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Metric-based ECE theory of electrodynamics

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The metric of ECE theory is incorporated into the theory of electrodynamics and gravitation through the field equations. The electric displacement ***D*** and magnetic field strength ***H*** are related to the electric field strength ***E*** and magnetic flux density ***B*** through metric elements. It is shown that the metric elements appear in the denominator of the driving term of the spin connection resonance equation. The electric displacement and polarization in a dielectric are shown to be determined by the metric, and similarly the magnetic field strength and magnetization. The equations for gravitation are similarly structured so the way in which gravitation and electromagnetism interact is determined by the metric, which is defined by the tetrads. The fundamental geometry is developed and the geometrical connection shown to be antisymmetric.

Keywords: ECE theory, metrical theory of electrodynamics and gravitation, spin connection resonance, counter gravitation, basic geometry, antisymmetry of connection.

1. Introduction

In this series of papers [1-10] the philosophy of relativity has been put on a firm geometrical footing by the use of a geometry that correctly includes spacetime torsion. In the obsolete Einsteinian general relativity a major error existed in the geometrical framework of the theory throughout the twentieth century - the axiomatic omission of spacetime torsion. Unfortunately this means that the claims of Einstenian general relativity are meaningless and this conclusion has been accepted [11]. However the use of the metric by Einstein is still valid, and in this paper the metric is incorporated specifically into the ECE theory of electrodynamics. The ECE theory is developed in a spacetime with torsion and curvature included, and the metric of this spacetime is used to raise and lower indices in the field equations. This method was implicit in previous developments of ECE theory for use in electrodynamics, but by considering the metric specifically, various properties in physics can be expressed in metrical format. This procedure has all the advantages of a unified description of natural philosophy, because both electromagnetism and gravitation are expressed as metrical properties.

In Section 2 the basic geometry is defined and summarized. This is a geometry

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that defines the Riemann curvature and Riemann torsion from the action of the commutator of covariant derivatives on a four vector (or more generally any tensor). For reasons unknown, Einstein and his contemporaries omitted consideration of torsion. This procedure is incorrect, and this is an error that was unfortunately repeated uncritically. The commutator is antisymmetric by definition, so the Riemann curvature is antisymmetric in the indices mu and nu of the commutator. If mu is the same as nu the curvature vanishes, meaning that the spacetime is flat and that the connection is zero. So the connection is non-zero if and only the commutator is non-zero. The commutator isolates the connection by its action on any tensor, so the connection is antisymmetric and its indices mu and nu are always different. The Riemann torsion is the antisymmetric difference of two connections, and the torsion is also antisymmetric in the indices mu and nu of the commutator. These simple facts of geometry form the basis of ECE theory, in which the electromagnetic and gravitational fields are built on the torsion. The latter is represented by Cartan geometry, which may be used to extend Riemann geometry in well known ways [12]. These fundamental geometrical properties are summarized in a table for ease of reference.

In Section 3 the metric based field equations of ECE are defined directly from the geometry of Section 2 and in Section 3 the field tensors are defined in a Table for ease of reference.

2. Summary of the fundamental geometry

In condensed notation [1–10] the fundamental geometry of ECE theory consists of the two well known Cartan Maurer structure equations:

$$T = D\wedge q, \tag{1}$$

$$\mathrm{R} = D\wedge \omega, \tag{2}$$

and the two identities of Cartans differential geometry:

$$D\wedge T := \mathrm{R}\wedge q, \tag{3}$$

should R be italic or normal, in the manuscript it is normal

$$D\wedge \tilde{T} := \tilde{R}\wedge q, \tag{4}$$

where T denotes the Cartan torsion, R denotes the Cartan curvature, q denotes the Cartan tetrad, ^ denotes the wedge product of Cartan, and the tilde denotes the Hodge dual. The Hodge dual identity was proven rigorously in UFT 137 of this series (www.aias.us). So the ECE theory adheres to the philosophy of the Ockham Razor and uses the simplest possible geometrical basis that is rigorously correct. It is incorrect to omit the torsion as in twentieth century general relativity. The geometry devised by Cartan is an extension of Riemann geometry, in which the covariant derivative is defined as follows:

$$D_\mu V^\rho := \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda} V^\lambda \tag{5}$$

Here V^ρ is a four vector, and $\Gamma^\rho_{\mu\lambda}$ is the geometrical connection. In a flat spacetime, the connection is zero, so in a flat spacetime:

$$D_\mu V^\rho = \partial_\mu V^\rho. \tag{6}$$

The commutator of covariant derivatives operates on any tensor, for example operates on a four vector, and is defined as being antisymmetric in its indices mu and nu:

$$\left[D_\mu, D_\nu\right] V^\rho = -\left[D_\nu, D_\mu\right] V^\rho. \tag{7}$$

In a flat spacetime the commutator is zero:

$$\left[D_\mu, D_\nu\right] V^\rho = \left[\partial_\mu, \partial_\nu\right] V^\rho = 0. \tag{8}$$

It can be shown [1–10,12] that:

$$\left[D_\mu, D_\nu\right] V^\rho = R^\rho_{\kappa\nu\mu} V^\kappa - T^\kappa_{\mu\nu} D_\kappa V^\rho \tag{9}$$

where the curvature tensor is defined as:

$$R^\rho_{\kappa\nu\mu} := \partial_\mu \Gamma^\rho_{\nu\kappa} - \partial_\nu \Gamma^\rho_{\mu\kappa} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\kappa} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\kappa} \tag{10}$$

and where the torsion tensor is defined as:

$$T^\kappa_{\mu\nu} := \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu}. \tag{11}$$

