

Central force fields described by m theory, Part I

Horst Eckardt¹,
A.I.A.S. and UPITEC
(www.aias.us, www.upitec.org)
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Abstract

m theory was introduced to replace Einstein's erroneous field equation. The new theory was applied in the macroscopic realm to problems of gravitation by using Lagrange theory. In this paper, we extend the underlying formalism to the full range of Cartan geometry, obtaining all internal quantities like spin connections for a given potential. The results are valid for both electromagnetic and gravitational central structures. We obtain a new central force that stems from the geometric structure of spacetime itself. In addition, a rotational field appears, although there are no rotational parts in the potential. The approach of m theory, which is based on the line element of general relativistic spacetime, can be generalized without essential changes to the results.

Keywords: Unified field theory; m theory; central symmetry; gravitation; electromagnetism.

1 Introduction

ECE theory is the physical interpretation of Cartan geometry [1–5]. It has been under development since 2003 for use in many fields of physics. It replaces Einstein's theory of general relativity, which is restricted to curvature, by introducing torsion into theoretical physics. Torsion was inferred by Cartan as a completion of Riemann geometry, which only contains curvature. Torsion cannot be neglected, because curvature is always inter-connected with torsion, and setting torsion to zero leads to contradictions. Therefore, Einstein's field equation is no longer tenable and can only be considered as an approximation in weak fields, where it flows into ECE theory under certain preconditions [7].

What has retained its value, however, is the metric of spacetime, which describes the curvilinear coordinates that are found both in Einstein's general theory of relativity and in ECE theory. m theory [5] was derived from this metric and describes the distortion of the coordinates in a centrosymmetric geometry. In the series of UFT papers, it has been applied to cosmology and the quantum world. Through m theory, the unification of the quantum theory with general relativity was achieved [6].

¹email: mail@horst-eckardt.de

In [5], the metric of m theory has been combined with Lagrangian mechanics. Valuable results in the field of relativistic mechanics have been obtained from this approach. In particular, the deflection of light by heavy celestial objects could now be described correctly in a parameter-free manner. In this paper, we extend the possible uses of m theory. In order to cover the complete range of physics described by Cartan geometry, we fully integrate m theory into this geometry. To accomplish this, the metric must first be expressed by the Cartan tetrad. This will allow the full range of physics covered by Cartan geometry to be calculated, from the potentials to the Christoffel symbols and spin connections to the force fields of mechanics and electrodynamics.

It has already been shown in [5] that the diagonal metric of m theory can be converted into a diagonal tetrad structure. To this tetrad we then apply the complete calculation mechanism of Cartan geometry, as described in [5] and [8]. This leads to interesting insights when, for example, the m function goes to zero at an event horizon or at the center of the coordinate system. The function $m(r)$ describes a radial space density or aether density, and a significant decrease in this density leads to unique effects.

2 Connection of the general relativistic line element with Cartan geometry

Within this paper, we will use the metric of a non-constant, centrally symmetric spacetime that is different from Minkowski space. We will base our development on a metric that is common in Einstein theory, but we will develop our method within ECE2 theory, i.e., Cartan geometry. Nonetheless, this a development of “true general relativity”, even in the sense of standard physics.

According to Section 2.1.3 of [5], the squared line element in a space with curvature and torsion is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (1)$$

where $g_{\mu\nu}$ is the symmetric metric and dx^μ is the differential of the spacetime coordinate x^μ . In a Minkowski space for a spherically symmetric spacetime, this takes the form

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2 \quad (2)$$

with a time coordinate $x_0 = ct$, radius coordinate r , polar angle θ and azimuthal angle ϕ . In a general spherically symmetric spacetime with torsion and curvature, the line element has to be generalized as described in Chapter 7 of [9]:

$$ds^2 = c^2 m(r, t) dt^2 - n(r, t) dr^2 - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2. \quad (3)$$

$m(r, t)$ and $n(r, t)$ are general functions describing the distortion of spacetime by a central point mass at $r = 0$. Only the radial and time coordinates are affected. The angular parts remain unchanged because of the rotational symmetry. It was shown in [9] that the line element can be simplified further by the replacement

$$n(r, t) = \frac{1}{m(r, t)}, \quad (4)$$

and that the time dependence of $m(r, t)$ can be rolled over to the time coordinate. Therefore, the simplified line element reads

$$ds^2 = c^2 m(r) dt^2 - \frac{dr^2}{m(r)} - r^2 d\theta^2 - r^2 \sin(\theta)^2 d\phi^2. \quad (5)$$

