CHAPTER 4: PHOTON MASS AND THE B(3) FIELD.

4.1 INTRODUCTION

The B(3) field was inferred in November 1991 \{1 - 10\} from a consideration of the conjugate product of nonlinear optics in the inverse Faraday effect. In physics before the great paradigm shift of ECE theory the conjugate product was thought to exist in free space only in a plane of two dimensions. This was absurd dogma necessitated by the need for a massless photon and the U(1) gauge invariance of the old theory \{13\}. The lagrangian had to be invariant under a certain type of gauge transformation. Therefore there could be no longitudinal components of the free electromagnetic field, meaning that the vector cross product known as the conjugate product could have no longitudinal component in free space, but as soon as it interacted with matter it produced an experimentally observable longitudinal magnetization. In retrospect this is grossly absurd, it defies basic geometry, the basic definition of the vector cross product in three dimensional space, or the space part of four dimensional spacetime.

The first papers on B(3) appeared in Physica B in 1992 and 1993 and can be seen in the Omnia Opera of www.aias.us. The discovery of B(3) was not immediately realized to be linked to the mass of the photon, an idea that goes back to the corpuscular theory of Newton and earlier. It was revived by Einstein as he developed the old quantum theory and special relativity, and with the inference of wave particle duality it became part of de Broglie’s school of thought in the Institut Henri Poincare in Paris. Members of this school included Proca and Vigier, whose life work was dedicated largely to the theory of photon mass and a type of quantum mechanics that rejected the Copenhagen indeterminacy. This is usually known as
causal or determinist quantum mechanics. The ECE theory has clearly refuted indeterminacy in favour of causal determinism, because ECE has shown that essentially all the valid equations of physics have their origin in geometry. Indeterminism asserts that some aspects of nature are absolutely unknowable, and that there is no cause to an effect, and that a particle for example can do anything it likes, go forward or backward in time. To the causal determinists this is absurd and anti Baconian dogma, so they have rejected it since it was proposed, about ninety years ago. This was the first great schism in physics. The second great schism follows the emergence of ECE theory, which has split physics into dogma (the standard model) and a perfectly logical development based on geometry (ECE theory). Every effect has a cause, and the wave equations of physics are derived from geometry in a rigorously logical manner.

Many aspects of the standard model have been refuted with astonishing ease. This suggests that the standard model was “not even wrong” in the words of Pauli, it was a plethora of ridiculous abstraction that could never be tested experimentally and which very few could understand. This plethora of nonsense is blasted out over the media as propaganda, doing immense harm to Baconian science. This book tries to redress some of that harm.

Vigier immediately accepted the B(3) field and in late 1992 suggested in a letter to M. W. Evans, the discoverer of B(3), that it implied photon mass because it was an experimentally observable longitudinal component of the free field and so refuted the dogma of U(1) gauge transformation. Vigier was well aware of the fact that the Proca lagrangian is not U(1) gauge invariant because of photon mass, and by 1992 had developed the subject in many directions. The subject of photon mass was as highly developed as anything in the standard physics. The two types of physics developed side by side, one being as valid as the other, but one (the standard model) being much better known. The de Broglie School of Thought was of course well known to Einstein, who invited Vigier to become his assistant, so
by implication Einstein favoured the determinist school of quantum mechanics as is well known. So did Schrödinger, who worked on photon mass for many years. One of Schrödinger's last papers, with Bass, is on photon mass, from the Dublin Institute for Advanced Studies in the mid fifties. So by implication, Einstein, de Broglie and Schrödinger all rejected the standard model's U(1) gauge invariance, so they would have rejected the Higgs boson today.

The B(3) field was also accepted by protagonists of higher topology electrodynamics, three or four of whose books appear in this World Scientific series "Contemporary Chemical Physics". For example books by Lehnert and Roy, Barrett, Harmuth et al., and Crowell, and it was also accepted by Kielich, a pioneer of non-linear optics. Other articles, notably by Reed {7} on the Beltrami fields and higher topology electrodynamics, appear in "Modern Nonlinear Optics", published in two editions and six volumes from 1992 to 2001. Piekara also worked in Paris and with Kielich, inferred the inverse Faraday effect (IFE). The latter was re-inferred by Pershan at Harvard in the early sixties and first observed experimentally in the Bloembergen School at Harvard in about 1964. The first observation used a visible frequency laser, and the IFE was confirmed at microwave frequencies by Deschamps et al. {7} in Paris in 1970 in electron plasma. So it was shown to be an ubiquitous effect that depended for its description on the conjugate product. The B(3) field was widely accepted as being a natural description of the longitudinal magnetization of the IFE.