In the twentieth century it was well accepted that there is a symmetry correspondence between the commutator, curvature and, in the rare cases when it was recognised to exist, torsion. They are all antisymmetric by definition:

$$\left[D_\mu, D_\nu\right] V^\rho = -\left[D_\nu, D_\mu\right] V^\rho, \tag{12}$$

$$R^{\rho}_{\kappa\mu\nu} = -R^{\rho}_{\kappa\nu\mu}, \tag{13}$$

$$T^{\kappa}_{\mu\nu} = -T^{\kappa}_{\nu\mu}. \tag{14}$$

It follows that the connection is also antisymmetric in its lower two indices, because these are the same indices as in Eqs.(12) to (14):

$$\Gamma^{\kappa}_{\mu\nu} = -\Gamma^{\kappa}_{\nu\mu}. \tag{15}$$

The catastrophic error made in the Einsteinian general relativity was to assert axiomatically that the torsion is zero and that the connection is both symmetric and non-zero:

$$\Gamma^{\kappa}_{\mu\nu} = ?\Gamma^{\kappa}_{\nu\mu} \neq 0 \tag{16}$$

ECE corrects this error by basing general relativity on the rigorously correct Eqs. (1) to (4).

So in the twentieth century it was assumed that:

$$\left[D_{\mu}, D_{\nu}\right]V^{\rho} = ?R^{\rho}_{\kappa\nu\mu}V^{\kappa} \tag{17}$$

and textbooks on general relativity almost always start with this equation without mentioning torsion. The only clear exception is the book by Carroll [12], chapter 3. Eq. (17) can be disproven straightforwardly because if:

$$\mu = \nu, \tag{18}$$

then

$$\left[D_{\mu}, D_{\nu}\right]V^{\rho} = \left[\partial_{\mu}, \partial_{\nu}\right]V^{\rho} = 0 \tag{19}$$

and the spacetime is a flat spacetime, defined as a spacetime with zero curvature. For a flat spacetime:

$$R^{\rho}_{\kappa\nu\mu} = 0 \tag{20}$$

and the connection is zero by definition. Therefore if the curvature is zero the connection is zero, and this is always the case when Eq. (18) is true. So for the curvature to be non-zero the connection must be antisymmetric and the torsion must be non-zero. Eq. (17) is therefore incorrect because of its omission of torsion, QED. This means that mu can never be the same as nu in the connection. This conclusion is seen in another way by writing Eq. (9) as:

$$\left[D_\mu, D_\nu\right]V^\rho = -\Gamma^\kappa_{\mu\nu} D_\kappa V^\rho + \cdots \tag{21}$$

when it becomes clear that the indices of the connection, mu and nu, are the same as those of the commutator. So both must be antisymmetric in mu and nu in order to be non-zero. Similarly mu can never be the same as nu in the Riemann torsion and Riemann curvature, a fact that was accepted in the twentieth century.

The Einstein field equation was based erroneously on Eq. (18), and unfortunately is meaningless.

The Cartan geometry of Eqs. (1) to (4) has well known advantages over the Riemann geometry [1–10, 12]. Cartan geometry can be used with two different representations of the same space, so can be used with Cartans spinors or for any basis set. Riemann and Cartan geometry are inter-linked by the fact that the complete vector field is independent of the way in which its components and basis elements are represented using coordinate systems. This fundamental necessity leads to what is known as the tetrad postulate:

$$\partial_\mu q^a_\nu = \Gamma^\lambda_{\mu\nu} q^a_\lambda - \omega^a_{\mu b} q^b_\nu \tag{22}$$

in which the spin connection of Cartan is defined by the covariant derivative [1–10,12]:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \tag{23}$$

The tetrad postulate can be simplified to:

$$\partial_\mu q^a_\mu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \tag{24}$$

using the definitions:

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda, \tag{25}$$

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu. \tag{26}$$

Therefore, the need to keep the complete vector field constant results in the following expansion of the gamma connection used in Riemann geometry:

$$\Gamma^a_{\mu\nu} = \partial_\mu q^a_\nu + \omega^a_{\mu\nu}. \tag{27}$$

The gamma connection is antisymmetric, so:

$$\partial_\mu q_\nu^a + \omega_{\mu\nu}^a = -\left(\partial_\nu q_\mu^a + \omega_{\nu\mu}^a\right) \tag{28}$$

which is the ECE antisymmetry condition introduced and developed in UFT 132 onwards. Unlike the gamma connection, the spin connection itself is not antisymmetric in its lower two indices, because of Eq. (27). The Cartan torsion is a vector valued two-form of differential geometry [1–10, 12] and is defined by:

$$T_{\mu\nu}^a = \partial_\mu \, q_\nu^a - \partial_\nu \, q_\mu^a + \omega_{\mu b}^a \, q_\nu^b - \omega_{\nu b}^a \, q_\mu^b. \tag{29}$$

Use of the tetrad postulate reduces Eq. (29) to:

$$T_{\mu\nu}^a = \Gamma_{\mu\nu}^a - \Gamma_{\nu\mu}^a \tag{30}$$

which is equivalent to the Riemann torsion:

$$T_{\mu\nu}^\kappa = \Gamma_{\mu\nu}^\kappa - \Gamma_{\nu\mu}^\kappa \tag{31}$$

using the definition:

$$T_{\mu\nu}^\kappa = q_a^\kappa \, T_{\mu\nu}^a. \tag{32}$$

Using the definition of the wedge product [1-10, 12] the identity (3) becomes:

$$D_\mu \, T_{\nu\rho}^a + D_\rho \, T_{\mu\nu}^a + D_\nu \, T_{\rho\mu}^a := R_{\mu\nu\rho}^\rho + R_{\rho\mu\nu}^\rho + R_{\nu\rho\mu}^\rho \tag{33}$$

which may be rewritten as:

$$D_\mu \, \tilde{T}^{a\mu\nu} := \tilde{R}_\mu^{a\mu\nu} \tag{34}$$

using the definition of the Hodge dual of a two-form in four dimensions:

$$\tilde{T}^{a\mu\nu} := \frac{1}{2} \|g\|^{1/2} \epsilon^{\mu\nu\alpha\beta} \, T_{\alpha\beta}^a. \tag{35}$$

