Covariant format of the ECE fermion equation.

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Abstract.

The ECE fermion equation is developed in covariant format without negative energy being considered and using Pauli matrices only. The equation is the first single particle equation of the fermion and anti fermion, and dispenses with the need for Dirac matrices and the Dirac sea and the consequent need for multi particle interpretations. The equation gives a rigorously non negative Born probability and a conserved probability current. It is solved for the hamiltonian, the first few terms of which give the Landé and Thomas factors, spin orbit coupling, the interaction of the fermion with the magnetic field, ESR and NMR, the Darwin term of quantum electrodynamics, and the fine structure of the H atom. A new half-operator method of solution is developed.

Keywords: ECE fermion equation, covariant format, Landé factor, Thomas factor, spin-orbit coupling, fine structure of the H atom , half operator method.

1. Introduction.

The ECE fermion equation has been developed in previous papers of this series [1–10] and in the preceding paper UFT 172 was given in its final form in all detail. The fermion equation makes the Dirac equation obsolete in that it dispenses with the need for the 4 x 4 matrices of Dirac and does not use negative energy eigenstates. There is therefore no need for the Dirac sea concept or any concept that arises from negative energy. The latter is considered to be unphysical by this author and by other scholars. The fermion equation in wave format originates in the ECE wave equation of the general spacetime and is a special case of the ECE wave equation, whose wavefunction is a tetrad matrix. The Dirac equation was first derived from ECE theory in UFT 4 of this series (www.aias.us). That paper was developed with a 2 x 2 tetrad matrix which was re-arranged in a column vector of four entries, the upper two being components of the right handed Pauli spinor, the lower two being components of the left handed Pauli matrix. This rearrangement led to the Dirac equation. However, in the philosophy of relativity as developed in ECE theory, the eigenfunction of every valid wave equation of physics and quantum mechanics is a tetrad defined by Cartan geometry [11]. It cannot be a column vector as used by Dirac. The latter´s equation is Lorentz covariant only, while the fermion equation is generally covariant automatically, a major advantage. The fermion equation is also part of a generally covariant unified field theory based on geometry - ECE theory.

In UFT 129 and 130, the fermion equation was developed with a 2 x 2 matrix eigenfunction, and it was shown that the Pauli matrices are sufficient to describe a fermion and anti fermion in a rigorously correct single particle interpretation free of negative energy. The energy eigenstates of the fermion equation are rigorously positive, thus curing one of the major problems of the Dirac equation, negative energy and consequent impossibility of a single particle interpretation [12]. In UFT 172 the fermion equation was written out in all detail and shown to be equivalent to the chiral representation of the Dirac equation developed by Ryder [12]. The fermion equation is therefore the fundamental statement of the Lorentz transform applied to the right and left Pauli spinors. This conclusion emerges from Ryder´s demonstration that the chiral representation is given by this Lorentz transform. The latter´s structure is determined from very fundamental considerations of the Poincare group {12]. The original Dirac equation (the standard representation) does not emerge from this fundamental Lorentz transform. This shows that Dirac made an incorrect choice of gamma matrices, and this error led to negative energy. In the chiral representation the eigenstates of energy are positive.

This reasoning confirms that the structure of ECE theory is the most fundamental one known at present in physics, all wave equations having the same fundamental structure with tetrad eigenfunctions valid in any mathematical space of any dimension. The correct wave equation of the fermion is a special case of the ECE wave equation. Dirac´s original procedure was to use Clifford algebra to factorise the d´Alembertian with 4 x 4 matrices and that procedure was not part of a unified field theory as needed. Most of the successes attributed to the Dirac equation can also be viewed as minor advances, for example the Landé factor, ESR and NMR can be obtained from the Schroedinger equation using Pauli matrices. The Thomas factor can be obtained in other ways, notably the original derivation by Thomas in 1926 / 1927, and spin orbit coupling and fine structure in spectra can be described with non relativistic methods to a high degree of precision. The successes of the Dirac equation include its ability to give a non negative Born probability and rigorously conserved probability current, and its relativistic correction of the fine structure of spectra. These successes were achieved at great cost: Dirac´s incorrect choice of gamma matrices led to negative energy and many problems of interpretation. It led for example to the idea of particles moving backwards in time being equivalent to antiparticles moving forward in time. This idea was introduced by Stueckelberg in 1941 [13], but is almost always attributed to Feynman [14]. This author and other scholars reject the ideas of negative energy and motion backwards in time because the fermion equation makes these ideas unnecessary and so is preferred by the fundamental principle of natural philosophy: Ockham´s Razor. The fermion equation becomes the basis for relativistic quantum mechanics, quantum electrodynamics and quantum field theory and also the basis for computation of the fine and hyperfine structure of spectra.

