (e_1, e_2, 0) = (\cos\varphi , \sin\varphi , 0) \tag{149}$$

$$\boldsymbol{e}_\varphi = (e_1, e_2, 0) = (- \sin\varphi , \cos\varphi , 0) \tag{150}$$

$$\boldsymbol{e}_Z = (0 , 0 , e_3) = (0 , 0 , 1) \ . \tag{151}$$

By definition:

$$(e^{(1)}, 0 , 0) = (1 , 0 , 0) \tag{152}$$

$$(0 , e^{(2)}, 0) = (0 , 1 , 0) \tag{153}$$

$$(0 , 0 , e^{(3)}) = (0 , 0 , 1) \tag{154}$$

and applying Eq. (145):

$$e^{(1)} = q^{(1)}_1 e^1 + q^{(1)}_2 e^2 \quad , \tag{155}$$

$$e^{(2)} = q_1^{(2)} e^1 + q_2^{(2)} e^2 \quad , \tag{156}$$

i.e.

$$q_1^{(1)} \cos\varphi + q_2^{(1)} \sin\varphi = 1 \tag{157}$$

$$- q_1^{(2)} \sin\varphi + q_2^{(2)} \cos\varphi = 1 \tag{158}$$

A possible solution is:

$$q_1^{(1)} = \cos\varphi \, , \, q_2^{(1)} = \sin\varphi \quad , \tag{159}$$

$$q_1^{(2)} = - \sin\varphi \, , \, q_2^{(2)} = \cos\varphi \quad , \tag{160}$$

$$q_3^{(3)} = 1 \tag{161}$$

so the tetrad is:

$$q_\mu^a = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ - \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad . \tag{162}$$

The tetrad matrix is the transformation matrix from one coordinate system to the other [16].

The tetrad vectors [1-10] are defined by the unit vectors as follows:

$$\boldsymbol{q}^{(1)} = \boldsymbol{e}^{(1)} = \boldsymbol{e}_1 = \boldsymbol{e}_r = q_1^{(1)} \boldsymbol{i} + q_2^{(1)} \boldsymbol{k} \quad , \tag{163}$$

$$\boldsymbol{q}^{(2)} = \boldsymbol{e}^{(2)} = \boldsymbol{e}_2 = \boldsymbol{e}_\varphi = q_1^{(2)} \boldsymbol{i} + q_2^{(2)} \boldsymbol{k} \quad , \tag{164}$$

$$\boldsymbol{q}^{(3)} = \boldsymbol{e}^{(3)} = \boldsymbol{e}_3 = \boldsymbol{e}_Z = \boldsymbol{k} \quad . \tag{165}$$

So any vector can be defined by a tetrad vector, because the latter is a unit vector. This concept is extended in ECE theory to any spacetime. For example the electromagnetic potential in ECE theory is:

$$A_\mu^a = A^{(0)} \, q_\mu^a \tag{166}$$

where $A^{(0)}$ is a scalar valued magnitude, and the gravitational potential is [1-10]:

$$\Phi_\mu^a = \Phi^{(0)} \, q_\mu^a \quad . \tag{167}$$

So ECE theory IS Cartan geometry, which has been well known and accepted since the nineteen twenties [12]. ECE theory shows that the whole of known physics may be unified with Cartan geometry, and this is a triumph of the philosophy of relativity, first proposed in the eighteen eighties by Heaviside and Fitzgerald. ECE theory has made the so called "standard model" of physics obsolete in numerous ways, so the standard model will never recover credibility as an attempt to unify physics. It has gone the way of phlogiston.

Metrics in two different coordinate systems ($g_{\mu\nu}$ and η_{ab}) are related by the tetrads as follows [12]:

$$g_{\mu\nu} = q^a_\mu \, q^b_\nu \, \eta_{ab} \quad . \tag{168}$$

So:

$$g_{11} = q_1^{(1)} \, q_1^{(1)} \, \eta_{(1)(1)} \; + \; q_1^{(2)} \, q_1^{(2)} \, \eta_{(2)(2)} \; , \tag{169}$$

$$g_{22} = q_2^{(1)} \, q_2^{(1)} \, \eta_{(1)(1)} \; + \; q_2^{(2)} \, q_2^{(2)} \, \eta_{(2)(2)} \; , \tag{170}$$