In this definition $||g||^{1/2}$ is the square root of the absolute value of the determinant of the metric and is used as a weighting factor in a spacetime [12] that is not a Minkowski spacetime. In Eq. (35) $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric unit tensor of Minkowski spacetime in four dimensions. The Hodge dual of a two form in four dimensions is another two form [12]. The covariant derivatives used in Eqs. (33) and (34) are defined by the spin connection:

$$D_\mu \tilde{T}^{a\mu\nu} := \partial_\mu \tilde{T}^{a\mu\nu} + \omega^a_{\mu b} \tilde{T}^{b\mu\nu} \tag{36}$$

So Eq. (34) can be written as:

$$\partial_\mu \tilde{T}^{a\mu\nu} := \tilde{R}^{a\mu\nu}_{\mu} - \omega^a_{\mu b} \tilde{T}^{b\mu\nu} \tag{37}$$

which is a correctly covariant equation because it originates in the covariant identity (34).

Equation (37) of Cartan geometry is the geometrical basis of the homogeneous field equation in ECE theory [1–10]. Its right hand side defines the homogeneous current:

$$j^{a\nu}_H := \tilde{R}^{a\mu\nu}_{\mu} - \omega^a_{\mu b} \tilde{T}^{b\mu\nu}. \tag{38}$$

The wave equation of ECE Theory [1–10] is a development of the tetrad postulate into:

$$\left(\square + R\right) q^a_\mu = 0 \tag{39}$$

where R is defined by:

$$R := q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right). \tag{40}$$

Eq. (39) has the structure of the fundamental wave equations of physics and R is the covariant mass:

$$R = \left(\frac{mc}{\hbar}\right)^2 \tag{41}$$

a concept used extensively in UFT 158 to UFT 166. Here $\hbar$ is the reduced Planck constant and c is known in standards laboratories as the speed of light in a vacuum, a fixed universal constant.

In order to derive the inhomogeneous field equation of ECE theory the original geometry of Cartan has been considerably extended during the course of development [1–10]. These are fundamental mathematical advances which check themselves as in UFT 137. These advances start with the definition of the Hodge dual of the connection used in Eq. (5):

$$\Lambda^{\kappa}_{\mu\nu} := \tilde{\Gamma}^{\kappa}_{\mu\nu} = \frac{1}{2}\|g\|^{1/2}\, \epsilon^{\alpha\beta}_{\mu\nu}\, \Gamma^{\kappa}_{\alpha\beta}. \tag{42}$$

This definition follows from the fact that $\Gamma^{\kappa}_{\alpha\beta}$ is antisymmetric in its lower two indices. The Hodge dual connection $\Lambda^{\kappa}_{\mu\nu}$ can be used to define the covariant derivative:

$$D_{\mu} V^{\kappa} = \partial_{\mu} V^{\kappa} + \Lambda^{\kappa}_{\mu\nu} V^{\nu}. \tag{43}$$

The Hodge dual of the torsion tensor is defined as:

$$\tilde{T}^{\kappa}_{\mu\nu} = \Lambda^{\kappa}_{\mu\nu} - \Lambda^{\kappa}_{\mu\nu}. \tag{44}$$

The Hodge dual torsion is generated from the commutator of covariant derivatives defined by Eq. (9) as follows:

$$\left[D_{\mu}, D_{\nu}\right]_{\mathrm{HD}} V^{\rho} = \tilde{R}^{\rho}_{\kappa\mu\nu} V^{\kappa} - \tilde{T}^{\kappa}_{\mu\nu} D_{\kappa} V^{\rho}. \tag{45}$$

It follows that this operation defines the Hodge dual curvature tensor:

$$\tilde{R}^{\rho}_{\kappa\mu\nu} := \partial_{\mu} \Lambda^{\rho}_{\nu\kappa} - \partial_{\nu} \Lambda^{\rho}_{\mu\kappa} + \Lambda^{\rho}_{\mu\lambda} \Lambda^{\lambda}_{\nu\kappa} - \Lambda^{\rho}_{\nu\lambda} \Lambda^{\lambda}_{\mu\kappa} \tag{46}$$

where $[D_{\mu}, D_{\nu}]_{\mathrm{HD}}$ is the Hodge dual of the commutator operator. These Hodge duals are defined self consistently by:

$$\left[D_{\mu}, D_{\nu}\right]_{\mathrm{HD}} = \frac{1}{2}\|g\|^{1/2}\, \epsilon^{\alpha\beta}_{\mu\nu} \left[D_{\alpha}, D_{\beta}\right], \tag{47}$$

$$\tilde{T}^{\kappa}_{\mu\nu} = \frac{1}{2}\|g\|^{1/2}\, \epsilon^{\alpha\beta}_{\mu\nu} T^{\kappa}_{\alpha\beta}, \tag{48}$$

$$\tilde{R}^{\rho}_{\kappa\mu\nu} = \frac{1}{2}\|g\|^{1/2}\, \epsilon^{\alpha\beta}_{\mu\nu} R^{\rho}_{\kappa\alpha\beta}. \tag{49}$$

The Hodge dual of the tetrad postulate follows from Eq. (27):

$$\tilde{\Gamma}^{a}_{\mu\nu} := \Lambda^{a}_{\mu\nu} = \left(\partial_{\mu} q^{a}_{\nu} + \omega^{a}_{\mu\nu}\right)_{\mathrm{HD}} \tag{50}$$

where the Hodge dual of the sum on the right hand side is taken. This is defined by the following notation:

$$\Lambda^a_{\mu\nu} := \left(\partial_\mu Q^a_\nu + \Omega^a_{\mu\nu}\right). \tag{51}$$

