A cosmology based exclusively on torsion

M. W. Evans*
Civil List and A.I.A.S.

and

H. Eckardt† and D.W.Lindstrom‡
A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

Abstract

The metric compatibility condition of Riemann geometry is used as the basis of a new method of finding the antisymmetric connection from a given metric. This method shows that the force of gravitation is directly proportional to Riemann torsion, and that all elements of the Riemann curvature vanish. Computer algebra is used to produce examples of antisymmetric connection elements. The Einstein field equation is nowhere used in this new cosmology, based on a rigorously correct geometry with considerations of metric compatibility and the correct Bianchi identity. All cosmology is reduced to spacetime torsion, and spacetime curvature eliminated from the subject.

Keywords: ECE theory, torsion based cosmology

1 Introduction

In recent publications [1]- [10] the use of a symmetric connection in cosmology has been shown to be incorrect geometrically. The commutator method [11], well known in geometry, shows that the symmetric connection implies a symmetric commutator, so that the Riemann curvature and torsion both vanish. The only physically meaningful connection must be antisymmetric in its lower two indices. This means that the spacetime torsion is always non-zero and that the Einstein field equation is incorrect [1]- [10] irretrievably. In Section 2 it is shown that the antisymmetric connection can be obtained straightforwardly using the equation of metric compatibility. For a diagonal metric the

*email: emyrone@aol.com
†email: horsteck@aol.com
‡email: dwlindstrom@gmail.com

connections are obtained very simply by hand calculation. For additional and non-diagonal metrics, computer algebra is used in Section 3 to deduce the connections. Only one equation of metric compatibility is needed. In the early days of tensor analysis three such equations were used in cyclic permutation to give a well known [11] expression for the symmetric connection in terms of metric derivatives, but no account was taken of the commutator of covariant derivatives, which shows that the connection must be antisymmetric. An important result of this paper is that all elements of the Riemann curvature vanish, so that cosmology becomes a subject based exclusively on torsion. This is a major feature of the post Einstein paradigm shift. The antisymmetric connection and the force of universal gravitation are shown to be directly proportional through the rest energy of the attracted particle.

2 The connection from a diagonal metric

The metric compatibility condition is [1]- [11]:

$$D_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} \tag{1}$$

where $g_{\mu\nu}$ is the symmetric metric and $\Gamma^\lambda_{\rho\mu}$ the Christoffel connections. Consider a metric in cylindrical polar coordinates in a plane of the type:

$$g_{\mu\nu} = \begin{bmatrix} 1-\frac{r_0}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{r_0}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\vartheta \end{bmatrix} \tag{2}$$

where:

$$r_0 = \frac{2MG}{c^2}. \tag{3}$$

Here M is the mass of the attracting object, c the vacuum speed of light and G the Newton constant. This metric is one possible solution of the ECE Orbital Theorem of UFT 111 (www.aias.us). It gives an accurate description of the relativistic Kepler problem [12].

For

$$\mu = \nu = 0 \tag{4}$$

Eq. (1) becomes:

$$\partial_1 g_{00} - \Gamma^0_{10}\, g_{00} - \Gamma^0_{10}\, g_{00} = 0 \tag{5}$$

so

$$\Gamma^0_{10} = -\Gamma^0_{01} = \frac{r_0}{2r^2(1-\frac{r_0}{r})}. \tag{6}$$

It is already clear that the connection can be obtained from the metric very easily. Yet this fact has been overlooked for 110 years. Similarly the non vanishing connections are:

$$\Gamma^1_{11} = \frac{1}{2g_{11}} \partial_1 g_{11}, \tag{7}$$

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r},\ \Gamma^3_{23} = \cot\theta. \tag{8}$$

Table 1: Non-Zero Connection and Torsion Elements of Metric (2).