Following upon the suggestion by Vigier that B(3) implied the existence of photon mass, the first attempts were made to develop O(3) electrodynamics {1 - 10}, in which the indices of the complex circular basis, (1), (2) and (3), were incorporated into electrodynamics as described in earlier chapters of this book. Many aspects of U(1) gauge invariance were rejected, as described in the Omnia Opera on www.aias.us from 1993 to 2003, a decade of
development. During this time, five volumes were produced by Evans and Vigier \{1-10\} in the famous van der Merwe series of "The Enigmatic Photon", a title suggested by van der Merwe himself. These are available in the Omnia Opera of www.aias.us. In the mid nineties van der Merwe had published a review article on the implications of B(3) at Vigier's suggestion, in "Foundations of Physics". This was a famous journal of avant garde physics, one of the very few to allow publication of ideas that were not those of the standard physics.

The O(3) electrodynamics was a higher topology electrodynamics that was transitional between early B(3) theory and ECE theory, in which the photon mass and B(3) were both derived from Cartan geometry.

4.2 DERIVATION OF THE PROCA EQUATIONS FROM ECE THEORY.

The Proca equation as discussed briefly in Chapter Three is the fundamental equation of photon mass theory and in this section it is derived from the tetrad postulate. The latter always gives finite photon mass in ECE theory and consider it in the format:

\[ \partial_{\mu} q^a = \partial_{\mu} q^a + \omega^a_{\mu b} q^b - \Gamma^\lambda_{\mu \nu} q^a = 0 \quad - (1) \]

where \( q^a \) is the Cartan tetrad, where \( \omega^a_{\mu b} \) is the spin connection and \( \Gamma^\lambda_{\mu \nu} \) is the gamma connection. Define:

\[ \omega^a_{\mu} = \omega^a_{\mu b} q^b, \quad - (2) \]

\[ \Gamma^a_{\mu \nu} = \Gamma^a_{\mu \nu} q^a, \quad - (3) \]

then:

\[ \partial_{\mu} q^a = \Gamma^a_{\mu \nu} - \omega^a_{\mu} = - \partial^a_{\mu}. \quad - (4) \]

Differentiate both sides:
\[ \partial_\mu a^\alpha = \square a^\alpha = \partial_\mu \Omega_\mu^\alpha - (5) \]

and define:
\[ \partial_\mu \Omega_\mu^\alpha := -R a^\alpha - (6) \]

to find the ECE wave equation:
\[ (\square + R) a^\alpha = 0 - (7) \]

and the equation:
\[ \partial_\mu \Omega_\mu^\alpha + R a^\alpha = 0, - (8) \]

where the curvature is:
\[ R = -a^\alpha d_\mu \Omega_\mu^\alpha - (9) \]

Now use the ECE postulate and define an electromagnetic field:
\[ F_\mu^\alpha := A_\\(0)\Omega_\mu^\alpha - (10) \]

to find:
\[ (\square + R) A_\mu^\alpha = 0 - (11) \]

and
\[ \partial_\mu F_\mu^\alpha + RA_\mu^\alpha = 0. - (12) \]
These are the Proca wave and field equations, Q. E. D.

The photon mass is defined by the curvature:

$$ R = \left( \frac{mc}{k} \right)^2 $$  

Therefore:

$$ \left( \Box + \left( \frac{mc}{k} \right)^2 \right) A^a_\mu = 0 $$  

and

$$ \partial_\mu F^a_\mu + \left( \frac{mc}{k} \right)^2 A^a_\nu = 0. $$

For each state of polarization $a$ these are the Proca equations of the mid thirties. They are not $U(1)$ gauge invariant and refute Higgs boson theory immediately, because Higgs boson theory is $U(1)$ gauge invariant. Eq. (10) can be regarded as a postulate of ECE theory in which the electromagnetic field is defined by the connection $\Omega^a_\mu$. By antisymmetry:

$$ F^a_\mu = - F^a_\mu $$

and from the first Cartan structure equation:

$$ T^a_\mu = \partial_\nu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_\mu - \omega^a_\nu. $$

The fundamental postulates of ECE theory are:

$$ A^a_\mu = A^{(0)} \dot{q}_\mu^a, $$

$$ F^a_\mu = A^{(0)} T^a_\mu, $$

so:
\[
F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + A^{(0)}(\omega_{\mu} - \omega_{\nu})
\]
\[
= A^{(0)}\left(\Gamma_{\mu \nu}^a - \Gamma_{\nu \mu}^a\right) - (20)
\]

By antisymmetry:
\[
F_{\mu \nu} = 2\left(\partial_{\mu} A_{\nu} + A^{(0)}\omega_{\nu}\right) - (21)
\]

so:
\[
F_{\mu \nu}^{\text{(original)}} = 2\left(F_{\mu \nu}^{\text{(new)}} + A^{(0)}\omega_{\nu}\right) - (22)
\]