$$g_{33} = q_3^{(3)} \, q_3^{(3)} \, \eta_{(3)(3)} \quad . \tag{171}$$

This means (note 153(10)) that the metrics in the Cartesian and cylindrical polar systems are unit diagonal:

$$g_{\mu\nu} = \eta_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{172}$$

if the definition (168) is implemented. Note carefully that if the curvilinear definition [15] of the metric is used:

$$g_{ij} = \frac{\partial \boldsymbol{r}}{\partial u_1} \cdot \frac{\partial \boldsymbol{r}}{\partial u_2} \tag{173}$$

the result for the cylindrical polar coordinate system is:

$$g_{ij} = \; g_{ji} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{174}$$

and this is different from the result from the Cartan definition. The latter system is more elegant and powerful and is coordinate free as can be seen from Eq. (172). The curvilinear definition is not coordinate free because of the presence of r^2in the metric. This appears to be the first time that this discrepancy has been noticed.

Extending this method as in note 153(11) some tetrads in various three dimensional coordinate systems compared with the Cartesian system are given in Table 1.

Table 1: Tetrads in Three Dimensional Coordinate Systems

Coordinate System	Tetrad q^a_μ
Cartesian	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Complex circular	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 \\ 1 & i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$
Cylindrical Polar	$\begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Spherical Polar	$\begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\varphi \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix}$

In each case the tetrad matrix is the transformation matrix. As in note 153(12), consider now the rotation of a vector $\boldsymbol{V}$ about the Z axis in the XY plane [16]:

$$\begin{pmatrix} V'_X \\ V'_Y \\ V'_Z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix} . \tag{175}$$

It is seen that the transformation matrix from the Cartesian to cylindrical polar coordinate system is the rotation matrix. Consider the rotation to be the passive rotation [16]. The passive rotation is defined as one in which the vector is kept constant but the axes are rotated anticlockwise [16]. The rotation of the axes defines the connection of geometry and this is one of the simplest examples of the meaning of the connection. The rotation generator [16] is defined as:

$$J_Z = \frac{1}{i} \frac{dR_Z(\varphi)}{d\varphi} \quad (\text{at } \varphi = 0) \; = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{176}$$

$$R_Z(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{177}$$

and so is defined by the tetrad matrix, which is the rotation matrix. The rotation generators obey the cyclic equations [16]:

$$\left[J_X , J_Y \right] = i\, J_Z \quad , \tag{178}$$

et cyclicum.

These are also the equations of the angular momentum operators in quantum mechanics within a factor ħ, the reduced Planck constant:

$$\left[J_X , J_Y \right] = i\hbar\, J_Z \quad , \tag{179}$$

et cyclicum.

So the angular momenta of quantum mechanics are defined by Cartan geometry , Q.E.D. specifically they are defined by tetrads such as:

$$q^a_\mu = \begin{pmatrix} q_1^{(1)} & q_2^{(1)} & 0 \\ q_1^{(2)} & q_2^{(2)} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{180}$$

Rotation conserves the vector field [16], and in Cartan geometry the conservation of the vector field is the tetrad postulate [1-10,12]:

$$D_\mu\, q^a_\nu = \partial_\mu\, q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu}\, q^a_\lambda = 0 \tag{181}$$

which is simply:

$$\partial_\mu\, q^a_\nu = \alpha^a_{\mu\nu} \tag{182}$$

where:

$$\alpha^a_{\mu\nu} = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \tag{183}$$

In the notation of Eq. (180) the lower indices without brackets are indices of the Cartesian system, and the upper indices with brackets those of the cylindrical polar system defined by:

$$\cos\varphi = \frac{X}{r} \quad , \quad \sin\varphi = \frac{Y}{r} \quad , \quad Z = Z \tag{184}$$

where r is defined to be a constant:

$$r = (X^2 + Y^2)^{½} \tag{185}$$

Therefore:

$$\partial_2\, q^a_\mu = \frac{\partial q^a_\mu}{\partial Y} = \frac{1}{r}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad \partial_1\, q^a_\mu = \frac{\partial q^a_\mu}{\partial Y} = \frac{1}{r}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{186}$$

