Development of the ECE Fermion equation.

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Abstract.

The ECE fermion equation replaces the Dirac equation by use of 2 x 2 Pauli matrices in factorizing the ECE wave equation instead of the 4 x 4 Dirac matrices. The fermion equation is part of a generally covariant unified field theory and is preferred by Ockham´s Razor to the Dirac equation. The ECE fermion equation is shown to produce half integral fermion spin, the Landé factor (g = 2) of the fermion, ESR and NMR, fine and hyperfine spectra structure, the Thomas factor and the Darwin term. It produces positive energy eigenvalues, so there is no negative energy problem as in the Dirac equation. There is therefore no Dirac sea problem. The ECE antifermion equation is produced straightforwardly by application of parity inversion. The basis set of Pauli matrices used in the fermion equation is the mirror image of that used in the Dirac equation, whose 4 x 4 matrices are shown to be superfluous and obsolete. The fermion equation can be used as the basis for development of quantum electrodynamics and quantum field theory, together with a new particle theory and computational quantum chemistry.

Keywords: ECE fermion equation, quantum electrodynamics, quantum field theory, particle theory.

1. Introduction.

The Dirac equation was a cornerstone of theoretical physics and chemistry in the twentieth century and originated in an attempt to factorize second order wave equations into first order differential equations. It was thought originally that this could be done by taking the square root of the Einstein energy equation of special relativity and applying the operator relations of the then new quantum mechanics. This was thought to be the route to relativistic quantum mechanics. Dirac´s contribution was based on his study of the matrix methods of Heisenberg. It is thought now that Dirac discovered the Pauli matrices independently. These were formulated originally by Pauli to produce a cyclic permutation of matrices (elements of a basis set) with a factor 1 / 2 (half integral spin or half integral angular momentum), building on the discovery of spinors by Cartan in 1913 and attempting to explain the Stern Gerlach experiment which showed that the electron had two types of spin. This procedure is now described in terms of the SU(2) representation space. The original formulation of the 2 x 2 matrices by Pauli was purely phenomenological. Dirac´s contribution was essentially to factorize the d´Alembertian operator of the wave equation with his well known 4 x 4 matrices, the Dirac matrices, and the Minkowski metric. This procedure produced a first order equation, the Dirac equation of the electron. The eigenfunction of this equation is the Dirac spinor, a column vector of four components, and is often described as the superposition of two Pauli spinors, right and left handed. The Pauli spinor is a two component column vector and is an example of a Cartan spinor.

In his original work, Dirac chose a combination of Dirac or gamma matrices now known as the standard representation [1, 2]. There is freedom of choice of Dirac matrices, they are not entirely fixed by experimental observation. Similarly there is freedom of choice in the Pauli matrices as is well known. There is also freedom of choice in the Cartesian representation, which can be right or left handed. Ryder [1] developed another choice of gamma matrices known as the chiral representation. The chiral representation is the correct choice because it does not produce negative energy eigenstates. The ECE fermion equation [3-12] was first developed in UFT 4 of this series (www.aias.us) and then in UFT 129 and 130. The fermion equation was deduced from Cartan´s geometry of the nineteen twenties, and is part of a generally covariant unified field theory, the well known ECE theory. The eigenfunction of the fermion equation is a 2 x 2 matrix in SU(2) representation space, and is an example of a Cartan tetrad. Therefore the fermion equation is written from the outset in the general spacetime with torsion and curvature, the two fundamental properties that characterize any space in any dimension, properties defined by the two Cartan structure equations. The wave format of the fermion equation is obtained from the most fundamental statement of Cartan´s differential geometry, the tetrad postulate. The latter is an expression of the fact that the complete vector field is invariant, a property which is true in any dimension and any mathematical space. The tetrad of the wave fermion equation is the matrix that links two two-dimensional spinors in accordance with Cartan´s definition of the tetrad as the matrix linking two vectors in two different representations, labelled usually as a and μ. The covariant derivatives are defined in the a and μ representation in terms of connections. The fermion wave equation therefore applies in the general space of two dimensions, whereas the wave format of the Dirac equation applies only in the Minkowski spacetime as is well known. Dirac used the Minkowski metric of "flat spacetime" to factorize the d´Alembertian with his gamma matrices.