The postulate (\text{10}) is a convenient way of deriving the two Proca equations from the tetrad postulate. In so doing:
\[
R_\mu = \left(\frac{m_0 c}{\kappa}\right)^2 - (23)
\]

where \(m_0\) is the rest mass of the photon. More generally define:
\[
R = \left(\frac{m c}{\kappa}\right)^2 - (24)
\]

where:
\[
m = \gamma m_0 - (25)
\]

then the de Broglie equation is generalized to:
\[
E = \hbar \omega = mc^2 = \kappa c R^{1/2} - (26)
\]

and the square of the mass of the moving photon is defined by the curvature:
\[
m^2 = \left(\frac{\hbar}{c}\right)^2 R = \left(\frac{\hbar}{c}\right)^2 \sqrt{g} \, a_{\mu \nu} \left(\omega_{\mu}^a - \Gamma_{\mu \nu}^a\right) - (27)
\]
The Proca equations are discussed further in Chapter three. The dogmatic U(1) gauge transformation of the standard physics is:

\[ A^\mu \rightarrow A^\mu + j^\mu x \quad -(28) \]

but the Proca Lagrangian in the usual standard model units is:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^\mu A^\mu \quad -(29) \]

and this lagrangian is not U(1) gauge invariant because the transformation \((28)\) changes it.

This fundamental problem for U(1) gauge invariance has never been resolved, and the current theory behind the Higgs boson still uses U(1) gauge invariance after many logical refutations. The result is a deep schism in physics between the scientific ECE theory and the dogmatic standard theory.

4.3 LINK BETWEEN PHOTON MASS AND B(3).

The complete electromagnetic field tensor of ECE theory can be defined by:

\[ F^{a\mu\nu} = \delta^{a\mu\nu} - \delta^{a\mu\nu} + \omega_{\mu\nu} A^a - \omega_{\mu\nu} A^a \quad -(30) \]

where:

\[ A^a_{\mu} = A_{\mu}^{(0)} A^a_{\mu}, \quad \delta^{a\mu\nu} = \delta_{\mu\nu} A^a_{\mu} \quad -(31) \]

Consider now the tetrad postulate in the format:

\[ \mathfrak{D}^a_{\mu} \gamma^a_{\nu\rho} = \Gamma^a_{\mu\nu\rho} - \omega^a_{\mu\nu} = \Omega^a_{\mu\nu} \quad -(32) \]

Eq. \((31)\) follows directly from the subsidiary postulate:
and as shown already in this chapter gives the Proca wave and field equations in generally covariant format. It is seen that the Proca equations are subsidiary structures of the more general nonlinear structure (\(30\)).

The B(3) field that is the basis of unified field theory is defined by:

\[
B_{\mu\nu}^a = -i g \left( A^c A_{\mu\nu}^b - A^c A_{\nu\mu}^b \right) = \omega_{\mu b} A_{\nu}^b - \omega_{\nu b} A_{\mu}^b.
\]

and is derived from the non-linear part of the complete field tensor \((30)\). In the B(3) theory:

\[
\omega_{\mu b} = -i g A^c_{\mu} \varepsilon_{abc}.
\]

Now define for each polarization index \(a\):

\[
\mathcal{J}^a_{\mu\nu} = J^a_{\mu} A^a_{\nu} - J^a_{\nu} A^a_{\mu}.
\]

It follows that:

\[
\mathcal{J}^a_{\mu\nu} + \mathcal{J}^a_{\nu\mu} + \mathcal{J}^a_{\mu\mu} = 0.
\]

This equation is the same as:

\[
\mathcal{J}^a_{\mu\nu} = 0
\]

where the tilde denotes the Hodge dual. It follows that:

\[
\mathcal{J}^a_{\mu\nu} = 0
\]

which is the homogeneous field equation of the Proca structure. Eq. (32) allows the description of the Aharonov Bohm effects \{1-10\} with the assumption:
With this assumption the potential is non zero when the field is zero. In UFT 157 on www.aias.us the following relation was derived for each polarization index \( a \):

\[
\Gamma \alpha = \omega \gamma \mu \nu - (40)
\]

where the charge current density is:

\[
j \gamma \nu = R A \gamma \mu - (41)
\]

and where:

\[
A \gamma \mu = \left( \frac{\phi}{c} \right) \left( \frac{A}{A} \right) - (43)
\]

Here \( \mu_0 \) is the vacuum permeability and \( \epsilon_0 \) is the vacuum permittivity. So:

\[
\rho = -\epsilon_0 R \phi - (44)
\]

and:

\[
J = -\frac{R A}{\mu_0} - (45)
\]

where \( \rho \) is the charge density, \( \phi \) is the scalar potential, \( J \) is the current density and \( A \) is the vector potential. A list of S. I. Units was given earlier in this book, and the units of the vacuum permeability are:

\[
\mu_0 = \frac{J s C^{-2} m^{-1}}{} - (46)
\]
Now define the field tensor and its Hodge dual as:

\[ B_{\mu\nu} = \nabla \times E_{\mu\nu} = - \nabla \times H_{\mu\nu} = \mu_0 \, j_{\mu\nu} \quad -(50) \]

\[ j_{\mu\nu} = \nabla \times B_{\mu\nu} = - \nabla \times H_{\mu\nu} = \mu_0 \, j_{\mu\nu} \quad -(51) \]

\[ j_{\mu} = - \frac{\mu}{\rho} A_{\mu} \quad -(52) \]