In Section 2 the fermion equation is expressed in a succinct and simple covariant format with the introduction of the fermion operator π_μ in its momentum representation, and this covariant format is shown to lead to a rigorously non negative Born probability which is the same as that obtained from the chiral representation of the Dirac equation [12]. However, the fermion equation derives it with the advantages described already. It is shown that the probability current of the fermion equation is conserved. In Section 3 a new half-operator method is developed to solve the fermion equation for the hamiltonian. Again, the same result is obtained as for the Dirac equation, but without the Dirac gamma matrices and without negative energy eigenstates being present in the calculation. The fermion equation´s hamiltonian gives all that is usually attributed to Dirac, notably the Landé (g = 2) factor, the Thomas factor of two, spin orbit coupling and the Darwin term. Finally in Section 4 the relativistic corrections of the fine structure in the H atom are calculated analytically from the fermion equation, again without use of Dirac gamma matrices and negative energy eigenstates.

2. Covariant format.

The covariant format of the fermion equation is:

$$\pi_\mu \, \psi \, \sigma^\mu = m \, c \sigma^1 \, \psi \qquad (1)$$

where the fermion operator in its momentum representation [12] is defined as:

$$\pi_\mu = (\pi_0 \, , \pi_1 \, , \pi_2 \, , \pi_3 \,) \quad . \qquad (2)$$

Here:

$$\pi_0 = \sigma^0 \, p_0 \quad , \quad \pi_i = \sigma^3 \, p_i \qquad (3)$$

where p_μ is the energy momentum four vector:

$$p_\mu = (p_0 \, , p_1 \, , p_2 \, , p_3 \,) = \left(\frac{E}{c} \, , -\boldsymbol{p} \,\right) . \qquad (4)$$

The Pauli matrices are defined by:

$$\sigma^\mu = (\sigma^0 , \sigma^1 , \sigma^2 , \sigma^3) \tag{5}$$

where

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} , \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \tag{6}$$

The eigenfunction of Eq. (1) is the tetrad [1–10]:

$$\psi = \begin{bmatrix} \psi_1^R & \psi_2^R \\ \psi_1^L & \psi_2^L \end{bmatrix} . \tag{7}$$

whose entries are defined by the right and left Pauli spinors:

$$\Phi^R = \begin{bmatrix} \psi_1^R \\ \psi_2^R \end{bmatrix} , \quad \Phi^L = \begin{bmatrix} \psi_1^L \\ \psi_2^L \end{bmatrix} . \tag{8}$$

This eigenfunction is referred to as "the fermion spinor".

The position representation [12] of the fermion operator is defined by the symbol δ and is:

$$\delta_\mu = -\frac{i}{\hbar} \pi_\mu . \tag{9}$$

Therefore the fermion equation is a first order differential equation:

$$i \hbar \delta_\mu \psi \sigma^\mu = m c \sigma^1 \psi . \tag{10}$$

For purposes of comparison the covariant format of the Dirac equation in chiral representation [12] is:

$$\gamma^\mu \delta_\mu \psi_D = m c \psi_D \tag{11}$$

where:

$$\psi_D = \begin{bmatrix} \Phi^R \\ \Phi^L \end{bmatrix} \tag{12}$$

is a column vector (see UFT 172) with four entries, and where the Dirac matrices in chiral representation are [12]:

$$\gamma^{\mu} = (\gamma^0 , \gamma^1 , \gamma^2 , \gamma^3) . \tag{13}$$

The complete details of the development of Eq. (1) are given in UFT 172 and particular note 172(8) (www.aias.us). Note that the ordering of terms in Eq. (1) is important because matrices do not commute and ψ is a 2 x 2 matrix. Above all, the energy eigenvalue of Eq. (1) is rigorously positive in value, never negative. The complex conjugate of the adjoint matrix of the fermion spinor is referred to as the "adjoint spinor" of the fermion equation, and is defined by:

$$\psi^{+} = \begin{bmatrix} \psi_1^{R*} & \psi_1^{L*} \\ \psi_2^{R*} & \psi_2^{L*} \end{bmatrix} . \tag{14}$$

The adjoint equation of Eq. (1) is defined as:

$$- i\, \hbar \delta_{\mu}\, \psi^{+}\, \sigma^{\mu} = m\, c \sigma^1\, \psi^{+} \tag{15}$$

where the complex conjugate of $i\,\hbar$ has been used. These equations have well known counterparts in the Dirac theory [12], but in that theory the gamma matrices are used and the definition of the adjoint spinor is more complicated.

The probability four-current of the fermion equation is defined as:

$$j^{\mu} := \frac{1}{2}\, \mathrm{Tr}\, (\psi\, \sigma^{\mu}\, \psi^{+} + \psi^{+} \sigma^{\mu}\, \psi\,) . \tag{16}$$

The Born probability is therefore:

$$j^0 = \psi_1^R\, \psi_1^{R*} + \psi_2^R\, \psi_2^{R*} + \psi_1^L\, \psi_1^{L*} + \psi_2^L\, \psi_2^{L*} \tag{17}$$

and is rigorously positive. It is the same as the Born probability [12] of the chiral representation of the Dirac equation. In the latter the four current is defined as:

$$j_D^{\mu} = \overline{\psi}_D\, \gamma^{\mu}\, \psi_D \tag{18}$$

and the adjoint Dirac spinor is a four entry row vector defined by:

$$\overline{\psi}_D = \psi_D^{+}\, \gamma^0 . \tag{19}$$

It is shown as follows that the probability four- current of the fermion equation is conserved:

$$\delta_{\mu} j^{\mu} = 0 . \tag{20}$$

To prove this result multiply both sides of Eq. (1) from the right with ψ^{+} :

$$i\,\hbar\delta_\mu\,\psi\,\sigma^\mu\,\psi^{+} = m\,c\sigma^1\,\psi\,\psi^{+}\ . \tag{21}$$

Multiply both sides of Eq. (15) from the right by ψ :

$$-\,i\,\hbar\delta_\mu\,\psi^{+}\,\sigma^\mu\psi = m\,c\sigma^1\,\psi^{+}\,\psi\ . \tag{22}$$

Subtract Eq. (22) from Eq. (21):

$$i\,\hbar\delta_\mu\,(\psi\,\sigma^\mu\,\psi^{+} + \psi^{+}\,\sigma^\mu\psi\,) = m\,c\sigma^1\,(\psi\,\psi^{+} - \psi^{+}\,\psi\,)\ . \tag{23}$$

By definition:

$$\psi\,\psi^{+} - \psi^{+}\,\psi = \begin{bmatrix}\psi_1^R & \psi_2^R\\ \psi_1^L & \psi_2^L\end{bmatrix}\begin{bmatrix}\psi_1^{R*} & \psi_1^{L*}\\ \psi_2^{R*} & \psi_2^{L*}\end{bmatrix} - \begin{bmatrix}\psi_1^{R*} & \psi_1^{L*}\\ \psi_2^{R*} & \psi_2^{L*}\end{bmatrix}\begin{bmatrix}\psi_1^R & \psi_2^R\\ \psi_1^L & \psi_2^L\end{bmatrix} \tag{24}$$

so:

$$\text{Trace}\,(\psi\,\psi^{+} - \psi^{+}\,\psi\,) = 0\ . \tag{25}$$

Therefore:

$$\text{Trace}\,(\delta_\mu\,(\psi\,\sigma^\mu\,\psi^{+} + \psi^{+}\,\sigma^\mu\psi\,)\,) = 0 \tag{26}$$

and therefore:

$$\delta_\mu j^\mu = 0 \tag{27}$$

which is Eq. (20), Q.E.D. Note that the fermion differential operator δ_μ is used.