In Section 2 this famous procedure by Dirac is shown to be superfluous. The factorization can be achieved straightforwardly using only the 2 x 2 Pauli matrices to produce the fermion equation. The latter automatically takes the place of the Dirac equation by Ockham´s Razor. It achieves all that Dirac achieves but in a simpler and more powerful manner. The fermion equation is written in the general mathematical space, and is an equation of general relativity, being part of a generally covariant unified field theory [3-12], the ECE theory that applies in any space of any dimension and for any fundamental field or combination of fields. ECE theory is far simpler than the obsolete attempts hitherto at producing a unified field theory. The fermion equation is shown to produce positive eigenstates of energy throughout. The reason why Dirac thought that he had produced "negative energy" is his choice of the wrong combination of gamma matrices. Negative energy does not exist in nature. Dirac realized this in the late twenties and attempted a cure with the Dirac sea. This concept was quickly rejected in the early thirties. The fermion equation does not suffer from the drawback of negative energy, and is therefore preferred over the Dirac equation. The anti fermion emerges not from negative energy and Dirac sea, but by straightforward application of parity inversion and conservation of CPT symmetry. Dirac did not in fact predict the positron, he predicted that the proton would be the anti particle of the electron.

In Section 3 the fermion equation is used to produce the probability current and the Landé factor, (g factor) of the electron. Not only this, but also in a simpler and more transparent manner than the Dirac equation. Similarly the fermion equation produces straightforwardly all known spectral properties such as fine structure (spin orbit coupling and Thomas factor of 2) and the Darwin term, ESR and NMR. This is achieved without getting into the entirely unnecessary difficulties of "negative energy". Therefore the fermion equation takes the place of the Dirac equation and can be developed extensively in computational quantum chemistry, quantum electrodynamics, second quantization, quantum field theory and particle theory. Following upon the dramatic collapse of the twentieth century particle theory in UFT 158 onwards, an entirely new particle theory has been proposed based on the eigenvalues R of the ECE wave equation.

2. The Fermion equation.

The fermion equation is:

$$\sigma^0\ \hat{E}\psi\,\sigma^0 - c\,\sigma^3\,(\,\hat{p}_X\,\psi\,\sigma^1 - \hat{p}_Y\,\psi\,\sigma^2 + \hat{p}_Z\,\psi\,\sigma^3\,) = m\,c^2\,\sigma^1\,\psi \tag{1}$$

in which the eigenfunction is a tetrad defined by:

$$\psi = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} . \tag{2}$$

The standard Pauli matrices are [1]:

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ,\quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ,\quad \sigma^2 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} ,\quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \tag{3}$$

The operators of quantum mechanics are:

$$\hat{E} = i\,\hbar\,\frac{\partial}{\partial t} \quad , \quad \hat{\boldsymbol{p}} = -\,i\,\hbar\,\nabla \tag{4}$$

so Eq. (1) is a first order differential equation of relativistic quantum mechanics, an equation which has been derived without the use of the 4 x 4 Dirac matrices. Written out in full, Eq. (1) is:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \hat{E} \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - c \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \left\{ \hat{p}_X \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right.$$

$$-\,\hat{p}_Y \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + \hat{p}_Z \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Bigg\} = m\,c^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} \tag{5}$$

i.e.:

$$\hat{E} \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} - c\,\hat{p}_X \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} + c\,\hat{p}_Y \begin{pmatrix} -\,i\psi_2^R & i\,\psi_1^R \\ i\psi_2^L & -\,i\psi_1^L \end{pmatrix} - c\,\hat{p}_Z \begin{pmatrix} \psi_1^R & -\psi_2^R \\ -\psi_1^L & \psi_2^L \end{pmatrix} = m\,c^2 \begin{pmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{pmatrix}. \tag{6}$$