So:

$$\begin{pmatrix} 0 & \alpha^{(1)}_{22} & 0 \\ \alpha^{(2)}_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{r}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{187}$$

i.e.

$$\alpha^{(1)}_{22} = -\,\alpha^{(2)}_{21} = \frac{1}{r} \quad . \tag{188}$$

As in UFT 63 on www.aias.us, the connections are inversely proportional to the radial component r . It is found that the $\alpha^{a}_{\mu\nu}$ matrix is a rotation generator within $i\,/\,r$:

$$\begin{bmatrix} 0 & \alpha^{(1)}_{22} & 0 \\ \alpha^{(2)}_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{i}{r}\, J_Z \quad . \tag{189}$$

The tetrad postulate (182) is very fundamental, it is the mathematical expression of the fact that a passive rotation is equivalent to the usual way in which rotation is defined, as the active rotation [16] of a vector keeping the coordinates fixed. The active rotation in Eq. (182) is $\partial_\mu\, q^{a}_{\nu}$ and the passive rotation is $\alpha^{a}_{\mu\nu}$. The tetrad postulate shows that the active and passive rotations are identically equal. This is always the case in natural philosophy and nearly the whole of mathematics. In the usual way of dealing with rotations in a plane XY about Z the idea of connection is not considered, but the connection is always inherent in the analysis. For the purposes of creating a unified field theory [1-10], Cartan geometry is entirely sufficient.

4. Maximization by resonance of the effect of electromagnetism on gravitation.

This section is condensed from notes 153(13) to 153(15) on www.aias.us and produces a very simple practical design for on board counter gravitation, a problem of great practical importance as humankind rapidly runs out of fuel for aircraft. It is demonstrated first that there exists a cross term in the hamiltonian relevant to the problem, a cross term that defines the effect of electromagnetism on gravitation. Usually this is very tiny, so tiny that it cannot be observed under usual conditions when for example testing the Coulomb law. However it exists from fundamentals of the minimal prescription and can be amplified by resonance. It is simple to show these facts as follows.

Define the gravitational four potential:

$$\Phi^{\mu} = (\, \Phi \,,\, c\mathbf{\Phi}\,) \tag{190}$$

and the electromagnetic four potential:

$$A^{\mu} = (\varphi \,,\, c\boldsymbol{A}\,) \tag{191}$$

where φ is the scalar potential and $\boldsymbol{A}$ the vector potential. In conventional gravitational theory only the scalar Φ is considered. Define the four momentum of a particle of mass m and charge e_1 by:

$$p^{\mu} = (\,\frac{E}{c} \,,\, \boldsymbol{p}\,) \tag{192}$$

where E is the energy and $\boldsymbol{p}$ the linear momentum. The hamiltonian is [1-10,16]:

$$\mathrm{H} = \frac{1}{2m} p^{\mu} p_{\mu} \tag{193}$$

and is an invariant. If E_0 is the initial energy and if we restrict attention for the moment to the scalar potentials, then:

$$p^{\mu} \longrightarrow p^{\mu} + m\, \Phi^{\mu} + e_1 A^{\mu} \tag{194}$$

$$H_1 = \frac{1}{2mc^2} H_1 = \frac{1}{2mc^2} \left(E_0 + e_1 \varphi + m\, \Phi\right)^2 \quad . \tag{195}$$

If for simplicity of argument and illustration:

$$E_0 = 0 \tag{196}$$

then:

$$H_1 = \frac{1}{2mc} (m^2 \Phi^2 + {e_1}^2 \varphi^2 + 2\, m e_1\, \Phi\, \varphi\,) \tag{197}$$

and the cross term is:

$$H_1 \text{ (cross term)} = e_1\, \Phi\, \varphi \,/\, c^2 \quad . \tag{198}$$

Similarly, for the momentum part of p^{μ}:

$$H_2 = \frac{1}{2m} \left({e_1}^2\, \boldsymbol{A} \,.\, \boldsymbol{A} + m^2\, \boldsymbol{\Phi} .\, \boldsymbol{\Phi} + 2\, m e_1\, \boldsymbol{A} \,.\, \boldsymbol{\Phi}\, \right) \tag{199}$$

so the complete cross term is:

$$H \text{(cross term)} = e_1 \left(\frac{\Phi\, \varphi}{c^2} + \boldsymbol{A} \,.\, \boldsymbol{\Phi}\, \right) \tag{200}$$

and depends only on e_1 and not on m.