3. Half-operator solution of the fermion equation.

The fermion Eq. (1) may be expanded into two simultaneous equations (see UFT 172 and its background notes on www.aias.us and advance postings on the diary or blog of www.aias.us):

$$(\hat{E} + c\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = mc^2\Phi^R \tag{28}$$

$$(\hat{E} - c\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = mc^2\Phi^L \tag{29}$$

in which $\hat{E}$ and $\hat{p}$ are the Schroedinger operators of quantum mechanics:

$$\hat{E} = i\,\hbar\,\frac{\partial}{\partial t} \quad , \quad \hat{\boldsymbol{p}} = -\,i\,\hbar\,\nabla \quad . \tag{30}$$

They have a momentum and position representation [12]. The momentum representation uses E and p as in classical physics and the position representation uses the differential operators $\frac{\partial}{\partial t}$ and ∇ which act on the eigenfunction. When $\hat{E}$ and $\hat{p}$ are used in the position representation they are denoted $\hat{E}$ and $\hat{p}$, otherwise they are denoted E and p. Eqs. (28) and (29) are Lorentz covariant automatically because they are derived directly [12] from the Lorentz transformations of φ^R and φ^L. The fermion Eq. (1) combines these equations in a succinct format which is not only Lorentz covariant but generally covariant.

Eqs. (28) and (29) may be written as:

$$(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m^2\,c^4\boldsymbol{\Phi}^L \tag{31}$$
$$(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m^2\,c^4\boldsymbol{\Phi}^R \quad . \tag{32}$$

i.e.:

$$(\hat{E}^2 - \mathrm{c}^2\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m^2\,c^4\boldsymbol{\Phi}^R \tag{33}$$
$$(\hat{E}^2 - \mathrm{c}^2\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m^2\,c^4\boldsymbol{\Phi}^L \quad . \tag{34}$$

The half-operator method of solving Eqs. (33) and (34) starts from considering:

$$\hat{E}^2 := E\,\hat{E} \quad . \tag{35}$$

By definition, the classical E is:

$$E = \gamma\, m\, c^2 \tag{36}$$

so the half operator is:

$$\hat{E}^2 = i\,\hbar\gamma\, m\, c^2\,\frac{\partial}{\partial t} \quad . \tag{37}$$

It follows that Eqs. (33) and (34) become:

$$(\gamma\, m\, c^2\hat{E} - \mathrm{c}^2\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m^2\,c^4\boldsymbol{\Phi}^R \tag{38}$$
$$(\gamma\, m\, c^2\hat{E}^2 - \mathrm{c}^2\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m^2\,c^4\boldsymbol{\Phi}^L \quad . \tag{39}$$

These are time dependent Schroedinger type equations [15] which may be written as:

$$\hat{E}\varphi^R = \hat{H}\,\varphi^R \tag{40}$$

$$\hat{E}\,\varphi^L = \hat{H}\,\varphi^L \qquad (41)$$

where the hamiltonian operator is defined as:

$$\hat{H} = \frac{1}{E}\left(c^2\, \boldsymbol{\sigma}.\hat{\boldsymbol{p}}\, \boldsymbol{\sigma}.\hat{\boldsymbol{p}} + m^2 c^4\right) \ . \qquad (42)$$

Note carefully that Eqs. (40) and (41) are derived without use of the Dirac gamma matrices and with rigorously positive energy eigenstates.