The complete set of equations of the Proca structure is therefore:

\[ \nabla^\alpha (F^{\alpha\beta}) = \nabla^\alpha (F^{\alpha\beta}) = - (47) \]

\[ \nabla^\alpha (F^{\alpha\beta}) + RA_{\alpha\beta} = 0 \quad -(48) \]

\[ (\Box + R) F_{\mu\nu} = 0 \quad -(49) \]

These definitions give the inhomogeneous Proca field equation under all conditions, including the vacuum:

\[ \nabla \cdot E = \rho / \varepsilon_0 = - \nabla \phi \quad -(54) \]

\[ \nabla \times B = - \frac{1}{c^2} \frac{\partial E}{\partial t} = \mu_0 j = - RA \quad -(55) \]

and the homogeneous field equations:

\[ \nabla \cdot B = 0 \quad -(56) \]

\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \quad -(57) \]
under all conditions.

The solution of Eq. (54) is:

$$\phi = \frac{1}{\varepsilon_0} \int \frac{\rho d^3x}{|x - x'|} \tag{58}$$

and from Eqs. (54) and (58):

$$\phi = -\frac{\rho}{\varepsilon_0 R} = \frac{1}{\varepsilon_0} \int \frac{\rho d^3x}{|x - x'|} \tag{59}$$

so:

$$\int \frac{\rho d^3x}{|x - x'|} = -\frac{\rho}{R} \tag{60}$$

where:

$$R = -qV a \rho \left( \Gamma^a_{\mu
u} - \omega^a_{\mu
u} \right) \tag{61}$$

Therefore:

$$\int \frac{\rho (x') d^3x'}{|x - x'|} = \frac{qV a \rho \left( \Gamma^a_{\mu
u} - \omega^a_{\mu
u} \right)}{\sqrt{a d^\mu \left( \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right)}} \tag{62}$$

The original Proca equation of the thirties assumed that:

$$qV a \rho \left( \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right) = \left( \frac{m_0 c}{\sqrt{E^2 - \gamma^2 m^2}} \right)^2 \tag{63}$$

where $m_0$ is the rest mass. For electromagnetic fields in the vacuum this was assumed to be the photon rest mass, so the Proca equations were assumed to be equations of a boson with finite mass. More generally in particle physics this can be any boson. In Proca theory therefore the electromagnetic field is associated with a massive boson (i.e. a photon that has mass). Therefore the original Proca equations of the thirties assumed:
\[ \phi = \frac{1}{\epsilon_0} \left( \frac{\hbar}{m_0 c} \right)^2 \rho \]  

It follows that:

\[ \int \rho \frac{d^3 x}{\sqrt{1 - \beta^2}} = \left( \frac{\hbar}{m_0 c} \right)^2 \rho \]  

From Eqs. (59) and (65):

\[ \phi_{\text{vac}} = \frac{1}{\epsilon_0} \left( \frac{\hbar}{m_0 c} \right)^2 \rho_{\text{vac}} \]  

giving the photon rest mass as the ratio:

\[ m_0^2 = \left( \frac{\hbar}{c} \right)^2 \rho_{\text{vac}} \phi_{\text{vac}} = 1.4 \times 10^{-74} \left( \frac{\rho_{\text{vac}}}{\phi_{\text{vac}}} \right) \]  

Two independent experiments are needed to find \( \rho_{\text{vac}} \) and \( \phi_{\text{vac}} \). A list of experiments used to determine photon mass is given in ref. (1). However, in this Section the assumptions used in these determinations are examined carefully, and in the main, they are shown to be untenable. Later in this chapter a new method of determining photon mass, based on Compton scattering, will be given.

Conservation of charge current density for each polarization index \( a \) means that:

\[ \partial_\mu j^\mu = 0 \]  

From Eqs. (68) and (62):

\[ \partial_\mu A^\mu = 0 \]
In the standard physics Eq. (69) is known as the Lorenz gauge, an arbitrary assumption. In the Proca photon mass theory the Lorenz gauge is derive analytically. In the Proca theory the four potential is physical, and the $U(1)$ gauge invariance is refuted completely. In consequence, Higgs boson theory collapses.