The engineering problem is to maximize the cross term by resonance, in order to decrease **g,** the Earth´s acceleration due to gravity.

In the simplest theory, the combined force on the particle is:

$$\boldsymbol{F} = e_1\, \boldsymbol{E} + m\, \mathbf{g} \tag{201}$$

For simplicity of argument, define this to be in the Z axis:

$$\boldsymbol{F} = F\, \mathbf{k} \quad . \tag{202}$$

So we can use scalar notation:

$$F = e_1\, E + m\, \mathrm{g} \quad . \tag{203}$$

Here:

$$\mathrm{g} = \dot{v} = \ddot{r} \tag{204}$$

so

$$F = e_1\, E + m\, \ddot{r} \quad . \tag{205}$$

The simplest possible type of Euler Bernoulli resonance [11] occurs when:

$$\mathrm{F} + \mathrm{k}\, r = F_0 \cos \omega t \tag{206}$$

(see Section 1). So, from Eqs. (205) and (206):

$$m\, \ddot{r} + e_1\, E + \mathrm{k}\, r = F_0 \cos \omega t \tag{207}$$

and

$$m\, \ddot{r} + \mathrm{k}\, r = F_0 \cos \omega t - e_1\, E \quad . \tag{208}$$

Now assume that:

$$\mathrm{E} = E_0 \cos \omega t \tag{209}$$

so Eq. (208) becomes:

$$m\, \ddot{r} + \mathrm{k}\, r = 2\, e_1 E_0 \cos \omega t \quad . \tag{210}$$

Rewrite this as [11]:

$$\ddot{r} + \omega_0{}^2\, r = A \cos \omega t \tag{211}$$

with, in this notation:

$$\omega_0{}^2 = \frac{\mathrm{k}}{m} \quad , \quad A = 2\frac{e_1}{m} E_0 \quad . \tag{212}$$

Amplitude resonance [11] occurs at:

$$r\,(\mathrm{t}) = \left(\frac{A}{\omega_0{}^2 - \omega^2}\right) \cos \omega t \quad . \tag{213}$$

Define the kinetic energy as:

$$T = \frac{1}{2} m \ \dot{r}^2 \quad . \tag{214}$$

Kinetic energy resonance occurs at:

$$T = \frac{mA^2}{2} \left(\frac{\omega^2}{\omega_0{}^2 - \omega^2} \right) \sin^2 \omega t \quad , \quad \omega = \omega_0 \quad . \tag{215}$$

On average:

$$< T > = \frac{mA^2}{4} \left(\frac{\omega^2}{\omega_0{}^2 - \omega^2} \right) \tag{216}$$

because [11]:

$$< \sin^2 \omega t > = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin^2 \omega t \ dt = \frac{1}{2} \quad . \tag{217}$$

So amplitude and kinetic energy resonance occur at the same frequency:

$$\omega = \omega_0 \quad . \tag{218}$$

Resonance in the cross term (200) occurs at the frequency (218) and can be induced by an alternating electric field.

The force due to gravity is defined as:

$$F = m \ \mathrm{g} = m \ \ddot{r} \tag{219}$$

and is defined by:

$$F = -\left(\frac{\omega^2}{\omega_0{}^2 - \omega^2} \right) mA \cos \omega t \quad . \tag{220}$$

It goes to negative infinity at resonance induced by an alternating electric field. The device is put on board an aircraft or spacecraft to maximize at resonance a **g** that opposes the Earth´s **g**.