For a static fermion Eqs. (40) and (41) reduce to:

$$i\,\hbar \frac{\partial \varphi^R}{\partial t} = m\,c^2 \varphi^R \qquad (43)$$

$$i\,\hbar \frac{\partial \varphi^L}{\partial t} = m\,c^2 \varphi^L \qquad (44)$$

in which

$$\varphi^R\,(0) = \varphi^L\,(0) \qquad (45)$$

and in which the two eigenstates of rest energy, $m\,c^2$, are both positive. For a moving particle:

$$E = \gamma\, m\, c^2 \qquad (46)$$

$$\boldsymbol{p} = \gamma\, m\, \boldsymbol{v} \qquad (47)$$

where the Lorentz factor is given by:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \qquad (48)$$

where v is the fermion velocity. The hamiltonian operator is therefore:

$$\hat{H} = \frac{\gamma}{m}\ \boldsymbol{\sigma}.\hat{\boldsymbol{v}}\ \ \boldsymbol{\sigma}.\hat{\boldsymbol{v}} + \frac{mc^2}{\gamma} \qquad (49)$$

and Eqs. (40) and (41) become:

$$i\,\hbar \frac{\partial \varphi^R}{\partial t} = \hat{H}\,\varphi^R \qquad (50)$$

$$i\,\hbar \frac{\partial \varphi^L}{\partial t} = \hat{H}\,\varphi^L \qquad (51)$$

in which the eigenstates of rest energy are again both positive: mc^2

.

The interaction of the fermion with the electromagnetic field is defined by the minimal prescription and is:

$$((E - e\,\varphi)(\hat{E} - e\,\varphi) - c^2\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ )\ \varphi^R = m^2\, c^4 \Phi^R \tag{52}$$

$$((E - e\,\varphi)(\hat{E} - e\,\varphi) - c^2\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ )\ \varphi^L = m^2\, c^4 \Phi^L \tag{53}$$

where $\hat{\boldsymbol{\pi}}$ (not to be confused with π_μ) is defined by:

$$\hat{\boldsymbol{\pi}} = \hat{\boldsymbol{p}} - e\,\boldsymbol{A}\ . \tag{54}$$

Here φ and $\boldsymbol{A}$ are the scalar and vector potentials in S. I, units. More generally, Eqs. (52) and (53) contain the spin connection of ECE theory [1-10]. The spin connection enters into the way the potentials are related to the magnetic flux density ***B*** and electric field strength ***E***. Eqs. (52) and (53) are time dependent Schroedinger equations:

$$\hat{E}\varphi^R = \hat{H}\ \varphi^R \tag{55}$$
$$\hat{E}\varphi^L = \hat{H}\ \varphi^L \tag{56}$$

in which the hamiltonian operator is

$$\hat{H} = \frac{m^2\, c^4}{E - e\,\varphi} + e\,\varphi + \frac{c^2 \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}}{E - e\,\varphi}\quad . \tag{57}$$

Operators that commute with *H* give expectation values that are constants of motion (see ref. [15] and the accompanying note 173(6)). The correctly relativistic Landé and Thomas terms are obtained from:

$$\begin{aligned} \hat{H}_3 &= c^2 \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ (E - e\,\varphi\ )^{-1}\ \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}} \\ &= \frac{c^2}{E}\ \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ (\ 1 - \frac{e\,\varphi}{E}\ )^{-1}\ \boldsymbol{\sigma}.\boldsymbol{\pi} \\ &\backsim \frac{c^2}{E}\ \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ (\ 1 + \frac{e\,\varphi}{E}\ )\ \boldsymbol{\sigma}.\boldsymbol{\pi} \end{aligned} \tag{58}$$

where *E* is defined by Eq. (46), so:

$$\hat{H}_3 = \frac{1}{\gamma m}\ (\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ )\ (\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ ) + \frac{e}{\gamma^2 m^2 c^2}\ \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ \varphi\ \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\ . \tag{59}$$