Eq. (6) can be written as the following four equations:

$$\hat{E}\psi_1^R - c\,(\hat{p}_Z\psi_1^R + (\hat{p}_X + i\hat{p}_Y)\,\psi_2^R\,) = m\,c^2\psi_1^L \tag{7}$$

$$\hat{E}\psi_2^R - c\,((\hat{p}_X - i\hat{p}_Y\,)\,\psi_1^R - \hat{p}_Z\psi_2^R\,) = m\,c^2\psi_2^L \tag{8}$$

$$\hat{E}\psi_1^L + c\,(\hat{p}_Z\psi_1^L + (\hat{p}_X + \hat{p}_Y\,)\,\psi_2^L\,) = m\,c^2\psi_1^R \tag{9}$$

$$\hat{E}\psi_2^L + c\,((\hat{p}_X - \hat{p}_Y\,)\,\psi_1^L - \hat{p}_Z\psi_2^L\,) = m\,c^2\psi_2^R\quad . \tag{10}$$

These four equations can in turn be written as the following two matrix equations:

$$\hat{E} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \mathrm{c} \begin{pmatrix} \hat{p}_Z & (\hat{p}_X + i\hat{p}_Y) \\ (\hat{p}_X - \hat{p}_Y\,) & -\,\hat{p}_Z \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \end{pmatrix} = m\,c^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_1^L \\ \psi_2^L \end{pmatrix} \tag{11}$$

and

$$\hat{E} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \mathrm{c} \begin{pmatrix} \hat{p}_Z & (\hat{p}_X + i\hat{p}_Y) \\ (\hat{p}_X - \hat{p}_Y\,) & -\,\hat{p}_Z \end{pmatrix} \begin{pmatrix} \psi_1^L \\ \psi_2^L \end{pmatrix} = m\,c^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \end{pmatrix} \tag{12}$$

which may be written in condensed notation as:

$$(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m\,c^2\boldsymbol{\Phi}^L \tag{13}$$

$$(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m\,c^2\boldsymbol{\Phi}^R \qquad . \tag{14}$$

These make up the Dirac equation in the chiral representation [1], Q.E.D.

From Eqs. (13) and (14):

$$(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m^2\,c^4\boldsymbol{\Phi}^L \tag{15}$$

$$(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m^2\,c^4\boldsymbol{\Phi}^R\ \ . \tag{16}$$

The d´Alembertian operator is defined as:

$$\Box = \partial^{\mu} \partial_{\mu} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{17}$$

and so it is found that:

$$(\Box + (mc/\hbar)^2)\, \psi_1^R = 0 \tag{18}$$

$$(\Box + (mc/\hbar)^2)\, \psi_2^R = 0 \tag{19}$$

$$(\Box + (mc/\hbar)^2)\, \psi_1^L = 0 \tag{20}$$

$$(\Box + (mc/\hbar)^2)\, \psi_2^L = 0 \tag{21}$$

i.e.

$$(\Box + (mc/\hbar)^2) \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} = 0 \tag{22}$$

which is the wave fermion equation, Q.E.D. Eq. (1) is a factorization of Eq. (22), and the latter is an example of the ECE wave equation [3 -12] of generally covariant unified field theory:

$$(\Box + R)\, q_{\mu}^{a} = 0 \tag{23}$$

with tetrad eigenfunction:

$$q_{\mu}^{a} = \begin{pmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{pmatrix} \tag{24}$$

and eigenvalues:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad . \tag{25}$$

Note carefully that the eigenoperator is the d´Alembertian in any spacetime. This fact allows the use of the Pauli matrices in any spacetime. In general R is defined in terms of connections, and Eq. (22) uses a limit of R for the free fermion regarded as a single particle.