The Hodge dual Cartan torsion is therefore defined by:

$$\tilde{T}^a_{\mu\nu} = \tilde{T}^\kappa_{\mu\nu} Q^a_\kappa \tag{52}$$

With these definitions it can be shown that the following identity is true:

$$D_\mu \tilde{T}^a_{\nu\rho} + D_\rho \tilde{T}^a_{\mu\nu} + D_\mu \tilde{T}^a_{\rho\mu} = \tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} \tag{53}$$

where:

$$\tilde{R}^a_{\mu\nu\rho} = Q^a_\kappa \tilde{R}^\kappa_{\mu\nu\rho} \tag{54}$$

and so on. The precise proof of Eq. (53) was given in UFT 137. Eq. (53) can be rewritten as:

$$D_\mu T^{a\mu\nu} := R^{a\mu\nu}_\mu . \tag{55}$$

This equation was used in Ref. (2) to disprove metrics of the Einstein field equation, and is the geometrical basis of the inhomogeneous field equation of ECE theory. The inhomogeneous current is defined from Eq. (55) as:

$$\partial_\mu T^{a\mu\nu} = j^{a\nu}_l = R^{a\mu\nu}_\mu - \omega^a_{\mu b} T^{b\mu\nu}. \tag{56}$$

Finally, in this section the Hodge dual of the tetrad postulate can be developed into the Hodge dual of the wave equation (39), giving the result:

$$\left(\square + R_1\right) Q^a_\mu = 0 \tag{57}$$

where

$$R_1 := Q^\nu_a \partial^\mu \left(\Omega^a_{\mu\nu} - \Lambda^a_{\mu\nu}\right). \tag{58}$$

Therefore, a Hodge dual structure exists for the whole of Cartan's geometry. These findings are summarized for ease of reference in Table 1.

3. The metrical structure of the ECE field equations

In the well known classical electrodynamics [13, 14] of the nineteenth century, there are four laws which in tensor notation become two equations covariant under the Lorentz transform and written in Minkowski spacetime. These two equations are the homogeneous and inhomogeneous field equations. The former is written in terms of the electric field strength $\boldsymbol{E}$ and magnetic flux density $\boldsymbol{B}$. It is almost always assumed that the homogenous charge current density is zero. The inhomogeneous field equation is written in terms of the electric displacement $\boldsymbol{D}$ and magnetic field strength $\boldsymbol{H}$, charge density ρ and current density $\boldsymbol{J}$. In vector notation the complete set of field equations is [13, 14]:

$$\nabla \cdot \boldsymbol{B} = 0 \tag{59}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \tag{60}$$

$$\nabla \cdot \boldsymbol{D} = \rho \tag{61}$$

$$\nabla \times \boldsymbol{H} + \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \tag{62}$$

The constitutive equations introduce the polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{M}$, defined as follows [14], where ϵ_0 and μ_0 are the vacuum permittivity and permeability respectively:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \tag{63}$$

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M} \right). \tag{64}$$

The $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are expressed classically [13, 14] in terms of the scalar and vector potentials Φ, and $\boldsymbol{A}$, as follows:

$$\boldsymbol{E} = -\nabla \Phi - \frac{\partial \boldsymbol{A}}{\partial t}, \tag{65}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{66}$$

The vector structure of the homogeneous field equation can be summarized elegantly in the notation of differential geometry as follows:

$$F = \mathrm{d}^\wedge A \tag{67}$$

$$\mathrm{d}^\wedge F = 0 \tag{68}$$

where A is a scalar-valued one form, and F a scalar-valued two form.

In ECE theory the field equations of classical electrodynamics become equations

Table 1

Homogeneous Geometry	**Inhomogeneous Geometry**
$T = D \wedge q,$	$\tilde{T} = \left(D \wedge q\right)_{\mathrm{HD}}$
$D \wedge T = \mathrm{R} \wedge q$	$D \wedge \tilde{T} = \tilde{R} \wedge q$
$D_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\lambda} V^\lambda$	$D_\mu V^\rho = \partial_\mu V^\rho + \Lambda^\rho_{\mu\lambda} V^\lambda$
$\left[D_\mu, D_\nu\right] V^\rho = R^\rho_{\kappa\nu\mu} V^\kappa - T^\kappa_{\mu\nu} D_\kappa V^\rho$	$\left[D_\mu, D_\nu\right]_{\mathrm{HD}} V^\rho = \tilde{R}^\rho_{\kappa\nu\mu} V^\kappa - \tilde{T}^\kappa_{\mu\nu} D_\kappa V^\rho$
$T^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu}$	$\tilde{T}^\kappa_{\mu\nu} = \tilde{\Gamma}^\kappa_{\mu\nu} - \tilde{\Gamma}^\kappa_{\nu\mu} = \Lambda^\kappa_{\mu\nu} - \Lambda^\kappa_{\nu\mu}$
$R^\lambda_{\mu\nu\rho} = \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho}$	$\tilde{R}^\lambda_{\mu\nu\rho} = \partial_\mu \Lambda^\lambda_{\nu\rho} - \partial_\nu \Lambda^\lambda_{\mu\rho} + \Lambda^\lambda_{\mu\nu} \Lambda^\sigma_{\nu\rho} - \Lambda^\lambda_{\nu\sigma} \Lambda^\sigma_{\mu\rho}$
$\partial_\mu q^a_\nu = \Gamma^\lambda_{\mu\nu} q^a_\lambda - \omega^a_{\mu b} q^b_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu}$	$\partial_\mu Q^a_\mu = \Lambda^a_{\mu\nu} - \Omega^a_{\mu\nu}$
$\left(\square + R\right) q^a_\mu = 0$	$\left(\square + R_1\right) Q^a_\mu = 0$
$R = q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right)$	$R_1 = Q^\nu_a \partial^\mu \left(\Omega^a_{\mu\nu} - \Lambda^a_{\mu\nu}\right)$
$D_\mu T^a_{\nu\rho} + D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} = R^\rho_{\mu\nu\rho} + R^\rho_{\rho\mu\nu} + R^\rho_{\nu\rho\mu}$	$D_\mu \tilde{T}^a_{\nu\rho} + D_\rho \tilde{T}^a_{\mu\nu} + D_\mu \tilde{T}^a_{\rho\mu} = \tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu}$
$D_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^{a\mu\nu}_\mu$	$D_\mu T^{a\mu\nu} = R^{a\mu\nu}_\mu$
$\partial_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^{a\mu\nu}_\mu - \omega^a_{\mu b} \tilde{T}^{b\mu\nu} = 0\left(\text{experimentally}\right)$	$\partial_\mu T^{a\mu\nu} = R^{a\mu\nu}_\mu - \omega^a_{\mu b} T^{b\mu\nu} = \mathrm{J}^{a\nu}\left(\text{experimentally}\right)$
$\partial_\mu \tilde{F}^{\mu\nu} = 0$ for each a	$\partial_\mu G^{\mu\nu} = \mu_0 A^{(0)} \mathrm{J}^\nu = \mu_0 j^\nu$ for each a
$\square A^a_\mu = 0$	$\square A^a_\mu = \mu_0 j^a_\mu = -R_1 A^a_\mu$
$R = 0$	$R_1 A^a_\mu = -\mu_0 j^a_\mu$