In ECE theory [1-10] the electric field strength in volts per metre is:

$$\mathrm{E} = -\nabla \varphi - \frac{d\boldsymbol{A}}{dt} + \varphi \ \boldsymbol{\omega}_E - \omega_E \boldsymbol{A} \tag{221}$$

and the acceleration due to gravity is:

$$\mathbf{g} = -\nabla \Phi - \frac{d\boldsymbol{\Phi}}{dt} + \varphi \ \boldsymbol{\omega}_g - \omega_g \boldsymbol{\Phi} \tag{222}$$

where the electromagnetic and gravitational spin connections are:

$$\omega_E{}^{\mu} = (\omega_E \, , \boldsymbol{\omega}_E \,) \tag{223}$$

$$\omega_g{}^{\mu} = (\omega_g \, , \boldsymbol{\omega}_g \,) \quad . \tag{224}$$

In the standard model:

$$\mathbf{E} = -\nabla \varphi - \frac{d\boldsymbol{A}}{dt} \tag{225}$$

$$\mathbf{g} = -\nabla \Phi - \frac{d\boldsymbol{\Phi}}{dt} \tag{226}$$

and $\boldsymbol{\Phi}$ is also omitted, so:

$$\mathbf{g} = -\nabla \Phi \tag{227}$$

Elementary considerations of commutator antisymmetry [1-10] mean that:

$$\nabla \varphi = \frac{d\boldsymbol{A}}{dt} \tag{228}$$

so in gravitation, even on the level of the standard model:

$$\nabla \Phi = \frac{d\boldsymbol{\Phi}}{dt} \tag{229}$$

The gravitational potential is:

$$\Phi = -\frac{MG}{r} \tag{230}$$

so:

$$F = m \, \mathrm{g} = -m \left| \frac{d\boldsymbol{\Phi}}{dt} \right| = -m \frac{d\Phi}{dr} = -\frac{mMG}{r^2} \tag{231}$$

which is the equivalence principle. This has been derived theoretically from antisymmetry [1-10]. Neither Newton nor Einstein DERIVED the equivalence principle, (they assumed it), but it is a direct and very simple result of commutator antisymmetry. In ECE theory [1-10] this derivation is referred to as one of the ECE principle of antisymmetry, and the spin connection of Eqs. (223) and (224) brought into consideration.

Similarly, it is recognized for the first time in this section that the electric equivalence principle is:

$$F = e_1 \, E = -\frac{e_1 e_1}{4\pi\epsilon_0 r^2} \tag{232}$$

and is the equivalence of the Lorentz and Coulomb forces. This is again DERIVABLE simply and directly from ECE antisymmetry. Both gravitational and electric equivalence principles are true experimentally to many orders of magnitude of precision. There is also however, a cross term to be taken into consideration, as described already.

Finally, as shown in 153(15) on www.aias.us, the definition of the Cartan torsion may be simplified, so that it becomes better understood by and more useful to engineers. The original definition by Cartan [1-10,12] is:

$$T^a_{\mu\nu} = \partial_\mu \, q^a_\nu - \partial_\nu \, q^a_\mu + \omega^a_{\mu b} \, q^b_\nu - \omega^a_{\nu b} \, q^b_\nu \tag{233}$$

Using [12]:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \tag{234}$$

Eq. (233) simplifies to:

$$\tau^a_{\mu\nu} = \partial_\mu \, q^a_\nu - \partial_\nu \, q^a_\mu \tag{235}$$

where:

$$\tau^a_{\mu\nu} = T^a_{\mu\nu} - \Omega^a_{\mu\nu} \tag{236}$$

and:

$$\Omega^a_{\mu\nu} = \omega^a_{\mu\nu} - \omega^a_{\nu\mu} \ . \tag{237}$$

Summing over a indices we obtain:

$$\tau_{\mu\nu} = \partial_\mu \, q_\nu - \partial_\nu \, q_\mu \ . \tag{238}$$

The electromagnetic field may be defined as:

$$F_{\mu\nu} = A^{(0)} \tau_{\mu\nu} = \partial_\mu \, A_\nu - \partial_\nu \, A_\mu \tag{239}$$

so

$$A_\mu = A^{(0)} q_\mu \tag{240}$$

which looks the same as the standard model definition for purposes of engineering. Of course it is not, because the standard definition omits the connection and indices a . However, for some practical purposes Eq. (239) suffices. Antisymmetry [1-10] means:

$$\partial_\mu A_\nu = - \partial_\nu A_\mu \quad (241)$$

i.e. Eq. (228). Note 153(15) then develops the definitions of the gravitational and electromagnetic fields.

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