For purposes of comparison only, the following gives details of the Dirac equation in chiral representation [1]:

$$(\gamma^{\mu}\, \hat{p}_{\mu} - mc)\, \psi_D = 0 \tag{26}$$

in which:

$$\gamma^{\mu}\, \hat{p}_{\mu} = \gamma^{0}\, \hat{p}_0 + \gamma^{i}\, \hat{p}_i \tag{27}$$

and in which the eigenfunction is the Dirac spinor:

$$\psi_D = \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix} \quad . \tag{28}$$

The Dirac or gamma matrices in chiral representation are different from the ones originally used by Dirac, and are 4 x 4 matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \ \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \ \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \ \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{29}$$

usually written in condensed notation as:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad , \tag{30}$$

So Eq. (26) is:

$$\hat{E} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix} - c\,\hat{p}_X \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix} - c\,\hat{p}_Y \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix}$$

$$- c\,\hat{p}_Z \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix} = m\,c^2 \begin{pmatrix} \psi_1^R \\ \psi_2^R \\ \psi_1^L \\ \psi_2^L \end{pmatrix} \tag{31}$$

and gives Eqs. (7) to (10) again. The wave format of the Dirac equation (26) is:

$$(\Box + (mc/\hbar)^2)\ \psi_D = 0 \tag{32}$$

and gives Eqs. (18) to (21) again.

Clearly, the fermion equation is the simpler equation and uses 2 x 2 matrices throughout. So the fermion equation is preferred by Ockham´s Razor. The Dirac spinor is not a tetrad and cannot be written in the general spacetime. Attempts have been made to write the Dirac equation in the general spacetime, using different factorizations of the d´Alembertian. These attempts do not fit in to Cartan geometry and cannot be part of a unified field theory. Eq. (1) can be generalized immediately to the general spacetime by writing it as:

$$\sigma^0 \hat{E}\psi\,\sigma^0 - c\,\sigma^3\,(\hat{p}_X\,\psi\,\sigma^1 - \hat{p}_Y\,\psi\,\sigma^2 + \hat{p}_Z\,\psi\,\sigma^3) = \hbar\,R^{½}\,\sigma^1\,\psi\ . \tag{33}$$

The fermion equation has interesting mathematical properties. Notably, it uses the mirror image system of Pauli matrices defined by:

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\ ,\ \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\ ,\ \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\ ,\ \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\ . \tag{34}$$

in which the sign of σ^2 is the opposite of that in Eq. (3). The mirror image Pauli matrices have the cyclic property:

$$\left[\frac{\sigma^1}{2}\ ,-\frac{\sigma^2}{2}\right] = i\,\frac{\sigma^3}{2} \tag{35}$$

$$\left[\frac{\sigma^3}{2}\ ,\ \frac{\sigma^1}{2}\right] = -\,i\,\frac{\sigma^2}{2} \tag{36}$$

$$\left[-\frac{\sigma^2}{2}\ ,\ \frac{\sigma^3}{2}\right] = i\,\frac{\sigma^1}{2} \tag{37}$$

with SU(2) symmetry [1 -12]. In the mirror image system (34) the fermion equation is:

$$\boxed{\sigma^0\,\hat{E}\psi\,\sigma^0 + c\,\sigma^3\hat{p}_i\,\psi\,\sigma^i = m\,c^2\,\sigma^1\,\psi} \tag{38}$$

where contravariant covariant summation is implied as follows:

$$\begin{aligned}\hat{p}_i\,\sigma^i &= \hat{p}_1\,\psi\,\sigma^1 + \hat{p}_2\,\psi\,\sigma^2 + \hat{p}_3\,\psi\,\sigma^3 \\ &= -\,\hat{p}_X \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \hat{p}_Y \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} - \hat{p}_Z \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.\end{aligned} \tag{39}$$

Note that the $\hat{E}$ and $\hat{p}$ operators act on the wavefunction ψ , in which the components of the two Pauli column spinors:

$$\Phi^R = \begin{bmatrix} \psi_1^R \\ \psi_2^R \end{bmatrix} \quad , \quad \Phi^L = \begin{bmatrix} \psi_1^L \\ \psi_2^L \end{bmatrix} \tag{40}$$

appear in the top and bottom rows of the 2 x 2 matrix. We denote ψ as "the ECE spinor". In the Dirac spinor the two Pauli spinors appear in column vector format one on top of the other.