of general relativity derived from Eqs. (37) and (56) of geometry. This procedure adheres rigorously to the philosophy of relativity, in which all the equations of physics derive from the equations of geometry within a unified structure. The basic hypothesis of ECE electrodynamics defines the vector potential as a vector valued one form as follows:

$$A^a_\mu = A^{(0)}\, q^a_\mu \tag{69}$$

where q^a_μ is the Cartan tetrad. The homogeneous field is then defined as the vector valued two form obtained from the first Cartan Maurer structure equation (1) with the same basic hypothesis as in Eq.(69):

$$F^a_{\mu\nu} = A^{(0)}\, T^a_{\mu\nu} \tag{70}$$

where

$$T^a_{\mu\nu} = \partial_\mu\, q^a_\nu - \partial_\nu\, q^a_\mu + \omega^a_{\mu b}\, q^b_\nu - \omega^a_{\nu b}\, q^b_\mu \tag{71}$$

These definitions are based on the Ockham Razor, i.e. are the simplest possible. In tensor notation Eq.(70) is:

$$F^a_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + \omega^a_{\mu b}\, A^b_\nu - \omega^a_{\nu b}\, A^b_\mu \tag{72}$$

while the nineteenth century classical electrodynamics [13, 14] uses:

$$F_{\mu\nu} = \partial_\mu\, A_\nu - \partial_\nu\, A_\mu . \tag{73}$$

Therefore ECE electrodynamics contains the spin connection, indicating that it is written in a more general spacetime. In vector notation Eq. (72) becomes [1–10]:

$$\boldsymbol{E}^a = -c\nabla A^a_0 - \frac{\partial \boldsymbol{A}^a}{\partial t} - c\,\omega^a_{0b}\,\boldsymbol{A}^b + c\,A^a_0\,\boldsymbol{\omega}^a_b , \tag{74}$$

$$\boldsymbol{B}^a = \nabla\times\boldsymbol{A}^a - \boldsymbol{\omega}^a_b \times \boldsymbol{A}^b , \tag{75}$$

where the potential and spin connection terms are defined as follows:

$$A^a_\mu = \left(A^a_0, \boldsymbol{A}^a\right),\ \omega^a_{\mu b} = \left(\omega^a_{0b}, \boldsymbol{\omega}^a_b\right) \tag{76}$$

By application of the antisymmetry law (28) it is found that:

$$\partial_\mu A_\nu^a + \omega_{\mu b}^a A_\nu^b + \partial_\nu A_\mu^a + \omega_{\nu b}^a A_\mu^b = 0 \tag{77}$$

which in vector notation becomes:

$$-c\nabla A_0^a - c\,\omega_{0b}^a \boldsymbol{A}^b = -\frac{\partial \boldsymbol{A}^a}{\partial t} + c\,A_0^a \boldsymbol{\omega}_b^a. \tag{78}$$

The antisymmetry law was shown in UFT 131 onwards to be a rigorous test of the nineteenth century classical electrodynamics, because the law implies:

$$\partial_\mu A_\nu = -\partial_\nu A_\mu \tag{79}$$

which means that the nineteenth century electrodynamics become untenable, a conclusion which has been accepted [11]. This result means that ECE electrodynamics is the only tenable electrodynamics.

From Eq. (37) the homogeneous field equation of ECE electrodynamics is:

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0 \tag{80}$$

which in vector notation is:

$$\nabla \cdot \boldsymbol{B}^a = 0, \tag{81}$$

$$\nabla \times \boldsymbol{E}^a + \frac{\partial \boldsymbol{B}^a}{\partial t} = 0. \tag{82}$$

The index a denotes polarization as follows:

$$a = (0),\ (1),\ (2),\ (3) \tag{83}$$

in which (0) is timelike and (1), (2) are transverse spacelike, (3) is longitudinal spacelike. For each :

$$\nabla \cdot \boldsymbol{B} = 0 \tag{84}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}. \tag{85}$$

These equations are the Gauss law of magnetism and the Faraday law of induction. Note carefully that these equations are equations of general relativity, and that their metric is not the Minkowski metric. The spin connection of general relativity enters into the definition of ***E*** and ***B*** as in Eq. (72). In the nineteenth century electrodynamics of Maxwell and Heaviside (MH equations), it can be seen from Eqs. (65) and (66) that there is no spin connection, indicating that the spacetime is flat – the Minkowski spacetime. Finally Eqs. (84) and (85) are generally covariant whereas the MH equations are covariant only under the Lorentz transform.