Antisymmetric Connection	Torsion
$\Gamma^0_{10} = -\Gamma^0_{01} = \frac{r_0}{2r(r-r_0)}$	$T^0{}_{10} = 2\Gamma^0_{10}$
$\Gamma^2_{12} = -\Gamma^2_{21} = \frac{1}{r}$	$T^2{}_{12} = -T^2{}_{21} = 2\Gamma^2_{12}$
$\Gamma^3_{13} = -\Gamma^3_{31} = \frac{1}{r}$	$T^3{}_{13} = 2\Gamma^3_{13}$
$\Gamma^3_{23} = -\Gamma^3_{32} = \cot\theta$	$T^3{}_{23} = 2\Gamma^3_{23}$

It is seen that there is one symmetric connection Γ^1_{11}, but as follows this connection results in zero torsion and curvature, so plays no part in universal gravitation. Consider the well known commutator equation of geometry [11]:

$$[D_\mu, D_\nu]V^\rho = -(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})D_\lambda V^\rho + R^\rho{}_{\sigma\mu\nu} \tag{9}$$

where D_μ is the covariant derivative, V^ρ a vector of any spacetime of any dimension, and where the Riemann torsion and curvature are defined respectively by:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{10}$$

and

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{11}$$

It is seen immediately from Eq. (9) that when:

$$\mu = \nu = 1 \tag{12}$$

the commutator, symmetric connection, torsion and curvature all vanish. It follows that the equation of metric compatibility (1) applies only to antisymmetric connections, i.e. the connection of any spacetime of any dimension is antisymmetric. Otherwise there is a contradiction between two fundamental equations (1) and (9).

From Eq. (6) it is seen that in the limit:

$$r >> r_0 \tag{13}$$

the inverse square law of gravitation is given by:

$$F = -\frac{mMG}{r^2} = -mc^2\Gamma^0_{10} \tag{14}$$

so universal gravitation is due exclusively to torsion. This is a central feature of the post Einstein paradigm shift. Table 1 summarises the results of this section. All elements of curvature vanish. Curvature plays no role in universal gravitation, a second major feature of the post Einstein paradigm shift.

3 Results for various diagonal metrics

In this section we discuss three examples of metrics and then specify the general solution method for diagonal metrics.

3.1 Examples for anatisymmetric connections

Three additional metrics have been analyzed by computer algebra:

- Crothers metric with generalized Schwarzschild parameters,
- general spherical metric,
- spherically symmetric metric with perturbation.

These are derivable from the theorem of orbits of UFT paper 111 and therefore do not rest upon the obsolete Einstein equation. A detailed analysis with conventional (symmetric) Christoffel connections is given in [1]. In the following we give the results for the antisymmetric Christoffel connections. Two independent computer algebra systems were used to ensure correctness of the results. For for the **Crothers metric with Schwarzschild parameters** these are:

$$g_{\mu\nu} = \begin{bmatrix} -\sqrt{(|r_0-r|^n+\alpha^n)^{\frac{2}{n}}}\,A & 0 & 0 & 0 \\ 0 & \sqrt{(|r_0-r|^n+\alpha^n)^{\frac{2}{n}}}\,B & 0 & 0 \\ 0 & 0 & (|r_0-r|^n+\alpha^n)^{\frac{2}{n}} & 0 \\ 0 & 0 & 0 & (|r_0-r|^n+\alpha^n)^{\frac{2}{n}}\sin^2\vartheta \end{bmatrix} \tag{15}$$

$$\Gamma^0{}_{10} = -\frac{|r_0-r|^n}{2\,(r_0-r)\,(|r_0-r|^n+\alpha^n)} \tag{16}$$

$$\Gamma^1{}_{11} = -\frac{|r_0-r|^n}{2\,(r_0-r)\,(|r_0-r|^n+\alpha^n)} \tag{17}$$

$$\Gamma^2{}_{12} = -\frac{|r_0-r|^n}{(r_0-r)\,(|r_0-r|^n+\alpha^n)} \tag{18}$$

$$\Gamma^3{}_{13} = -\frac{|r_0-r|^n}{(r_0-r)\,(|r_0-r|^n+\alpha^n)} \tag{19}$$

$$\Gamma^3{}_{23} = \frac{\cos\vartheta}{\sin\vartheta} \tag{20}$$

The results for the **general spherical metric** are as follows. α and β are functions of r and t.