The parity operator P acts on the ECE spinor as follows:

$$P\,\psi = \sigma^1\,\psi = \begin{bmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{bmatrix} \tag{41}$$

so on the right hand side of the fermion equation appears the parity inverted ECE spinor. Eq. (38) has a deep mathematical meaning and structure which should be capable of considerable development in pure and applied mathematics. Therefore the anti fermion equation is obtained straightforwardly from the fermion equation by operating on each term with *P* as follows:

$$P\,(\hat{E}) = \hat{E}\,, \quad P\,(\hat{p}\,) = -\,\hat{p}\,, \tag{42}$$

$$P \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} = \begin{bmatrix} \psi_1^L & \psi_2^L \\ \psi_1^R & \psi_2^R \end{bmatrix} .$$

Note carefully that the eigenstates of energy are always positive, both in the fermion and antifermion equations. The anti fermion is obtained from the fermion by reversing helicity:

$$\hat{p}\,(\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,) = -\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}} \quad . \tag{43}$$

The anti fermion has opposite parity to the fermion, the same mass, and the opposite electric charge. The static fermion is indistinguishable from the static anti-fermion [1]. So CPT symmetry is conserved as follows from fermion to anti fermion:

$$\mathrm{CPT} \rightarrow (-\,\mathrm{C}\,)\,\mathrm{T}\,(-\,\mathrm{P}\,) \tag{44}$$

where C is the charge conjugation operator and T the motion reversal operator. Note carefully that there is no negative energy anywhere in the analysis.

3. Some properties of the Fermion equation.

The probability four-current j^μ of an equation of relativistic quantum mechanics must be conserved:

$$\partial_\mu j^\mu = 0 \quad . \tag{45}$$

Define the hermitian conjugate of the ECE spinor by:

$$\psi^{+} = \begin{bmatrix} \psi_1^{R*} & \psi_1^{L*} \\ \psi_2^{R*} & \psi_2^{L*} \end{bmatrix} \tag{46}$$

then a conserved probability current can be defined by:

$$j^{\mu} := \frac{1}{2} \mathrm{Tr} \, (\psi \, \sigma^{\mu} \, \psi^{+} + \psi^{+} \sigma^{\mu} \, \psi \,) \quad . \tag{47}$$

The probability is defined by:

$$j^{0} = j^{\mu} \;\; (\mu \; = 0 \,) \tag{48}$$

a result which is always positive as required by the Born interpretation of quantum mechanics [2]. It is seen that j^0 is time independent and conserved:

$$\partial_0 j^0 = 0 \quad . \tag{49}$$

If it is assumed that:

$$\psi = \psi \, (0) \, e^{-i\Phi} \quad , \quad \psi^{+} = \psi^{+} \, (0) \, e^{i\Phi} \tag{50}$$

where Φ is a time and coordinate dependent phase factor, then:

$$\partial_{\mu} j^{\mu} = 0 \tag{51}$$

and the four-current is conserved, Q. E. D. The fermion equation therefore produces the correct characteristics of probability current, whereas the Klein Gordon equation for a spinless particle fails as is well known [1]. For purposes of comparison only, the probability current from the chiral representation of the Dirac equation is defined [1] as:

$$j^{\mu} = \overline{\psi}_D \, \gamma^{\mu} \, \psi_D \tag{52}$$

where the adjoint spinor is

$$\overline{\psi}_D = \psi_D^{+} \, \gamma^{0} \quad . \tag{53}$$

The conservation of j^{μ} is obtained by using the Dirac equation:

$$(i \, \gamma^{\mu} \, \partial_{\mu} - mc/\hbar \,) \, \psi_D = 0 \tag{54}$$

and the adjoint equation in which the operator $\overleftarrow{\partial}_{\mu}$ acts to the left:

$$\overline{\psi}_D \, (\, i \, \gamma^{\mu} \, \overleftarrow{\partial}_{\mu} + mc/\hbar \,) = 0 \tag{55}$$

so the probability is:

$$j^{0} = \overline{\psi}_D \, \gamma^{0} \, \psi_D = \psi_D^{+} \psi_D = \begin{bmatrix} \psi_1^{R*} \psi_2^{R*} \psi_1^{L*} \psi_2^{L*} \end{bmatrix} \begin{pmatrix} \psi_1^{R} \\ \psi_2^{R} \\ \psi_1^{L} \\ \psi_2^{L} \end{pmatrix} \tag{56}$$

$$= \psi_1^R \psi_1^{R*} + \psi_2^R \psi_2^{R*} + \psi_1^L \psi_1^{L*} + \psi_2^L \psi_2^{L*}$$

which is the same result as Eq. (48), Q.E.D. Eq. (48) is obtained more simply and more powerfully as explained in Section 2. The trace also has deep significance in group theory [2].

The fermion equation produces as follows the Lande factor (or electron g factor) observed in the anomalous Zeeman effect. Consider the fermion equation in the format (13) and (14), and use the well known minimal prescription to describe the effect of an electromagnetic field on the fermion. The scalar potential is Φ and the vector potential is $\boldsymbol{A}$. Then:

$$((\hat{E} - e\,\Phi\,) + c\,\boldsymbol{\sigma} \,.\, (\,\hat{\boldsymbol{p}} - e\,\boldsymbol{A}))\,\Phi^L = m\,c^2\,\Phi^R \tag{57}$$

$$((\hat{E} - e\,\Phi\,) - c\,\boldsymbol{\sigma} \,.\, (\,\hat{\boldsymbol{p}} - e\,\boldsymbol{A}))\,\Phi^R = m\,c^2\,\Phi^L \tag{58}$$

so:

$$((\hat{E} - e\,\Phi\,)^2 - c^2\,(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,\Phi^R = m^2 c^4\,\Phi^R \tag{59}$$

where:

$$\hat{\boldsymbol{\pi}} \, := \hat{\boldsymbol{p}} - e\,\boldsymbol{A} \quad . \tag{60}$$

Therefore:

$$((\hat{E} - e\,\Phi\,)^2 - m^2\,c^4\,)\,\Phi^R = \,c^2\,(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,\Phi^R \quad . \tag{61}$$

Now use:

$$(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,(\,\boldsymbol{\sigma} .\, \hat{\boldsymbol{\pi}}\,)\,\Phi^R = ((\,\hat{\boldsymbol{p}} - e\,\boldsymbol{A})^2 - i\,e\,\boldsymbol{\sigma} \,.\, (\hat{\boldsymbol{p}} \times \boldsymbol{A} + \boldsymbol{A} \times \hat{\boldsymbol{p}}\,)\,\Phi^R \tag{62}$$

where $\hat{p}$ is the operator:

$$\hat{\boldsymbol{p}} = -\,i\,\hbar\,\nabla \quad . \tag{63}$$

Therefore:

$$\begin{aligned} &\nabla \times (\boldsymbol{A}\Phi^R) + \boldsymbol{A} \times \nabla\Phi^R \\ &= (\nabla \times \boldsymbol{A})\,\Phi^R + (\nabla\Phi^R) \times \boldsymbol{A} + \boldsymbol{A} \times (\nabla\Phi^R) \\ &= (\nabla \times \boldsymbol{A})\,\Phi^R = \boldsymbol{B}\,\Phi^R \end{aligned} \tag{64}$$

where the magnetic flux density is defined by:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \quad . \tag{65}$$

So:

$$\hat{H}\,\Phi^R = \frac{1}{2m\,c^2}\,((\hat{E} - e\,\Phi)^2 - m^2\,c^4)\,\Phi^R \tag{66}$$

where:

$$\hat{H} = \frac{1}{2m} \left((\hat{\boldsymbol{p}} - e\,\boldsymbol{A})^2 - e\,\hbar\,\boldsymbol{\sigma} \,.\, \boldsymbol{B} \right) \quad . \tag{67}$$