From the well known radiative corrections $\{1 \cdots 10\}$ it is known experimentally that the vacuum contains charge current density. It follows directly from Eq. (52) that the vacuum also contains a four potential associated with photon mass. Therefore there are vacuum fields which in the non linear ECE theory include the $B(3)$ field. The latter therefore also exists in the vacuum and is linked to photon mass and Proca theory. In the standard dogma the assumption of zero photon mass means that the vacuum fields only have transverse components. This is of course geometrical nonsense, and leads to the unphysical $E(2)$ little group $\{13\}$ of the Poincaré group. The vacuum four potential is:

$$A^\mu (\text{vac}) = \left( \frac{\phi (\text{vac})}{c}, A (\text{vac}) \right)$$ (70)

It follows that a circuit can pick up the vacuum four potential via the inhomogeneous Proca equations

$$\nabla \cdot E = - R \phi (\text{vac})$$ (71)

and:

$$\nabla \times B - \frac{1}{c^2} \frac{dE}{dt} = - R A (\text{vac})$$ (72)

In this process, total energy is conserved through the relevant Poynting theorem derived as follows. Multiply Eq. (72) by $E$:

$$E \cdot (\nabla \times B) - \frac{1}{c^2} E \cdot \frac{dE}{dt} = - R E \cdot A (\text{vac})$$ (73)
Use:
\[ E \cdot \nabla \times B = -\nabla \cdot E \times B + B \cdot \nabla \times E \]  
\[ (74) \]
in Eq. (73) to find the Poynting theorem of conservation of total energy density:
\[ \frac{\partial W}{\partial t} + \nabla \cdot S = \frac{R}{\mu_0} E \cdot A \, (\text{vac}) \]  
\[ (75) \]

The electromagnetic energy density in joules per metres cubed is:
\[ W = \frac{1}{2} \left( \varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \]  
\[ (76) \]

and the Poynting vector is:
\[ S = \frac{1}{\mu_0} E \times B \]  
\[ (77) \]
Use:
\[
E \cdot \nabla \times B = - \nabla \cdot E \times B + B \cdot \nabla \times E \quad -(74)
\]

in Eq. (73) to find the Poynting theorem of conservation of total energy density:
\[
\frac{dW}{dt} + \nabla \cdot S = \frac{\mathcal{R}}{\mu_0} E \cdot A \quad -(75)
\]

The electromagnetic energy density in joules per metres cubed is:
\[
W = \frac{1}{2} (\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) \quad -(76)
\]

and the Poynting vector is:
\[
S = \frac{1}{\mu_0} E \times B \quad -(77)
\]

Eq. (76) defines the electromagnetic energy density available from the vacuum, more accurately spacetime. This process is governed by the Poynting Theorem (75) and therefore there is conservation of total energy, there being electromagnetic energy density in the vacuum. The relevant electromagnetic field tensor is:
\[
\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad -(78)
\]

so either:
\[
E = - \nabla \phi \quad -(79)
\]

or:
\[
E = - \frac{\partial A}{\partial t} \quad -(80)
\]

The antisymmetry of the Cartan torsion means that the complete non-linear field of Eq. (30) is antisymmetric:
The Cartan torsion is defined by:

\[ T^a_{\mu \nu} = \omega^a_{\mu \nu} - \omega^a_{\nu \mu}, \]  

(81)

The Cartan torsion is defined by:

\[ T^a_{\mu \nu} = \omega^a_{\mu \nu} - \omega^a_{\nu \mu}, \]  

(82)

where the antisymmetric torsion tensor \( T^a_{\mu \nu} \) is defined by the commutator of covariant derivatives:

\[ \left[ D_\mu, D_\nu \right] V^\rho = -T^\lambda_{\mu \nu} \partial_\lambda V^\rho + R^{\rho \gamma \lambda}_{\mu \nu \omega} V^\omega. \]  

(83)

The torsion tensor is defined by the difference of antisymmetric connections:

\[ T^\lambda_{\mu \nu} = \Gamma^\lambda_{\mu \nu} - \Gamma^\lambda_{\nu \mu}, \]  

(84)

and the tetrad postulate means that:

\[ \Gamma^a_{\mu \nu} = -\Gamma^a_{\nu \mu} = \partial_\mu \omega^a_{\nu \lambda} + \omega^a_{\nu \lambda} \omega^\lambda_{\mu \nu}. \]  

(85)

It follows that the antisymmetry in Eq. (30) is defined by:

\[ \int \omega^a_{\mu \nu} + \omega^a_{\nu \mu} A^b_{\lambda} = -\left( \int \omega^a_{\mu \nu} + \omega^a_{\nu \mu} A^b_{\lambda} \right). \]  

(86)

If Eq. (79) is used for the sake of argument then the Poynting Theorem becomes:

\[ \frac{dW}{dt} + \nabla \cdot S = -\frac{1}{2} \frac{R}{\mu_0} \frac{1}{\mu_0} \left( A^2 (\text{vac}) \right). \]  

(87)

From Eq. (45):
\[ A_{\text{vac}} = -\frac{\mu_0}{R} J_{\text{vac}} \]  

so we arrive at:

\[ \frac{dW}{dt} + \nabla \cdot S = -\frac{1}{2} \mu_0 R \frac{d}{dt} \left( \frac{J^2_{\text{vac}}}{R} \right) \]  

which shows that the vacuum energy density and vacuum Poynting vector are derived from the time derivative of the vacuum current density squared divided by R.