$$g_{\mu\nu} = \begin{bmatrix} -e^{2\,\alpha} & 0 & 0 & 0 \\ 0 & e^{2\,\beta} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\vartheta \end{bmatrix} \tag{21}$$

$$\Gamma^0{}_{00} = \frac{d}{d\,t}\,\alpha \tag{22}$$

$$\Gamma^0{}_{10} = \frac{d}{d\,r}\,\alpha \tag{23}$$

$$\Gamma^1{}_{01} = \frac{d}{d\,t}\,\beta \tag{24}$$

$$\Gamma^1{}_{11} = \frac{d}{d\,r}\,\beta \tag{25}$$

$$\Gamma^2{}_{12} = \frac{1}{r} \tag{26}$$

$$\Gamma^3{}_{13} = \frac{1}{r} \tag{27}$$

$$\Gamma^3{}_{23} = \frac{\cos\vartheta}{\sin\vartheta} \tag{28}$$

The results for the **spherically symmetric metric with perturbation** a/r (a being a constant) are:

$$g_{\mu\nu} = \begin{bmatrix} -\frac{\frac{a}{r}+r_0}{r} - 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\frac{\frac{a}{r}+r_0}{r}+1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\sin^2\vartheta \end{bmatrix} \tag{29}$$

$$\Gamma^0{}_{10} = -\frac{r_0\,r + 2\,a}{2\,r\,\left(r^2 + r_0\,r + a\right)} \tag{30}$$

$$\Gamma^1{}_{11} = \frac{r_0\,r + 2\,a}{2\,r\,\left(r^2 + r_0\,r + a\right)} \tag{31}$$

$$\Gamma^2{}_{12} = \frac{1}{r} \tag{32}$$

$$\Gamma^3{}_{13} = \frac{1}{r} \tag{33}$$

$$\Gamma^3{}_{23} = \frac{\cos\vartheta}{\sin\vartheta} \tag{34}$$

In some cases symmetric Christoffel symbols appear, these have to be discarded for torsion as described in the first section. In all cases the Riemann tensor is zero. This is an important difference to the results for the symmetric connections listed in [1]. Torsion alone is responsible for gravitation.

3.2 The general solution for a diagonal metric

The equation for metric compatibility (1) has three independent indices, this gives $4^3 = 64$ equations in combination. In case of a diagonal metric the number of index conbinations on the left hand side is reduced to 16, but at the right hand side metric elements other than $g_{\mu\mu}$ appear so that there are terms also for $\mu \neq \nu$. In the latter case linear equations arise. The coordinates are denoted by x_0 ... x_3 here. For example the second five out of the 64 equations read:

$$Eq.(6) : 2\,\Gamma^1_{01}\, g_{11} = \frac{\partial}{\partial\, x_0}\, g_{11} \tag{35}$$

$$Eq.(7) : \Gamma^2_{01}\, g_{22} + \Gamma^1_{02}\, g_{11} = 0 \tag{36}$$

$$Eq.(8) : \Gamma^3_{01}\, g_{33} + \Gamma^1_{03}\, g_{11} = 0 \tag{37}$$

$$Eq.(9) : \Gamma^2_{00}\, g_{22} + \Gamma^0_{02}\, g_{00} = 0 \tag{38}$$

$$Eq.(10) : \Gamma^2_{01}\, g_{22} + \Gamma^1_{02}\, g_{11} = 0 \tag{39}$$

As has been concluded from the examples for gravitation, the most important Christoffel connections follow from the case $\mu = \nu$. Then the following 16 equations appear:

$$Eq.(1) :\ 2\,\Gamma^0_{00}\, g_{00} = \frac{\partial}{\partial\, x_0}\, g_{00} \tag{40}$$

$$Eq.(2) :\ 2\,\Gamma^1_{01}\, g_{11} = \frac{\partial}{\partial\, x_0}\, g_{11} \tag{41}$$

$$Eq.(3) :\ 2\,\Gamma^2_{02}\, g_{22} = \frac{\partial}{\partial\, x_0}\, g_{22} \tag{42}$$

$$Eq.(4) :\ 2\,\Gamma^3_{03}\, g_{33} = \frac{\partial}{\partial\, x_0}\, g_{33} \tag{43}$$

$$Eq.(5) :\ 2\,\Gamma^0_{10}\, g_{00} = \frac{\partial}{\partial\, x_1}\, g_{00} \tag{44}$$

$$Eq.(6) :\ 2\,\Gamma^1_{11}\, g_{11} = \frac{\partial}{\partial\, x_1}\, g_{11} \tag{45}$$

$$Eq.(7) :\ 2\,\Gamma^2_{12}\, g_{22} = \frac{\partial}{\partial\, x_1}\, g_{22} \tag{46}$$

$$Eq.(8) :\ 2\,\Gamma^3_{13}\, g_{33} = \frac{\partial}{\partial\, x_1}\, g_{33} \tag{47}$$

