

# Conservation of energy in electric power from spacetime

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The relativistic Hamilton Jacobi equation and other methods are used to show that the acquisition of electric power from spacetime is a process that conserves total energy, momentum and angular momentum. Energy is simply transferred from spacetime to a device in the laboratory or in engineering. The concept of spacetime is the foundation of the theory of relativity, the Hamiltonian being defined directly from the metric.

*Keywords:* Relativistic Hamilton Jacobi equation, electric power from spacetime, conservation of energy, momentum and angular momentum.

## 1. Introduction

The general theory of relativity is based on the geometry of spacetime. Energy is contained within spacetime and the Hamiltonian in general relativity is defined by the geometry in the format of the line element. The Hamiltonian is half the rest energy of a given mass  $m$ , and is always constant and independent of the motion. It is known therefore as a constant of motion, and defining the Hamiltonian in this way also means that it is an invariant under the general coordinate transformation [11, 12]. All forms of energy are interconvertible, so the energy within spacetime can be of any form, notably gravitational and electromagnetic. The Einstein–Cartan–Evans (ECE) theory is a suggestion [1–10] for a practical unified field theory that is capable of describing physics in a geometrical framework. Therefore, ECE has been accepted as being the first unified field theory in the history of physics. String theory and hugely complicated twentieth century theories are not workable, and have not succeeded in producing a unified field theory. Indeed it is known that they are riddled with basic errors, some of them elementary and easy to demonstrate [1–10]. Accordingly they are not used outside a narrowly defined speciality of physics, they are not used in the rest of physics, in chemistry, the life sciences, or engineering.

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In Section 2 the relativistic Hamilton Jacobi equation is defined in general relativity in terms of the action  $S$ , and the Hamiltonian  $H$  defined directly by the relativistic Hamilton Jacobi equation. These well known concepts of classical physics can therefore be transferred logically to general relativity, as is well known. It becomes immediately clear that general relativity conserves energy and momentum, and all equations and processes of general relativity do so too. In Section 2 these fundamental properties of the theory of general relativity are used to demonstrate the existence of electric power in spacetime. There is an infinite amount of such power that can be used for practical purposes without violating any principle of conservation. The process of taking energy from spacetime is governed as for any dynamical process by classical physics, the relativistic Hamilton Jacobi equation and the Hamiltonian. The key concept is that spacetime contains energy, and is an unlimited reservoir of energy. Spacetime is not to be confused with the vacuum of relativity, the vacuum of relativity contains no mass, no charge and no energy. Every time a stone drops to the ground the gravitational energy of spacetime is revealed. Therefore spacetime is also an infinite reservoir of gravitational energy.

## 2. Relativistic Hamilton Jacobi equation and Hamiltonian

In the mathematical space with torsion and curvature the relativistic Hamilton Jacobi (HJ) equation is:

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = m^2 c^2 \quad (1)$$

where  $g^{\mu\nu}$  is the inverse metric tensor,  $S$  the action,  $m$  the mass and  $c$  the speed of light in a vacuum. In the general theory of relativity  $c$  is a universal constant with an exact value fixed in standards laboratories by agreement. The mathematical space of ECE theory is the physical spacetime of four dimensions [1–10]. ECE is therefore preferred to string theory by Ockham's Razor because ECE uses only the four well accepted and well observed dimensions of relativity, while string theory uses a multitude of speculative, unobservable, unproven, parameters masquerading as 'dimensions'. String theory has been constantly and heavily criticised for violating the fundamentals of natural philosophy in that it is deliberately constructed to be hugely complex and unobservable. Genuine natural philosophy is designed to be simple and all theories must be manifestly observable in the simplest possible way. Recent scholarship [1–10] has shown that the entire edifice of string theory is based on deeply flawed concepts such as the  $U(1)$  sector symmetry of electrodynamics, the massless photon, the incorrect symmetric connection of geometry, and above all the use of fictitious dimensions that do not exist in nature.

The four momentum is defined from the action by:

$$p_{\mu} = \frac{\partial S}{\partial x^{\mu}}, \quad (2)$$

so

$$g^{\mu\nu} p_{\mu} p_{\nu} = m^2 c^2, \quad (3)$$

which may be written as:

$$p^{\mu} p_{\mu} = m^2 c^2, \quad (4)$$

because:

$$p^{\mu} = g^{\mu\nu} p_{\nu}. \quad (5)$$

The Hamiltonian is conserved under all conditions and is:

$$H = \frac{1}{2m} p^{\mu} p_{\mu} \quad (6)$$

and the metric  $g_{\mu\nu}$  and inverse metric  $g^{\mu\nu}$  are related by:

$$g^{\mu\nu} g_{\mu\nu} = 4. \quad (7)$$

The Hamiltonian of general relativity is also defined in terms of the line element  $ds^2$  [1–10]. If the general line element of spherical spacetime is written as:

$$ds^2 = c^2 d\tau^2 = c^2 x(r, t) dt^2 - y(r, t) dr^2 - r^2 d\phi^2, \quad (8)$$

then the Hamiltonian is:

$$H = \frac{1}{2} mc^2 = \frac{1}{2} m \left( \frac{dS}{d\tau} \right)^2 = \frac{1}{2} m \left( xc^2 \left( \frac{dt}{d\tau} \right)^2 - y \left( \frac{dr}{d\tau} \right)^2 - r^2 \left( \frac{d\phi}{d\tau} \right)^2 \right). \quad (9)$$

The units of the Hamiltonian are energy (joules), so it becomes immediately obvious that energy is contained in spacetime. In the vacuum, on the other hand:

$$m = 0 \quad (10)$$

so the Hamiltonian  $H$  of the vacuum is zero. There is no energy in the vacuum, but there is energy in spacetime.

In these equations,  $d\tau$  is the infinitesimal of proper time, which is the time in the coordinate system fixed on the moving particle. As shown below, Eq. (9) generalizes the Einstein energy equation, the classical limit of the Dirac equation. The Minkowski line element is defined as a limit of (9) as follows:

$$x = y = 1. \tag{11}$$

The Lagrangian of general relativity is defined by the integral over the action  $S$ . The Euler Lagrange equation may be applied to Eq. (9) to give three constants of motion. These are the total energy, denoted  $E_1$ , the radial component of linear momentum, denoted  $\pi_r$ , and the total angular momentum denoted  $L$ . They are defined as follows:

$$E_1 = xmc^2 dt / d\tau, \tag{12}$$

$$\pi_r = ym dr / d\tau, \tag{13}$$

$$L = mr^2 d\phi / d\tau, \tag{14}$$

$$\pi_\phi = mr d\phi / d\tau. \tag{15}$$

The total linear momentum is the sum of radial and angular components [11, 12]:

$$\boldsymbol{\pi} = \pi_r \mathbf{e}_r + \pi_\phi \mathbf{e}_\phi \tag{16}$$

using the unit vectors of the cylindrical polar system of coordinates. All these quantities are constants of motion and are therefore conserved both for the free particle and for field particle interaction. All aspects of energy are therefore defined by spacetime, together with all conservation theorems.

These conserved quantities are defined in terms of  $d\tau$ , the infinitesimal of proper time. Writing the line element as:

$$c^2 d\tau^2 = x c^2 dt^2 - d\mathbf{r} \cdot d\mathbf{r} \tag{17}$$

then by definition:

$$d\mathbf{r} \cdot d\mathbf{r} = y dr^2 + r^2 d\phi^2 \tag{18}$$

where the velocity is defined by:

$$v = \frac{dr}{dt} \quad (19)$$

Here  $dt$  is the infinitesimal of time as measured in the laboratory frame. This is the frame of the observer, and the particle moves at a velocity  $v$  in this frame. The infinitesimal of proper time  $d\tau$  is defined in a frame of reference fixed on the particle. We have:

$$c^2 d\tau^2 = c^2 dt^2 - v^2 dt^2 \quad (20)$$

so:

$$d\tau = \left( c^2 - v^2 \right)^{1/2} dt \quad (21)$$

This is the relation between proper time and observer time in a spherically symmetric spacetime.

An example of the line element of spherically symmetric spacetime is the gravitational line element of the orbital theorem of UFT 111 ([www.aias.us](http://www.aias.us)):

$$x = 1 - \frac{r_0}{r} \quad (22)$$

$$y = \left( 1 - \frac{r_0}{r} \right)^{-1} \quad (23)$$

where

$$r_0 = \frac{2MG}{c^2} \quad (24)$$

Here  $M$  is the mass of an object that attracts  $m$  through gravitational interaction,  $G$  is Newton's constant and  $r$  is the distance between  $m$  and  $M$ . In this case:

$$d\tau = \left( 1 - \frac{r_0}{r} - \frac{v^2}{c^2} \right)^{1/2} dt. \quad (25)$$

If:

$$r_0 \ll r, \quad (26)$$

$$v \ll c, \quad (27)$$

then:

$$\left(1 - \frac{r_0}{r} - \frac{v^2}{c^2}\right) = \left(1 - \frac{r_0}{r}\right) \left(1 - \frac{v^2}{c^2}\right) - \frac{r_0 v^2}{r c^2} \sim \left(1 - \frac{r_0}{r}\right) \left(1 - \frac{v^2}{c^2}\right) \quad (28)$$

so Eq. (25) simplifies to:

$$d\tau \sim \left(1 - \frac{r_0}{r}\right)^{1/2} \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt. \quad (29)$$