The inhomogeneous field equation of ECE is derived from the geometrical structure of Eq. (56) and is:

$$\partial_\mu G^{a\mu\nu} = J^{a\mu} \tag{86}$$

where $G^{a\mu\nu}$ denotes the inhomogeneous field tensor. In general $G^{a\mu\nu}$ contains polarization and magnetization, as well as electric displacement and magnetic field strength. For simplicity consider for illustration a material in which there is no polarization or magnetization. Then:

$$G^{a\mu\nu} = \epsilon_0 \, g^{\mu\rho} g^{\nu\sigma} F^{a}_{\rho\sigma} \tag{87}$$

where the inverse metrics $g^{\mu\rho}$ and $g^{\nu\sigma}$ have been used to raise indices in the general four dimensional spacetime. Written out in full for each a:

$$G^{\mu\nu} = \begin{pmatrix} 0 & -D^1 & -D^2 & -D^3 \\ D^1 & 0 & -H^3/\mathrm{c} & H^2/\mathrm{c} \\ D^2 & H^3/\mathrm{c} & 0 & -H^1/\mathrm{c} \\ D^3 & -H^2/\mathrm{c} & H^1/\mathrm{c} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -D_X & -D_Y & -D_Z \\ D_X & 0 & -H_Z/\mathrm{c} & H_Y/\mathrm{c} \\ D_Y & H_Z/\mathrm{c} & 0 & -H_X/\mathrm{c} \\ D_Z & -H_Y/\mathrm{c} & H_X/\mathrm{c} & 0 \end{pmatrix}. \tag{88}$$

If the metric is assumed to be diagonal:

$$g^{\mu\rho} = \begin{pmatrix} g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & -g^{22} & 0 \\ 0 & 0 & 0 & -g^{33} \end{pmatrix} \tag{89}$$

then the component equations are:

$$\left.\begin{aligned} D_X &= \epsilon_0 g^{00} g^{11} E_X, \quad H_X = g^{22} g^{33} B_X / \mu_0 \\ D_Y &= \epsilon_0 g^{00} g^{22} E_Y, \quad H_X = g^{11} g^{33} B_Y / \mu_0 \\ D_Z &= \epsilon_0 g^{00} g^{33} E_Z, \quad H_X = g^{22} g^{11} B_z / \mu_0 \end{aligned}\right\} \tag{90}$$

and the field equations in vector notation are, for each a:

$$\nabla \cdot \boldsymbol{D} = \rho, \tag{91}$$

$$\nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \mathbf{J}. \tag{92}$$

As for the homogeneous ECE equations, these are equations of general relativity in which the metric is the metric of the four dimensional spacetime with torsion and curvature. This is a great advance on MH theory because it shows that the quantities of classical electrodynamics are based on the metric in the same overall manner as in the theory of gravitation [1–10]. The definition of $G^{a\mu\nu}$ is easily extended if necessary to include polarization and magnetization.

For ease of reference the complete set of equations is summarized as follows:

$$\left.\begin{aligned} &\nabla \cdot \boldsymbol{B} = 0, \qquad \nabla \cdot \boldsymbol{D} = \rho \\ &\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{0}, \quad \nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \mathbf{J} \\ &\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \qquad \boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right), \end{aligned}\right\} \tag{93}$$

where the fields are as follows.

$\boldsymbol{E}$ = electric field strength in units of J C^{-1} m^{1} = Volt m^{1}

$\boldsymbol{B}$ = magnetic flux density in units of tesla

$\boldsymbol{D}$ = electric displacement in units of C m^{-2}

$\boldsymbol{H}$ = magnetic field strength in units of A m^{-1}

$\boldsymbol{P}$ = polarization, $\boldsymbol{M}$ = magnetization.

For each a, the four potential and four current are defined as:

$$A_\mu = \left(\frac{\Phi}{c}, -\boldsymbol{A}\right), \quad J_\mu = \left(c\rho, -\boldsymbol{J}\right). \tag{94}$$

The simplest example of these equations is their application to a dielectric of permittivity ϵ and permeability μ in which the four current is zero, and in which there is no polarization or magnetization. Then, for each a:

$$\left.\begin{array}{ll}\nabla \cdot \boldsymbol{B} = 0, & \nabla \cdot \boldsymbol{D} = 0, \\ \nabla \times \boldsymbol{E} + \dfrac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, & \nabla \times \boldsymbol{H} - \dfrac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{0},\end{array}\right\} \tag{95}$$

in which the constitutive equations are [13]:

$$\boldsymbol{D} = \epsilon \boldsymbol{E}, \qquad \boldsymbol{B} = \mu \boldsymbol{H}. \tag{96}$$

Assuming solutions with time dependence [13] exp $(-i\omega t)$ gives the Helmholtz wave equations for each:

$$\left(\nabla^2 + \epsilon\mu\omega^2\right)\boldsymbol{E} = \boldsymbol{0}, \tag{97}$$

$$\left(\nabla^2 + \epsilon\mu\omega^2\right)\boldsymbol{B} = \boldsymbol{0}. \tag{98}$$

For plane waves with phase:

$$\Phi = \omega t - \kappa Z \tag{99}$$

Eqs. (95) mean:

$$\kappa = \left(\mu\epsilon\right)^{1/2} \omega \tag{100}$$

where κ is the wavenumber and where ω is the angular frequency. The phase velocity is:

$$v = \frac{\omega}{\kappa} = \left(\mu\epsilon\right)^{-1/2} = \frac{c}{n} \tag{101}$$

where n is the refractive index.

For ease of reference the field tensors used in these equations are written out in full in Table 2, with relevant components of the totally antisymmetric unit tensor in four dimensions.