$$Eq.(9) :\ 2\,\Gamma^0_{20}\, g_{00} = \frac{\partial}{\partial\, x_2}\, g_{00} \tag{48}$$

$$Eq.(10) :\ 2\,\Gamma^1_{21}\, g_{11} = \frac{\partial}{\partial\, x_2}\, g_{11} \tag{49}$$

$$Eq.(11) :\ 2\,\Gamma^2_{22}\, g_{22} = \frac{\partial}{\partial\, x_2}\, g_{22} \tag{50}$$

$$Eq.(12) :\ 2\,\Gamma^3_{23}\, g_{33} = \frac{\partial}{\partial\, x_2}\, g_{33} \tag{51}$$

$$Eq.(13): \ 2\,\Gamma^0_{30}\,g_{00} = \frac{\partial}{\partial\,x_3}\,g_{00} \tag{52}$$

$$Eq.(14): \ 2\,\Gamma^1_{31}\,g_{11} = \frac{\partial}{\partial\,x_3}\,g_{11} \tag{53}$$

$$Eq.(15): \ 2\,\Gamma^2_{32}\,g_{22} = \frac{\partial}{\partial\,x_3}\,g_{22} \tag{54}$$

$$Eq.(16): \ 2\,\Gamma^3_{33}\,g_{33} = \frac{\partial}{\partial\,x_3}\,g_{33} \tag{55}$$

3.3 The general solution for non-diagonal metrics

As soon as non-diagonal elements in the metric are present, the equation set becomes much more complex. Numerical methods for solution have to be applied then. The resulting equation set can be computed by computer algebra. We did this in the simplest case, adding two non-diagonal elements

$$g_{01} = g_{10}. \tag{56}$$

This already increases the complexity significantly. The five exemplary equations (35)-(39) take the form

$$Eq.(6): 2\,\Gamma^1_{01}\,g_{11} + \Gamma^0_{01}\,g_{10} + \Gamma^0_{01}\,g_{01} = \frac{\partial}{\partial\,x_0}\,g_{11} \tag{57}$$

$$Eq.(7): \Gamma^2_{01}\,g_{22} + \Gamma^1_{02}\,g_{11} + \Gamma^0_{02}\,g_{10} = 0 \tag{58}$$

$$Eq.(8): \Gamma^3_{01}\,g_{33} + \Gamma^1_{03}\,g_{11} + \Gamma^0_{03}\,g_{10} = 0 \tag{59}$$

$$Eq.(9): \Gamma^2_{00}\,g_{22} + \Gamma^1_{02}\,g_{10} + \Gamma^0_{02}\,g_{00} = 0 \tag{60}$$

$$Eq.(10): \Gamma^2_{01}\,g_{22} + \Gamma^1_{02}\,g_{11} + \Gamma^0_{02}\,g_{01} = 0 \tag{61}$$

The rest of the equation set looks similar.

Acknowledgments

The British Government is thanked for a Civil List Pension and the staff of AIAS for many interesting discussions. Dave Burleigh is thanked for posting, Alex Hill for translation work, and Simon Clifford and Robert Cheshire for broadcasting help.

References

[1] M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation". (Cambridge International Science Publishing, 2011).

[2] M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis, 2005 onwards), in seven volumes.

[3] Kerry Pendergast, "The Life of Myron Evans" (Cambridge International Science Publishing, 2011).

[4] M .W. Evans, ed., Journal of Foundations of Physics and Chemistry (Cambridge International Science Publishing, June 2011 onwards).

[5] The ECE sites: www.webarchive.org.uk, www.aias.us, www.atomicprecision.com, www.upitec.org, www.et3m.net.

[6] L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, 2007).

[7] M. W. Evans, ed., "Modern Nonlinear Optics" (Wiley 2001, 2nd edition) in three volumes; ibid., M. W. Evans and S. Kielich (eds.), (Wiley, 1992, 1993, 1997, 1st edition), in three volumes.

[8] M . W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

[9] M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002), in ten volumes hardback and softback.

[10] M . W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).

[11] S. P. Carroll, "Spacetime and Geometry: an Introduction to General Relativity" (Addison Wesley, New York, 2004).

[12] J. B. Marion and S. T. Thornton, "Classical Dynamics" (HB College Publishing, New York, 1988, 3rd. Edition).