In practical applications we are interested in transferring the electromagnetic energy density of the vacuum to a circuit which can use the energy density. In an isolated circuit consider the equation:

\[ \square A^a_{\mu} = \mu_0 j^a_{\mu} \]  

When the circuit interacts with the vacuum:

\[ j^a_{\mu} \Rightarrow j^a_{\mu} + j^a_{\mu_{\text{vac}}} \]  

so the Proca equation becomes:

\[ \square A^a_{\mu} = \mu_0 (j^a_{\mu} + j^a_{\mu_{\text{vac}}}) \]  

and

\[ j^a_{\mu} F^a_{\mu\nu} = \mu_0 (j^a_{\mu} + j^a_{\mu_{\text{vac}}}) \]  

The Coulomb law is modified to:

\[ \nabla \cdot E = \frac{1}{\varepsilon_0} \left( \rho_{\text{circuit}} + \rho_{\text{vac}} \right) \]
The d’Alembertian operator is defined by:

\[
\square = \frac{1}{c^2} \frac{d^2}{dt^2} - \nabla^2
\]  

The time dependent part of

\[
\frac{1}{c^2} \frac{d^2 \phi}{dt^2} + R \phi = \rho(vac) \varepsilon_0
\]  

The most fundamental unit of mass of the circuit is the electron mass \(m_e\), whose rest angular frequency is defined by the de Broglie wave particle dualism:

\[
R_e = \left( \frac{m_e c}{\hbar} \right)^2 = \frac{\omega_e^2}{c^2} = \sqrt{\omega} \int \left( \omega^{\mu} - n^{\mu} \right)
\]  

So Eq. (97) becomes:

\[
\frac{d^2 \phi}{dt^2} + \omega_e^2 \phi = c^2 \frac{\rho(vac)}{\varepsilon_0}
\]  

which is an Euler Bernoulli resonance equation provided that:

\[
c^2 \frac{\rho(vac)}{\varepsilon_0} = A \cos \omega t
\]  

The solution of the Euler Bernoulli equation

\[
\frac{d^2 \phi}{dt^2} + \omega_e^2 \phi = A \cos \omega t
\]  

is well known to be:
At resonance:

$$\omega_e = \omega$$ \hspace{2cm} -(103)$$

and the circuit’s scalar potential becomes infinite for all A, however tiny in magnitude. This allows the circuit design of a device to pick up practical quantities of electromagnetic radiation density from the vacuum by resonance amplification. The condenser plates used to observe the well known Casimir effect can be incorporated in the circuit design as in previous work by Eckardt, Lindstrom and others.

From Eqs. (41) and (144)

$$\frac{\mathcal{E}^2 \varphi({\text{vac}})}{\varepsilon_0} = -\mathcal{C}^2 R \varphi({\text{vac}})$$ \hspace{2cm} -(104)$$

and if we consider the space part of the scalar potential $\varphi$ then:

$$\Box \rightarrow -\nabla^2$$ \hspace{2cm} -(105)$$

and for each polarization index a the Proca equation reduces to:

$$\nabla^2 \phi = \left( \frac{mc}{\mathcal{E}} \right)^2 \phi$$ \hspace{2cm} -(106)$$

The Laplacian in polar coordinates is defined by:

$$\nabla^2 \phi \rightarrow \frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr}$$ \hspace{2cm} -(107)$$

so there is a solution to Eq. (106) known as the Yukawa potential:
This solution was used in early particle physics but was discarded as unphysical. The early experiments to detect photon mass \{1-10\} all assume the validity of the Yukawa potential. However the basic equation:

$$\Box A_\mu = \mu_0 \mathbf{j}_\mu \quad -(109)$$

also has the solution:

$$\phi = \frac{e}{4\pi \varepsilon_0} \left( \left( 1 - \frac{n \cdot v}{c} \right) |s - s'| \right)^{-1} \quad -(110)$$

and

$$A = \frac{\mu_0 e v}{4\pi} \left( \left( 1 - \frac{n \cdot v}{c} \right) |s - s'| \right)^{-1} \quad -(111)$$

which are the well known Lienard Wiechert solutions. Here \( t_r \) is the retarded time defined by:

$$t_r = t - \frac{1}{c} |s - s'|, \quad c = \frac{|s - s'|}{t - t_r} \quad -(112)$$

Therefore the static potential of the Proca equation is given by Eq. (110) with:

$$\nu = 0 \quad -(113)$$

and the static vacuum charge density in coulombs per cubic metre is given by:

$$\rho_{\text{vac}} = - \left( \frac{mc}{\kappa} \right)^2 \frac{1}{4\pi} \left( \frac{e}{|s - s'|} \right)_{tr} \quad -(113.a)$$
which is the Coulomb law for any photon mass.