This form is used in Einstein's field equation, in which the Ricci tensor is zero and an expression is derived for $m(r)$ in the vacuum, leading to the Schwarzschild metric. In ECE2 theory, we use this form for simplicity, but we can freely define the function $m(r)$. Comparing Eq. (1) with Eq. (5), it follows that the metric is diagonal and the metric coefficients are

$$g_{00} = m(r), \quad g_{11} = -\frac{1}{m(r)}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin(\theta)^2. \quad (6)$$

This metric-based theory, which we have called *m theory* [5], can also be developed from Cartan geometry itself. In Cartan geometry, the basis element is the tetrad, and the metric follows from the tetrad [5] by

$$g_{\mu\nu} = n q^a{}_\mu q^b{}_\nu \eta_{ab}, \quad (7)$$

where $q^a{}_\mu$ are the tetrad elements, η_{ab} is the Minkowski metric of tangent space, and $n = 4$ is the dimension of the base manifold. The metric does not generally allow the tetrad to be determined uniquely. In this case, however, the metric is diagonal. We can assume that the tetrad matrix is diagonal also, because we do not consider specific polarization effects of Cartan geometry. Therefore, Eq. (7) reduces to the diagonal elements in both the base manifold and tangent space, and we obtain

$$q^{(0)}_0 = \frac{1}{2} \sqrt{m(r)}, \quad q^{(1)}_1 = \frac{1}{2\sqrt{m(r)}}, \quad q^{(2)}_2 = \frac{r}{2}, \quad q^{(3)}_3 = \frac{r \sin(\theta)}{2}. \quad (8)$$

This is the connection of *m theory* to Cartan geometry.

The position vector in *m* space is

$$\mathbf{r} = \frac{r}{m(r)^{1/2}} \mathbf{e}_r, \quad (9)$$

and the velocity in *m* space with two spatial dimensions is

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{1}{m(r)^{1/2}} \left(\dot{r} \mathbf{e}_r + r \dot{\phi} \mathbf{e}_\phi \right). \quad (10)$$

We can use the variable name

$$\mathbf{r}_1 = r_1 \mathbf{e}_r = \frac{r}{m(r)^{1/2}} \mathbf{e}_r, \quad (11)$$

so that the velocity becomes

$$\mathbf{v}_1 = \dot{\mathbf{r}}_1 = \dot{r}_1 \mathbf{e}_r + r_1 \dot{\phi} \mathbf{e}_\phi. \quad (12)$$

A new time variable can be defined by

$$t_1 = m(r)^{1/2} t. \quad (13)$$

\mathbf{r}_1 and t_1 are the characteristic variables of m space.

From the line element (5) of m space, it follows that

$$ds^2 = c^2 m(r) dt^2 - \left(\frac{d\mathbf{r}_1}{dt} \right)^2 dt^2 = c^2 dt_1^2 - \mathbf{v}_1^2 dt^2. \quad (14)$$

In plane polar coordinates related to the observer space, this becomes

$$\begin{aligned} ds^2 &= c^2 \left(m(r) - \frac{\dot{r}^2 + r^2 \dot{\phi}^2}{m(r)c^2} \right) dt^2 \\ &= \frac{c^2 dt^2}{\gamma^2}. \end{aligned} \quad (15)$$

Thus, the general relativistic γ factor of m space is defined by

$$\gamma = \left(m(r) - \frac{\dot{r}^2 + r^2 \dot{\phi}^2}{m(r)c^2} \right)^{-1/2}. \quad (16)$$

The linear momentum of m space is

$$\mathbf{p}_1 = \gamma m \mathbf{v}_1 = \gamma m \frac{\mathbf{v}}{m(r)^{1/2}}. \quad (17)$$

3 Computational basis and examples

In [8], it has been shown how all Christoffel symbols, spin connections, and curvature and torsion tensors are derived from the Cartan tetrad. Here, we only repeat the variable names and what they represent:

- $\Gamma^\rho_{\mu\nu}$: Christoffel connection
- $\omega^a_{\mu b}$: spin connection
- $R^\lambda_{\rho\mu\nu}$: Riemann (curvature) tensor
- $T^\lambda_{\mu\nu}$: torsion tensor
- $R^a_{b\mu\nu}$: curvature form
- $T^a_{\mu\nu}$: torsion form
- \mathbf{E}^a : electric field
- \mathbf{B}^a : magnetic field
- $\Lambda^\lambda_{\mu\nu}$: dual Christoffel connection
- $\omega_{(\Lambda)}^a{}_{\mu b}$: dual spin connection

According to Eq. (8), the tetrad matrix is

$$(q^a_\mu) = \begin{bmatrix} \frac{1}{2} \sqrt{m(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{m(r)}} & 0 & 0 \\ 0 & 0 & \frac{r}{2} & 0 \\ 0 & 0 & 0 & \frac{r \sin(\theta)}{2} \end{bmatrix}, \quad (18)$$

and from this the metric follows:

$$(g_{\mu\nu}) = \begin{bmatrix} m(r) & 0 & 0 & 0 \\ 0 & -\frac{1}{m(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}. \quad (19)$$

The determinant of the metric is

$$\det(g_{\mu\nu}) = -\frac{1}{r^4 \sin(\theta)^2}. \quad (20)$$

Obviously, it is independent of the m function.

The execution of the computer algebra code developed in [8] gives all curvature/torsion parameters and connections. More of them are zero than non-zero, because the tetrad is diagonal. Some non-vanishing results are

$$\Gamma^0_{01} = -\Gamma^0_{10} = -\frac{\frac{dm(r)}{dr}}{2m(r)} \quad (21)$$

$$\Gamma^2_{12} = -\Gamma^2_{21} = \frac{1}{r} \quad (22)$$

$$\omega^{(0)}_{0(1)} = \omega^{(1)}_{0(0)} = -\frac{\frac{dm(r)}{dr}}{2} \quad (23)$$

$$\omega^{(1)}_{2(2)} = -\omega^{(2)}_{2(1)} = \sqrt{m(r)} \quad (24)$$

$$\Lambda^2_{03} = -\Lambda^3_{02} = \frac{1}{r} \quad (25)$$

$$\omega_{(\Lambda)}^{(2)}{}_{1(2)} = -\frac{1}{r} \quad (26)$$

$$R^0_{202} = -R^0_{220} = -\frac{r \frac{dm(r)}{dr}}{2} \quad (27)$$

$$T^0_{01} = -T^0_{10} = -\frac{\frac{dm(r)}{dr}}{m(r)} \quad (28)$$

$$R^{(0)}_{202} = -R^{(0)}_{220} = -\frac{\sqrt{m(r)} \frac{dm(r)}{dr}}{2} \quad (29)$$

$$T^{(0)}_{01} = -T^{(0)}_{10} = -\frac{\frac{dm(r)}{dr}}{2\sqrt{m(r)}} \quad (30)$$

The resulting electric and magnetic force fields for the four polarization directions are

$$\mathbf{E}^{(0)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{dm(r)}{dr}}{2\sqrt{m(r)}} \\ 0 \\ 0 \end{bmatrix}, \quad (31)$$

$$\mathbf{E}^{(1)} = \mathbf{E}^{(2)} = \mathbf{E}^{(3)} = \mathbf{0}, \quad (32)$$

$$\mathbf{B}^{(0)} = \mathbf{B}^{(1)} = \mathbf{0}, \quad (33)$$

$$\mathbf{B}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ -B_0 \end{bmatrix}, \quad \mathbf{B}^{(3)} = \begin{bmatrix} -C_0 r \cos \theta \\ B_0 \sin \theta \\ 0 \end{bmatrix} \quad (34)$$

with constants A_0 , B_0 and C_0 . A_0 is the primordial vector potential $A^{(0)}$ of ECE theory, B_0 is a magnetic field constant and C_0 a constant with units Tesla/m. Averaging the polarizations allows us to write

$$\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \mathbf{E}^{(3)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{dm(r)}{dr} \\ \sqrt{m(r)} \\ 0 \\ 0 \end{bmatrix}, \quad (35)$$

$$\mathbf{B} = \mathbf{B}^{(0)} + \mathbf{B}^{(1)} + \mathbf{B}^{(2)} + \mathbf{B}^{(3)} = \begin{bmatrix} -C_0 r \cos \theta \\ B_0 \sin \theta \\ -B_0 \end{bmatrix}. \quad (36)$$

For gravitation, the \mathbf{E} field corresponds to the gravitational field \mathbf{g} , and the \mathbf{B} field corresponds to the gravitomagnetic field $\boldsymbol{\Omega}$. The \mathbf{E} field has a radial component only, while the \mathbf{B} field has r , θ and ϕ components.