As a second example of the use of the field tensor $G^{a\mu\nu}$ consider its use in spin connection resonance [1–10] in the Coulomb law or Newton law. In the Coulomb law the electric field strength is defined by:

$$\boldsymbol{E}^a = -c\nabla A_0^a - \frac{\partial \boldsymbol{A}^a}{\partial t} - c\,\omega_{0b}^a \boldsymbol{A}^b + c\,A_0^a \boldsymbol{\omega}_b^a \tag{102}$$

The antisymmetry law means:

$$-c\nabla A_0^a - c\,\omega_{0b}^a \boldsymbol{A}^b = -\frac{\partial \boldsymbol{A}^a}{\partial t} + c\,A_0^a \boldsymbol{\omega}_b^a. \tag{103}$$

If there are no vector potentials present:

$$-c\nabla A_0^a = c\,A_0^a \boldsymbol{\omega}_b^a \tag{104}$$

and the electric field simplifies to:

$$\boldsymbol{E}^a = -c\nabla A_0^a + c\,A_0^a \boldsymbol{\omega}_b^a \tag{105}$$

For each a:

$$\boldsymbol{E} = -\nabla\Phi + \Phi^b \boldsymbol{\omega}_b \tag{106}$$

where:

$$\Phi := \mathrm{c}\,A_0 \tag{107}$$

So summing up over b indices:

$$\boldsymbol{E}^a = -\nabla\Phi^a + \Phi^0 \boldsymbol{\omega}_0^a + \Phi^1 \boldsymbol{\omega}_1^a + \Phi^2 \boldsymbol{\omega}_2^a + \Phi^3 \boldsymbol{\omega}_3^a \tag{108}$$

where the indices a are spacelike because they refer to the electric field strength, a pure spacelike quantity. If these a indices are labelled 1, 2 and 3, then for example:

$$\boldsymbol{E}^1 = -\nabla\Phi^1 + \Phi^1 \boldsymbol{\omega}_1^1 \tag{109}$$

In this equation, $\boldsymbol{E}^1$ can be associated only with Φ^1 by definition. The meaning of Φ^1 is the scalar potential associated with polarization 1, the same polarization as the field it defines. It follows by similar arguments that

$$\boldsymbol{E}^2 = -\nabla\Phi^2 + \Phi^2 \boldsymbol{\omega}_2^2 \tag{110}$$

$$\boldsymbol{E}^3 = -\nabla\Phi^3 + \Phi^3 \boldsymbol{\omega}_3^3 \tag{111}$$

By antisymmetry:

Table 2

Standard definitions of field tensors

$$F_{\mu\nu}=\begin{pmatrix}0 & -E_1/c & -E_2/c & -E_3/c\\ E_1/c & 0 & B_3 & -B_2\\ E_2/c & -B_3 & 0 & B_1\\ E_3/c & B_2 & -B_1 & 0\end{pmatrix}=\begin{pmatrix}0 & E_X/c & E_Y/c & E_Z/c\\ -E_X/c & 0 & -B_Z & B_Y\\ -E_Y/c & B_Z & 0 & -B_X\\ -E_Z/c & -B_Y & B_X & 0\end{pmatrix},$$

$$F^{\mu\nu}=\begin{pmatrix}0 & -E^1/c & -E^2/c & -E^3/c\\ E^1/c & 0 & -B^3 & B^2\\ E^2/c & B^3 & 0 & -B^1\\ E^3/c & -B^2 & B^1 & 0\end{pmatrix}=\begin{pmatrix}0 & -E_X/c & -E_Y/c & -E_Z/c\\ E_X/c & 0 & -B_Z & B_Y\\ E_Y/c & B_Z & 0 & -B_X\\ E_Z/c & -B_Y & B_X & 0\end{pmatrix}.$$

$$\tilde{F}^{\mu\nu}=\begin{pmatrix}0 & -B^1 & -B^2 & -B^3\\ B^1 & 0 & E^3/c & -E^2/c\\ B^2 & -E^3/c & 0 & E^1/c\\ B^3 & E^2/c & -E^1/c & 0\end{pmatrix}=\begin{pmatrix}0 & -B_X & -B_Y & -B_Z\\ B_X & 0 & E_Z/c & -E_Y/c\\ B_Y & -E_Z/c & 0 & E_X/c\\ B_Z & E_Y/c & -E_X/c & 0\end{pmatrix}.$$

$$\tilde{F}_{\mu\nu}=\begin{pmatrix}0 & -B_1 & -B_2 & -B_3\\ B_1 & 0 & -E_3/c & E_2/c\\ B_2 & E_3/c & 0 & -E_1/c\\ B_3 & -E_2/c & E_1/c & 0\end{pmatrix}=\begin{pmatrix}0 & B_X & B_Y & B_Z\\ -B_X & 0 & E_Z/c & -E_Y/c\\ -B_Y & -E_Z/c & 0 & E_X/c\\ -B_Z & E_Y/c & -E_X/c & 0\end{pmatrix}.$$

Here $F^{\mu\nu}=g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}$ (1)

$$\tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \tag{2}$$

$$g_{\mu\nu}=g^{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}=\mathrm{diag}(1,-1,-1,-1) \tag{3}$$

$$\left.\begin{array}{l}\epsilon^{0123}=-\epsilon^{1230}=\epsilon^{2301}=-\epsilon^{3012}=1\\ \epsilon^{1023}=-\epsilon^{2130}=\epsilon^{3201}=-\epsilon^{0312}=-1\\ \\ \epsilon^{1023}=-\epsilon^{2130}=\epsilon^{3201}=-\epsilon^{0312}=1\\ \epsilon^{1302}=-\epsilon^{2013}=\epsilon^{3120}=-\epsilon^{0231}=-1\end{array}\right\} \tag{4}$$