This means that photon mass does not affect the Coulomb law, known to be one of the most precise laws in physics. Similarly the photon mass does not affect the Ampere Maxwell law or Ampere law. This is observed experimentally \( \{\{1-0\}\} \) with high precision, so it is concluded that the usual Lienard Wiechert solution is the physical solution, and that the Yukawa solution is mathematically correct but not physical. On the other hand the standard physics ignores the Lienard Wiechert solution, and other solutions, and asserts arbitrarily that the Yukawa solution must be used in photon mass theory. The use of the Yukawa potential means that there are deviations from the Coulomb and Ampere laws. These have never been observed so the standard physics concludes that the photon mass is zero for all practical purposes. This is an entirely arbitrary conclusion based on the anthropomorphic claim of zero photon mass, a circular argument that is invalid. The theory of this chapter shows that the Coulomb and Ampere laws are true for any photon mass, and the latter cannot be determined from these laws. In other words these laws are not affected by photon mass in the sense that their form remains the same. For example the inverse square dependence of the Coulomb law is the same for any photon mass. The concept of photon mass is not nearly as straightforward as it seems, for example UFT244 on www.aias.us shows that Compton scattering when correctly developed gives a photon mass much different from Eq. \( 67 \). These are unresolved questions in particle physics because UFT244 has shown violation of conservation of energy in the basic theory of particle scattering.

Before proceeding to the description of determination of photon mass by Compton scattering a mention is made of the origin of the idea of photon mass. This was by Henri Poincare in his Palermo memoir submitted on July 23rd 1905, (Henri Poincare, “Sur la Dynamique de l’Electron” Rendiconti del Circolo Matematico di Palermo, 21, 127 - 175
This paper suggested that the photon velocity $v$ could be less than $c$, which is the constant of the Lorentz transformation. Typically for Poincaré he introduced several new ideas in relativity, including new four vectors usually attributed to later papers of Einstein. So Poincaré can be regarded as a co-pioneer of special relativity with many others. Einstein himself suggested a zero photon mass as a first tentative idea, simply because an object moving at $c$ must have zero mass, otherwise the equations of special relativity become singular. Later, Einstein may have been persuaded by the de Broglie School in the Institut Henri Poincaré in Paris to consider finite photon mass, but this is not clear. It was therefore de Broglie who took up the idea of finite photon mass from Poincaré. He was influenced by the works of Henri Poincaré before inferring wave particle duality in 1923, when he suggested that particles such as the electron could be wave-like. Confusion arises sometimes when it is asserted that the vacuum speed of light is $c$. This is not the meaning of $c$ in special and general relativity, $c$ is the constant in the Lorentz transform. Lorentz and Poincaré had inferred the tensorial equations of electromagnetism much earlier than Einstein as is well known. They had shown that the Maxwell-Heaviside equations obey the Lorentz transform. ECE has developed equations of electromagnetism that are generally covariant, and therefore also Lorentz covariant in a well defined limit. It is well known that Einstein and others were impressed by the work of de Broglie, Einstein described him famously as having lifted a corner of the veil.

Louis de Broglie proceeded to develop the theory of photon mass and causal quantum mechanics until the 1927 Solvay Conference, when indeterminism was proposed, mainly by Bohr, Heisenberg and Pauli. It was rejected by Einstein, Schrödinger, de Broglie and others. Later de Broglie returned to deterministic quantum mechanics at the suggestion of Vigier. A minority of physicists have continued to develop finite photon mass theory, setting
upper limits on the magnitude of the photon mass. There are multiple problems with the idea of zero photon mass, as is well known \{13\}. These are discussed in comprehensive detail in the five volumes of "The Enigmatic Photon" (Kluwer, 1994 - 2002) by M. W. Evans and J.-P. Vigier. Wigner \{13\} for example showed that special relativity can be developed in terms of the Poincaré group, or extended Lorentz group. In this analysis the little group of the Poincaré group for a massless particle is the Euclidean \(E(2)\), the group of rotations and translations in a two dimensional plane. This is obviously incompatible with the four dimensions of spacetime or the three dimensions of space. The little group for a massive particle is three dimensional and physical, no longer two dimensional.