To show these fields graphically in a centrally symmetric geometry, we use the function $m(r)$ that we have used earlier [5]:

$$m(r) = 2 - \exp\left(\log(2) \exp\left(-\frac{r}{R}\right)\right) \quad (37)$$

with a radial range R . This function, its derivative and the field component E_r are graphed in Fig. 1 with all constants set to unity. We have $m(r) \rightarrow 0$ for $r \rightarrow 0$. However, the derivative of $m(r)$ goes to a final limit. Consequently, the electric (or gravitational) field diverges when r approaches zero. This behavior has already been identified in [5] as a “vacuum force” that appears by the radial variation of $m(r)$. Such a force does not exist in classical physics and is a consequence of general relativity based on Cartan geometry. It is attractive and this means that matter is pulled into the center as soon as it comes into the region where $m(r)$ deviates from unity. In a sense, this may be interpreted as the ECE version of “black holes” that are otherwise derived from Einsteinian general relativity in a mathematically incorrect way.

Another interesting result is that a rotational magnetic (or gravitomagnetic) field exists. This is surprising, because the tetrad, which corresponds to the potential, has no rotational components. This field is graphed in Fig. 2, which shows the field vectors on two spheres, whose back side is not shown in order to not obscure visibility. The figure shows a twist in each sphere that, in principle, represents a Torkado structure as was discussed in Example 8.15 of [5]. The Torkado has an additional back-path at its central axis that does not appear in our example, but may also be present in a more customized geometry.

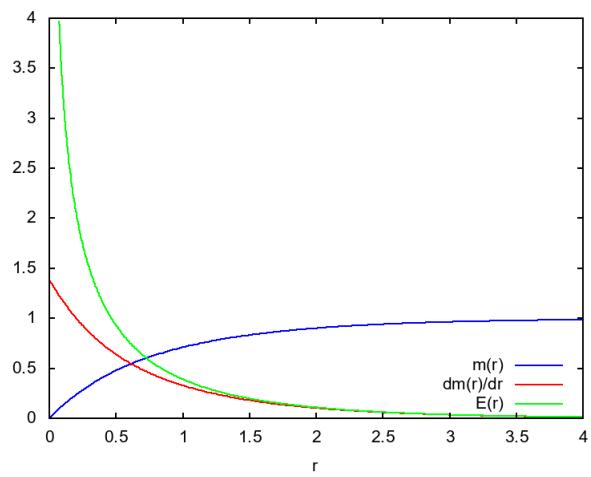


Figure 1: $m(r)$, $dm(r)/dr$ and $E_r(r)$ for the model function $m(r)$.

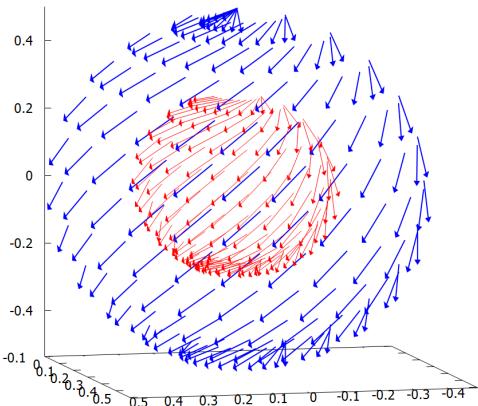


Figure 2: 3D representation of $\mathbf{B}(\mathbf{r})$.