In the limit of special relativity:

$$\frac{r_0}{r} \rightarrow 0 \quad (30)$$

so the relation between  $d\tau$  and  $dt$  simplifies to:

$$d\tau \rightarrow \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt. \quad (31)$$

The infinitesimal of proper time is shorter than  $dt$ . As  $v$  approaches  $c$ , the proper time infinitesimal approaches zero, when there is no interval of time, no change in time. The proper time is the shortest interval of time possible. A clock moving with the particle records  $dt$  in the frame of the observer and this appears longer than  $d\tau$  to the observer. The clock records  $d\tau$  when the clock and observer are fixed in the same frame. In the presence of a gravitational attraction, time is also affected by the field. From Eq. (6) however it is seen that the Hamiltonian  $H$  is always a constant. There is therefore energy in the spacetime that describes a gravitational field. As shown in recent UFT papers on [www.aias.us](http://www.aias.us) these concepts can be extended straightforwardly to the interaction of two charges,  $e_1$  and  $e_2$ . The distance  $r_0$  in that case becomes:

$$r_0 = \frac{2e_1 e_2}{4\pi m c^2 \epsilon_0} \quad (32)$$

where  $\epsilon_0$  is the permittivity of the vacuum. The Hamiltonian in this case shows that there is electrostatic energy in spacetime. ECE extends and unifies these

concepts, showing that the structures of dynamics and electrodynamics are the same. There exist Poynting Theorems for the gravitational and electromagnetic fields, and this is the conservation of energy theorem.

For a free particle of mass  $m$  in Minkowski spacetime:

$$p^\mu p_\mu = m^2 \gamma^2 \left( c^2 - \left( \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\phi}{dt} \right)^2 \right) \right) \quad (33)$$

where the Lorentz factor  $\gamma$  is defined as:

$$\gamma = \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \quad (34)$$

and the square of the total velocity as:

$$v^2 = \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\phi}{dt} \right)^2. \quad (35)$$

In this case the conserved total energy is:

$$E = \gamma mc^2 \quad (36)$$

and the conserved radial component of the total momentum is:

$$p_r = \gamma mv. \quad (37)$$

This is known as the relativistic momentum. The energy equation of Einstein follows as:

$$p^\mu p_\mu = \frac{E^2}{c^2} - p^2 = m^2 c^2 \quad (38)$$

In the contravariant covariant notation of Minkowski, the contravariant and covariant four momenta are:

$$p^\mu = \left( \frac{E}{c}, \mathbf{p} \right), p_\mu = \left( \frac{E}{c}, -\mathbf{p} \right) \quad (39)$$

and the Hamiltonian is the invariant:

$$H = \frac{1}{2m} p^\mu p_\mu = \frac{1}{2} mc^2. \quad (40)$$

When the particle interacts with a field of force, the line element is no longer a Minkowski line element, it becomes another type of line element defined by the orbital theorem of UFT 111. In the obsolete physics the line element was obtained from an incorrect equation proposed by Einstein in 1915, an equation based on the second Bianchi identity then thought to be correct. However it is now known and accepted [1–10] that the geometrical connection used in that second Bianchi identity is incorrectly symmetric in its lower two indices. It is straightforward to show (UFT 122 onwards on [www.aias.us](http://www.aias.us)) that the geometrical connection follows the antisymmetry of the defining commutator of covariant derivatives (UFT 137 and UFT 139). This glaring error was perpetrated uncritically because of the axiomatic neglect of spacetime torsion [1–11] in the old general relativity. If for the sake of illustration the field particle interaction is described by the general line element (8) of spherically symmetric spacetime, then the following equation of motion is the result:

$$\pi^\mu \pi_\mu = m^2 c^2 \quad (41)$$

and the Hamiltonian is conserved:

$$H = \frac{1}{2m} \pi^\mu \pi_\mu = \frac{1}{2} mc^2. \quad (42)$$

Here:

$$\pi^\mu = \left( \frac{E_1}{c}, \boldsymbol{\pi} \right) \quad (43)$$

where the conserved total energy is:

$$E_1 = xmc^2 \frac{dt}{d\tau}. \quad (44)$$

The total conserved momentum is:

$$\boldsymbol{\pi} = \pi_r \mathbf{e}_r + \pi_\phi \mathbf{e}_\phi. \quad (45)$$

The conserved angular momentum is:

$$L = mr^2 \frac{d\phi}{d\tau}, \quad (46)$$

and the infinitesimal of proper time is:

$$d\tau = \left( x - \frac{v^2}{c^2} \right)^{1/2} dt. \quad (47)$$

This is the equation for field particle interaction in a spherical spacetime with general  $x$  and  $y$ . The orbital theorem in UFT 111 means that:

$$x = \frac{1}{y}, \quad (48)$$

otherwise  $x$  and  $y$  are unconstrained and in general are both functions of  $r$  and  $t$ . It used to be thought that the gravitational metric:

$$x = \frac{1}{y} = 1 - \frac{r_0}{r} \quad (49)$$

gives a good account of solar system data, but recently the so-called ‘precision tests’ of general relativity have been found to be mathematically incorrect (UFT 150 and UFT 155 on [www.aias.us](http://www.aias.us)). Dark matter and string theory (both unscientific theories) do not correct these errors, because they are elementary errors perpetrated uncritically. They are easily revealed by use of contemporary computer precision and numerical integration of relatively simple integrals. It is also well known that the gravitational metric (49) fails drastically in whirlpool galaxies and other cosmological objects. ECE theory is able to correct these errors and is able to describe whirlpool galaxies straightforwardly using the torsion of spacetime. Indeed, the torsion of spacetime is glaringly visible in whirlpool galaxies. The pathology of the old physics chose to neglect this highly visible torsion and to use wholly fictitious and invisible dark matter to try to describe the very visible. This is obvious fallacy in the manner of epicycles or phlogiston, and the ludicrous paraphernalia of the old standard model – archetypical idols of Bacon's cave. This is why large areas of theoretical physics are rejected by entire disciplines of genuine science. The ideas of the old physics are kept alive by futile censorship of the ineluctable modality of the visible (apologies to James Joyce). Nothing could be darker.

Finally, in this paper we make the hypothesis:

$$\pi^\mu = P^\mu + m\Phi^\mu + e A^\mu + \dots \tag{50}$$

and use the concept of the minimal prescription [1–12]. The Hamiltonian is then conserved as:

$$H = \frac{1}{2m} (P^\mu + m\Phi^\mu + e A^\mu + \dots) (P_\mu + m\Phi_\mu + e A_\mu + \dots) = \frac{1}{2} mc^2. \tag{51}$$

In this example, a particle of mass  $m$  and charge  $e$  interacts simultaneously with the electromagnetic potential  $A_\mu$  and the gravitational potential  $\Phi_\mu$ . It is seen that the Hamiltonian develops cross terms between the gravitational and electromagnetic fields. These cross terms could be useful for counter gravitation. The quantization of these equations is carried out with:

$$\pi^\mu = i\hbar\partial^\mu, \quad \pi_\mu = i\hbar g_{\mu\nu}\partial^\nu, \tag{52}$$

so the d'Alembertian of flat spacetime  $\square$  is changed to the d'Alembertian  $\square'$  of the spacetime defined by the metric  $g_{\mu\nu}$ . The latter is used [1–11] to raise and lower indices. This process results in the wave equation:

$$\left( \square' + \left( \frac{mc}{\hbar} \right)^2 \right) \psi = 0 \tag{53}$$

which is equivalent to the ECE wave equation:

$$(\square + R)\psi = 0. \tag{54}$$

Note carefully that the d'Alembertian  $\square$  used in the latter is by construction that of flat spacetime:

$$\square = \partial^\mu \partial_\mu \tag{55}$$

but the ECE wave equation is one of the general spacetime. Similarly the covariant derivative of the general spacetime is defined in terms of the four derivative of flat spacetime, for example:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma_{\mu\lambda}^\nu V^\lambda \tag{56}$$

where  $\Gamma_{\mu\lambda}^\nu$  is the connection, antisymmetric in its lower two indices by definition via the commutator of covariant derivatives [1–11].

Quantization of the equation:

$$(P^\mu + eA^\mu)(P^\mu + eA^\mu) = m^2 c^2 \quad (57)$$

produces:

$$\left( \square + \left( \frac{mc}{\hbar} \right)^2 - \frac{ie}{\hbar} (A^\mu \partial_\mu + \partial^\mu A_\mu) - \frac{e^2}{\hbar^2} A^\mu A_\mu \right) \psi = 0 \quad (58)$$

so

$$R = \left( \frac{mc}{\hbar} \right)^2 - \frac{e^2}{\hbar^2} A^\mu A_\mu - \frac{ie}{\hbar} (A^\mu \partial_\mu + \partial^\mu A_\mu) \quad (59)$$

thus providing the link between Eqs. (53) and (54), Q.E.D.

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