For simplicity and illustration consider the case:

$$\Phi = 0 \tag{68}$$

where the fermion is in a static magnetic field as in ESR and NMR. Then the following Schroedinger structure is obtained:

$$\hat{H}\,\Phi^R = \frac{p^2}{2m}\,\Phi^R \tag{69}$$

where, from the Einstein energy equation:

$$p^2 = E^2 - m^2 c^4 \quad . \tag{70}$$

In the non-relativistic limit:

$$v << c \quad , \quad \gamma \rightarrow 1 \tag{71}$$

the non-relativistic kinetic energy T is obtained:

$$\hat{H}\,\Phi^R = T\,\Phi^R \,. \tag{72}$$

The interaction term of fermion and magnetic field is:

$$\hat{H}_{\text{int}} = \frac{e\,\hbar\,\boldsymbol{\sigma} \,.\, \boldsymbol{B}}{2m} \tag{73}$$

in which the g factor 2 of the electron has been incorporated correctly in accordance with experimental data from the anomalous Zeeman effect. The latter shows that the magnetic moment is:

$$\boldsymbol{m} = \frac{e\,\hbar\,\boldsymbol{\sigma}}{2m} = \frac{e}{m}\,\boldsymbol{S} \tag{74}$$

where the half integral spin angular momentum of the electron is:

$$\boldsymbol{S} = \frac{\hbar\,\boldsymbol{\sigma}}{2} \quad . \tag{75}$$

The classical magnetic moment from the orbital angular momentum $\boldsymbol{L}$ is [1, 2]:

$$\boldsymbol{m} = \frac{e}{2m}\,\boldsymbol{L} \tag{76}$$

and there is a factor of 2 difference between Eqs. (74) and (76). This difference shows up in the anomalous Zeeman effect. The experimental result is therefore:

$$\boldsymbol{m} = 2\,\frac{e}{2m}\,\boldsymbol{S} = \frac{e\,\hbar\,\boldsymbol{\sigma}}{2m} \tag{77}$$

and is described by Eq. (73) of the fermion equation, Q.E.D. The factor of 2 in the numerator of Eq. (77) is the Landé factor, or g = 2 factor of the electron. The factor 1 / 2 in Eq. (75) is the half integral spin of the electron. It is often claimed that Dirac was the first to produce the Landé factor but it is also produced from the Schroedinger equation with Pauli basis set as shown in notes 172(7) accompanying this paper on www.aias.us. Notes for UFT 172 contain full details of all calculations. Note carefully that the Landé factor is precisely 2 only in the non - relativistic limit, and as is well known, is also affected by radiative corrections [1 -12]. The fermion equation can be used to describe the radiative corrections.

Eq. (61) may be used to produce an elegant derivation from the fermion equation of the Thomas factor 2 of spin orbit coupling, and the concomitant Darwin term. This derivation is much simpler than that from the Dirac equation, and again does not suffer from "negative energy" that does not exist in nature. Consider again Eq. (61) in the case:

$$\boldsymbol{A} = \mathbf{0} \quad , \quad \Phi \neq 0 \tag{78}$$

then:

$$\frac{1}{2m}\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,\Phi^R = \frac{1}{2mc^2}\,(\,c^2p^2 - 2e\,\Phi E + e^2\,\Phi^2)\,\Phi^R \tag{79}$$

where we have used:

$$E^2 - m^2\,c^4 = p^2\,c^2 \quad . \tag{80}$$

So:

$$\frac{\widehat{\boldsymbol{p}}^2}{2m}\,\Phi^R = \left(\frac{1}{2m}\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) + \frac{e\,\Phi E}{mc^2} - \frac{e^2\Phi^2}{2mc^2}\right)\Phi^R \quad . \tag{81}$$