The Landé (g = 2) factor and the Thomas factor of 2 are obtained from an approximation and under fully relativistic experimental conditions must be described by the rigorous Eq. (59). Write Eq. (52) formally as:

$$(E - e\,\varphi - c^2\, \frac{\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\, \boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}}{\hat{E} - e\,\varphi}\ )\ \varphi^R\ = (\frac{m^2\, c^4}{E - e\,\varphi}\ )\ \varphi^R \tag{60}$$

and add $m\,c^2\ \varphi^R$ to each side to give:

$$(E + m\,c^2 - e\,\varphi - c^2\,\frac{\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\,\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}}{\hat{E} - e\,\varphi})\,\varphi^R = (\frac{m^2\,c^4}{E - e\,\varphi} + m\,c^2)\,\varphi^R \quad . \tag{61}$$

The g = 2 factor and Thomas factor of 2 are obtained in the non-relativistic approximation [12]:

$$\mathrm{E} \backsim m\,c^2 \tag{62}$$

so Eq. (61) becomes:

$$((\,2\,m\,c^2 - e\,\varphi)\,(\hat{E} - e\,\varphi\,) - c^2\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\,\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\,)\,\varphi^R = (m^2c^4 + m\,c^2\,(\hat{E} - e\,\varphi\,))\,\varphi^R$$
$$= (m\,c^2\,(\,2\,m\,c^2 - e\,\varphi\,))\,\varphi^R \tag{63}$$

i.e. the time dependent Schroedinger equation:

$$\hat{E}\varphi^R = \hat{H}\,\varphi^R \tag{64}$$

in which the hamiltonian is the same as that obtained from the Dirac equation and whose first five terms are the well known:

$$\hat{H} = m\,c^2 + e\,\varphi + \frac{1}{2m}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}} + \frac{1}{4m^2c^4}\,\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}}\,\varphi\,\boldsymbol{\sigma}.\hat{\boldsymbol{\pi}} \quad . \tag{65}$$

The great advantages of the fermion equation include the fact that it gives all these well known terms without negative energy and as part of a unified field theory that is generally covariant. This means that the interaction of the fermion with any other fundamental field, or combination of fields, can be developed using the methods of this section. The four fundamental fields: gravitation, electromagnetic, weak and strong nuclear, can be incorporated using the minimal prescription, a procedure that leads to experimentally observable terms similar to those in Eq. (65). The Landé (g = 2) factor from Eq. (65) is given by the operator component:

$$\hat{H}_3\,\varphi^R = \frac{1}{2m}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}}\,\varphi^R \tag{66}$$

and the Thomas factor of 2 appears in the denominator of the spin orbit term:

$$\hat{H}_4\,\varphi^R = \frac{1}{2m}\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}}\,\varphi\,\boldsymbol{\sigma}.\,\hat{\boldsymbol{\pi}}\,\varphi^R \tag{67}$$

Note carefully however that the fully relativistic result is given by Eq. (59) without use of the approximation (62). In the rigorously relativistic result the g factor and Thomas factors are no longer precisely two. The g factor is also affected by the well known radiative corrections which can be incorporated into the fermion equation (1) using the methods of quantum electrodynamics.

4. Fine structure of the H atom.

The solution of Eq. (1) for the H atom is obtained by first expanding it into Eqs. (28) and (29), and then applying the mathematical transformation:

$$\varphi_f^R \longrightarrow \frac{1}{\sqrt{2}}\,(\varphi^R + \varphi^L) \qquad (68)$$

$$\varphi_f^L \longrightarrow \frac{1}{\sqrt{2}}\,(\varphi^R - \varphi^L) \quad . \qquad (69)$$