4 Generalizations for spherical symmetry

We have been using the line element (5) of spherical symmetry, within which the approximation

$$n(r) \approx \frac{1}{m(r)} \quad (38)$$

was made for the function $n(r)$ appearing in the more basic line element (3). Using both functions, the tetrad (18) reads

$$(q^a_\mu) = \begin{bmatrix} \frac{1}{2}\sqrt{m(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{n(r)} & 0 & 0 \\ 0 & 0 & \frac{r}{2} & 0 \\ 0 & 0 & 0 & \frac{r\sin(\theta)}{2} \end{bmatrix}. \quad (39)$$

Evaluation by computer algebra gives the same results (31-34) for the force fields. Only the curvature and torsion parameters are affected. This means that Eq. (37) is not an approximation but an exact simplification of the line element. Another modification was to avoid the time dependence of the m function by rolling it over to the time coordinate. If we allow explicit time dependences in the form of $m(r, t)$ and $n(r, t)$, we obtain the original \mathbf{E} field $\mathbf{E}^{(0)}$ plus an additional field $\mathbf{E}^{(1)}$:

$$\mathbf{E}^{(0)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{dm(r,t)}{dr}}{\sqrt{m(r,t)}} \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{E}^{(1)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{dn(r,t)}{dt}}{\sqrt{n(r,t)}} \\ 0 \\ 0 \end{bmatrix}. \quad (40)$$

The result for $\mathbf{E}^{(0)}$ is not changed, but $\mathbf{E}^{(1)}$ contains the time derivative of $n(r)$ instead of the radial derivative of $m(r)$. In the case of

$$n(r, t) = \frac{1}{m(r, t)}, \quad (41)$$

we obtain for the second \mathbf{E} field polarization:

$$\mathbf{E}^{(1)} = \frac{A_0 c}{2} \begin{bmatrix} -\frac{\frac{dm(r,t)}{dt}}{(m(r,t)^{3/2})} \\ 0 \\ 0 \end{bmatrix}. \quad (42)$$

If only m depends on time but not n , the original result of Eqs. (31, 32) follows. We conclude that only a time dependence of the form $n(r, t)$ leads to an additional \mathbf{E} field.

We can experiment further by introducing non-diagonal terms in the tetrad. We have found that filling the first row or first column by elements different from zero is a very critical choice, because the equation system for solving the Christoffel symbols Γ then gives no solutions, in many cases. This means that couplings between time and space cannot be chosen arbitrarily and require close attention. Obviously, there are also physical restrictions to realizing such couplings.

In summary, we have found a strong vacuum force in spherical symmetry. An additional rotational structure appears as a magnetic or gravitomagnetic field, and the approximations in Einstein's theory with respect to the functions $m(r)$ and $n(r)$ are justified.

References

- [1] M. W. Evans, “Generally Covariant Unified Field Theory: The Geometrization of Physics”, Vols. 1 to 7, Abramis Academic, Bury St Edmunds, 2005 ff.
- [2] M. W. Evans, H. Eckardt (editor), D. W. Lindstrom, S. J. Crothers; “Principles of ECE Theory, Volume I: A New Paradigm of Physics”, New Generation Publishing, London, 2016, and ePubli, Berlin, 2019.
- [3] M. W. Evans, H. Eckardt (editor), D. W. Lindstrom, S. J. Crothers, U. Bruchholz “Principles of ECE Theory, Volume II: Closing the Gap between Experiment and Theory”, softcover and hardcover: ePubli Berlin, 2017.
- [4] M. W. Evans, S. J. Crothers, H. Eckardt, K. Pendergast, “Criticisms of the Einstein Field Equation - The End of the 20th Century Physics”, Cambridge Internetational Science Publishing, Great Abington, 2011.
- [5] H. Eckardt, “ECE UFT - The Geometrical Basis of Physics, Vol. I - Classical Physics”, textbook, epubli, Berlin, 2022, freely available as UFT Paper 438, Unified Field Theory (UFT) section of www.aias.us.
- [6] H. Eckardt, “ECE UFT - The Geometrical Basis of Physics, Vol. II - Quantum Physics”, textbook, in preparation, UFT Paper 448, Unified Field Theory (UFT) section of www.aias.us.
- [7] D. W. Lindstrom, H. Eckardt, M. W. Evans, “Ramifications of a Totally Antisymmetric Torsion Tensor I - A Bridge to Einstein’s Theory of General Relativity”, UFT Paper 445, Unified Field Theory (UFT) section of www.aias.us.
- [8] H. Eckardt, “The full path of calculation through Cartan geometry”, UFT Paper 439, Unified Field Theory (UFT) section of www.aias.us.
- [9] S. Carroll, “Spacetime and Geometry: Introduction to General Relativity” (Pearson Education Limited, 2014); manuscript online: <https://arxiv.org/pdf/gr-qc/9712019.pdf>.