Now use:

$$\mathrm{E} = \gamma\, mc^2 \tag{82}$$

and consider the non-relativistic limit (71) in which:

$$\mathrm{E} \longrightarrow mc^2 \quad . \tag{83}$$

Then:

$$\frac{\widehat{\boldsymbol{p}}^2}{2m}\,\Phi^R \longrightarrow \left(\frac{1}{2m}\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) + e\Phi - \frac{e^2\Phi^2}{2mc^2}\right)\Phi^R \quad . \tag{84}$$

However:

$$\frac{\widehat{\boldsymbol{p}}^2}{2m}\,\Phi^R = \frac{1}{2m}\,((\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}))\,\Phi^R \tag{85}$$

so

$$e\Phi \longrightarrow 2mc^2 \tag{86}$$

or

$$\frac{e\Phi}{2mc^2} \longrightarrow 1 \quad . \tag{87}$$

Therefore Eq. (84) can always be written as:

$$\frac{\widehat{\boldsymbol{p}}^{\,2}}{2m}\,\Phi^R \longrightarrow (\frac{1}{2m}\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,\frac{e\Phi}{2mc^2}\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,)\,\Phi^R \quad . \tag{88}$$

Using the fact that $\widehat{p}$ is the differential operator defined by Eq. (4) gives:

$$(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,\Phi(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) = \frac{\hbar}{i}\,\boldsymbol{\sigma}\,.\,((\nabla\,\Phi)\,\boldsymbol{\sigma}\,.\,\widehat{\boldsymbol{p}} + \sigma\,\nabla\,(\boldsymbol{\sigma}\,.\,\widehat{\boldsymbol{p}})) \quad . \tag{89}$$

The electric field strength is defined by:

$$E = -\nabla\,\Phi \quad . \tag{90}$$

Using the algebra of Pauli matrices:

$$(\boldsymbol{\sigma}.\widehat{\boldsymbol{E}})\,(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) = \boldsymbol{E}\,.\,\widehat{\boldsymbol{p}} + i\,\boldsymbol{\sigma}\,\boldsymbol{E} \times \widehat{\boldsymbol{p}} \tag{91}$$

so:

$$(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,\Phi(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) = -\hbar\,\boldsymbol{\sigma}\,.\,\boldsymbol{E} \times \widehat{\boldsymbol{p}} + i\,\hbar\,\boldsymbol{E}\,.\,\widehat{\boldsymbol{p}} + \Phi\,\,\widehat{\boldsymbol{p}}^{\,2} \quad . \tag{92}$$

The spin orbit term is:

$$A_{SO} = -\frac{e\hbar}{4m^2c^2}\,\boldsymbol{\sigma}\,.\,\boldsymbol{E} \times \widehat{\boldsymbol{p}} \quad . \tag{93}$$

The Thomas factor is the extra factor 2 in the denominator of Eq. (93), Q.E.D. If the Coulombic type potential is used:

$$\Phi = -\frac{e}{4\pi r\epsilon_0} \tag{94}$$

then:

$$\boldsymbol{E} = -\frac{e}{4\pi\epsilon_0}\,\frac{\boldsymbol{r}}{r^3} \tag{95}$$

and the spin orbit term is found in its usual format:

$$\widehat{H}_{\mathrm{SO}} = -\frac{e}{4\pi r^3\epsilon_0}\,\frac{\boldsymbol{S}.\,\boldsymbol{L}}{2m^2c^2} \quad . \tag{96}$$

Finally the Darwin term is found by using the result:

$$(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}})\,\Phi(\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}) = -\hbar^2\,\boldsymbol{\sigma}\,.\,\nabla\,\Phi\,\boldsymbol{\sigma}\,.\,\nabla\,\Phi^R \tag{97}$$

and is given by:

$$\hat{H}_{\text{Darwin}} = -\frac{\hbar^2}{4m^2c^2}\nabla\,\Phi\nabla\,\Phi^R\,. \tag{98}$$

The Darwin term is observed in the fine structure of spectra. So the fermion equation has been thoroughly tested and can be developed extensively in quantum field theory.

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