In Eqs. (68) and (69) the fundamental and physically meaningful eigenfunctions of the fermion equation appear on the left and are denoted by a subscript f to distinguish them from the eigenfunction combinations on the right. The latter are used to obtain the equations:

$$(\, E \; + \mathrm{c}\, \boldsymbol{\sigma}.\widehat{\boldsymbol{p}}\,)\,(\varphi^R - \varphi^L) = \; mc^2\,(\varphi^R + \varphi^L) \qquad (70)$$

$$(\, E \; - \mathrm{c}\, \boldsymbol{\sigma}.\widehat{\boldsymbol{p}}\,)\,(\varphi^R + \varphi^L) = \; mc^2\,(\varphi^R - \varphi^L) \qquad (71)$$

which by addition and subtraction give:

$$(E \; - mc^2)\,\varphi^R = \mathrm{c}\;\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}\;\varphi^L \qquad (72)$$

$$(E \; + mc^2)\,\varphi^L = \mathrm{c}\;\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}\;\varphi^R \quad . \qquad (73)$$

Consider these equations in the presence of a potential φ , which is the Coulomb attraction between the electron and proton of the H atom. The minimal prescription means replacing the operator *E* as follows:

$$E \longrightarrow E - e\,\varphi \quad . \qquad (74)$$

The vector potential ***A*** is considered to be absent for the sake of simplicity. Therefore:

$$\mathrm{c}\;.\;\boldsymbol{\sigma}.\widehat{\boldsymbol{p}}\;\begin{bmatrix}\varphi^L\\ \varphi^R\end{bmatrix} = \begin{bmatrix} E - e\,\varphi - mc^2 & 0 \\ 0 & \mathrm{E} - e\,\varphi + mc^2\end{bmatrix}\begin{bmatrix}\varphi^R\\ \varphi^L\end{bmatrix} \qquad (75)$$

which has a well known solution as described for example by Merzbacher [16]. To obtain this solution the following operator identity is used [16]:

$$\boldsymbol{\sigma}.\widehat{\boldsymbol{p}} = (\boldsymbol{\sigma}.\frac{\boldsymbol{r}}{r})(\frac{\boldsymbol{r}}{r}\,\bullet\,\widehat{\boldsymbol{p}} + i\,\frac{\boldsymbol{\sigma}.\widehat{\boldsymbol{L}}}{r}\,) \qquad (76)$$

where $\widehat{L}$ is the orbital angular momentum operator:

$$\boldsymbol{\sigma}.\widehat{\boldsymbol{L}}\;\;\varphi^R = \pm\,\hbar\,(\,\mathrm{j} + \frac{1}{2} \mp 1\,)\;\varphi^R \qquad (77)$$

$$\boldsymbol{\sigma}.\hat{\boldsymbol{L}}\ \varphi^{L} = \mp \hbar\,(j + \frac{1}{2} \pm 1)\ \varphi^{L}\ . \tag{78}$$

If we denote:

$$k = -(j + \frac{1}{2}) \tag{79}$$

then $\hat{H}$, $\hat{J}^2$, $\hat{J}_Z$ and $\hat{k}$ commute with $\hat{H}$ to give the quantum numbers n_r , j, m_j , and k of the H atom. The eigenfunctions are expanded as:

$$\varphi^{L} = i\, f(r)\, Y^{m_j}_{j l_L} \tag{80}$$

$$\varphi^{R} = g(r)\, Y^{m_j}_{j l_R} \tag{81}$$

and well known methods [16] (see note 173(7)) lead to the energy levels of the H atom:

$$E = mc^2 \left(1 + \frac{Z^2\alpha^2}{(n_r + ((j + \frac{1}{2})^2 - Z^2\alpha^2 j + \frac{1}{2})^{½})^2} \right)^{-½} \tag{82}$$

in which:

$$\frac{e\varphi}{\hbar c} = -\frac{Z\alpha}{r}\ . \tag{83}$$

The result from the Schroedinger equation of the H atom [14] is:

$$E = -\frac{Z^2\alpha^2}{n_r^2}\ . \tag{84}$$

Therefore the fermion equation (1) has been solved to give the fine structure of the H atom, giving the well known result (82) without the use of negative energy, a major advance in theoretical physics. For more complicated atoms and molecules, the highly developed methods of computational quantum chemistry can be applied directly to Eq. (1).

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