Define:

$$G^{\mu\nu} = \epsilon_0 \, g^{\mu\rho} \, g^{\nu\sigma} F_{\rho\sigma} \tag{5}$$

then

$$G^{\mu\nu} = \begin{pmatrix} 0 & -D^1 & -D^2 & -D^3 \\ D^1 & 0 & -H^3/\mathrm{c} & H^2/\mathrm{c} \\ D^2 & H^3/\mathrm{c} & 0 & -H^1/\mathrm{c} \\ D^3 & -H^2/\mathrm{c} & H^1/\mathrm{c} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -D_\mathrm{X} & -D_\mathrm{Y} & -D_\mathrm{Z} \\ D_\mathrm{X} & 0 & -H_\mathrm{Z}/\mathrm{c} & H_\mathrm{Y}/\mathrm{c} \\ D_\mathrm{Y} & H_\mathrm{Z}/\mathrm{c} & 0 & -H_\mathrm{X}/\mathrm{c} \\ D_\mathrm{Z} & -H_\mathrm{Y}/\mathrm{c} & H_\mathrm{X}/\mathrm{c} & 0 \end{pmatrix} \tag{6}$$

and

$$\left.\begin{aligned} D_\mathrm{X} &= \epsilon_0 \, g^{00} \, g^{11} \, E_\mathrm{X} \\ D_\mathrm{Y} &= \epsilon_0 \, g^{00} \, g^{22} \, E_\mathrm{Y} \\ D_\mathrm{Z} &= \epsilon_0 \, g^{00} \, g^{33} \, E_\mathrm{Z} \\ H_\mathrm{X} &= \frac{1}{\mu_0} g^{22} \, g^{33} \, B_\mathrm{X} \\ H_\mathrm{Y} &= \frac{1}{\mu_0} g^{11} \, g^{33} \, B_\mathrm{Y} \\ H_\mathrm{Z} &= \frac{1}{\mu_0} g^{22} \, g^{11} \, B_\mathrm{Z} \end{aligned}\right\} \tag{7}$$

Metric:

$$g^{\mu\nu} = \begin{pmatrix} g^{00} & 0 & 0 & 0 \\ 0 & -g^{11} & 0 & 0 \\ 0 & 0 & -g^{22} & 0 \\ 0 & 0 & 0 & -g^{33} \end{pmatrix} \tag{8}$$

Field Equations

$$\left.\begin{aligned} \nabla \cdot \boldsymbol{D} &= \rho \\ \nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} &= \mathbf{J} \end{aligned}\right\} \tag{9}$$

$$-\nabla \Phi^1 = \Phi^1 \boldsymbol{\omega}_1^1 \tag{112}$$

$$-\nabla \Phi^2 = \Phi^2 \boldsymbol{\omega}_2^2 \tag{113}$$

$$-\nabla \Phi^3 = \Phi^3 \boldsymbol{\omega}_3^3 \tag{114}$$

Therefore if Φ^1, Φ^2, Φ^3 are positive valued, then $\omega_1^1, \omega_2^2, \omega_3^3$ are negative valued. The displacements are defined by:

$$D^1 = \epsilon_0 g^{00} g^{11} E^1, \tag{115}$$

$$D^2 = \epsilon_0 g^{00} g^{22} E^2, \tag{116}$$

$$D^3 = \epsilon_0 g^{00} g^{33} E^3, \tag{117}$$

and the Coulomb law by:

$$\partial_1 D^1 + \partial_2 D^2 + \partial_3 D^3 = \rho \tag{118}$$

Any coordinate system can be introduced at this point. If attention is restricted to D^3 for simplicity then:

$$\partial_3 D^3 = \rho \tag{119}$$

Denote:

$$\boldsymbol{\omega}_3^3 = -\omega \boldsymbol{e}^3 \tag{120}$$

If the Cartesian coordinate system is used:

$$\boldsymbol{\omega}_3^3 = -\omega \mathbf{k} \tag{121}$$

and

$$\frac{\partial D_Z}{\partial Z} = \rho \tag{122}$$

where

$$D_Z = \epsilon_0 \, g^{00} \, g_{ZZ} \, E_Z \tag{123}$$

$$E_Z = -\frac{\partial \Phi_Z}{\partial Z} - \omega \Phi_Z \tag{124}$$

Therefore:

$$\frac{\partial^2}{\partial Z^2}\left(g^{00} \, g_{ZZ} \, \Phi_Z \right) + \frac{\partial}{\partial Z}\left(g^{00} \, g_{ZZ} \, \omega \Phi_Z \right) = -\rho / \epsilon_0. \tag{125}$$

This is an Euler Bernoulli resonance structure:

$$\frac{\partial^2}{\partial Z^2}\left(g^{00} \, g_{ZZ} \, \Phi_Z \right) + \frac{\partial}{\partial Z}\left(g^{00} \, g_{ZZ} \, \omega \right) \Phi_Z + g^{00} \, g_{ZZ} \, \omega \frac{\partial \Phi_Z}{\partial Z} = -\rho / \epsilon_0. \tag{126}$$

If g^{00} and g_{zz} are assumed to be independent of Z, for simplicity, then the spin connection resonance equation is:

$$\frac{\partial^2 \Phi_Z}{\partial Z^2} + \left(\frac{\partial \omega}{\partial Z} \right) \Phi_Z + \omega \frac{\partial \Phi_Z}{\partial Z} = -\rho / \epsilon_0 g^{00} \, g_{ZZ}. \tag{127}$$

This is a damped Euler Bernoulli resonance equation with metric elements incorporated in the denominator of the driving term. The resonance therefore depends on the metric, which in turn may be affected by gravitation. In the limit of vanishing spin connection, the Poisson equation is recovered as follows:

$$\frac{\partial^2 \Phi_Z}{\partial Z^2} + \frac{\rho}{\epsilon_0 g^{00} \, g_{ZZ}} = 0. \tag{128}$$

In the general coordinate system:

$$D^3 = -\epsilon_0 g^{00} \, g_{ZZ} \left(\partial_3 + \omega \right) \Phi^3, \qquad \partial_3 D^3 = \rho. \tag{129}$$

This entire structure may be transferred intact to dynamics, where it describes spin connection resonance in the Newton inverse square law.

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