This is the most obvious problem for a massless particle, and one of its manifestations is that the electromagnetic field in free space must be transverse and two dimensional, despite the fact that the theory of electromagnetism is built on four dimensional spacetime. The massless photon can have only two senses of polarization, labelled the transverse conjugates (1) and (2) in the complex circular basis \{1 - 10\} used in earlier chapters. This absurd dogma took hold because of the prestige of Einstein, but prestige is no substitute for logic. The idea of zero photon mass developed into \(U(1)\) gauge invariance, which became embedded into the standard model of physics. The electromagnetic sector of standard physics is still based on \(U(1)\) gauge invariance, refuted by the \(B(3)\) field in 1992 and in comprehensive developments since then. The idea of \(U(1)\) gauge invariance is in fact refuted by the Poincaré paper of 1905 described already, and by the work of Wigner, so it is merely dogmatic, not scientific. It is refuted by effects of nonlinear optics, notably the inverse Faraday effect, and in many other ways. It was refuted comprehensively in chapter three by the fact that the Beltrami equations of free space electromagnetism have intricate longitudinal solutions in free space. According to the \(U(1)\) dogma, these do not exist, an absurd conclusion.
Probably the most absurd idea of the U(1) dogma is the Gupta Bleuler condition, in which the time like (0) and longitudinal polarizations (3) are removed artificially \(^{13}\). There are also multiple well known problems of canonical quantization of the massless electromagnetic field. These are discussed in a standard text such as Ryder \(^{13}\), and in great detail in “The Enigmatic Photon” \(^{1-10}\). Finally the electroweak theory, which can be described as U(1) x SU(2), was refuted completely in UFT225.

The entire standard unified field theory depends on U(1) gauge invariance, so the entire theory is refuted as described above. Obviously there cannot be a Higgs boson.

4.4 MEASUREMENT OF PHOTON MASS BY COMPTON SCATTERING

The theory of particle scattering has been advanced greatly during the course of development of ECE theory in papers such as UFT155 to UFT171 on www.aias.us, reviewed in UFT200. It has been shown that the idea of zero photon mass is incompatible with a rigorously correct theory of scattering, for example Compton scattering. This is because of the numerous problems discussed at the end of Section 4.3 - zero photon mass is incompatible with special relativity, a theory upon which traditional Compton scattering is based. In UFT158 to UFT171 it was found that the Einstein de Broglie equations are not self consistent, a careful scholarly examination of the theory showed up wildly inconsistent results, which were also present in equal mass electron positron scattering.

The theory of Compton scattering with finite photon mass was first given in UFT158 to UFT171 and the notation of those papers is used here. The relativistic classical conservation of energy equation is:

\[
\gamma m_1 c^2 + m_2 c^2 = \gamma' m_1 c^2 + \gamma'' m_2 c^2
\]

\(\text{(114)}\)
where \( m_1 \) is the photon mass, \( m_2 \) is the electron mass, and where the Lorentz factors are defined by the velocities as usual. The photon mass is given by the equation first derived in UFT 160:

\[
m_1^2 = \left( \frac{\hbar}{c} \right)^2 \left[ \frac{1}{2a} \left( -b \pm \sqrt{b^2 - 4ac} \right) \right]^{1/2} - (115)
\]

where:

\[
a = 1 - \cos^2 \theta, \quad b = (\omega' + \omega \cos^2 \theta - 2A \cos \theta)
\]

\[
A = \omega \omega' - \chi \omega \omega', \quad c' = A^2 - \omega^2 \omega' \cos^2 \theta
\]

\( \omega \) is the scattered gamma ray frequency, \( \omega' \) the incident gamma ray frequency, and \( \Theta \) the scattering angle. Experimental data on Compton scattering can be used with the electron mass found in standards laboratories:

\[
m_2 = 9.10953 \times 10^{-31} \text{ kg} - (117)
\]

so:

\[
\chi_2 = 7.76343 \times 10^{20} \text{ rad s}^{-1} - (118)
\]

The two solutions for photon mass are given later in this section. One solution is always real valued and this root is usually taken to be the physical value of the mass of the photon.
$m_1 = \left( \frac{\hbar}{c^2} \right)^2 \left[ \frac{1}{2a} \left( -b \pm \sqrt{b^2 - 4ac} \right) \right]^{1/2}$

$a = 1 - \cos^2 \theta$

$b = \left( \omega' + \omega'' \right) \cos^2 \theta - 2A$

$A = \cos \omega' - \omega'' \cos^2 \theta$

$c' = A^2 - \omega^2 \omega'' \cos^2 \theta$

where $\omega'$ is the scattered gamma ray frequency, $\omega$ the incident gamma ray frequency, and where:

$x_2 = m_2 c^2$

Here $\hbar$ is the reduced Planck constant and $c$ the speed of light in vacuo. The scattering angle is $\theta$. Experimental data on Compton scattering can be used with the electron mass found in standards laboratories:

$m_2 = 9.10953 \times 10^{-31}$ kg

so:

$x_2 = 7.7634 \times 10^2 \text{ rad s}^{-1}$

The two solutions of Eq. (115) for photon mass are given later in this section. One solution is always real valued and this root is usually taken to be the physical value of the mass of the photon. It varies with scattering angle but is always close to the electron mass. The photon in this method is much heavier than thought previously. The other solution can be imaginary valued, and usually this solution would be discarded as unphysical. However R theory means that a real valued curvature can be found as follows:

$R = \frac{Km}{m} \left( \frac{c}{\hbar} \right)^2$
where * denotes complex conjugate. It is shown later that an imaginary valued mass can be interpreted in terms of superluminal propagation.

The velocity of the photon after it has been scattered from a stationary electron is given by the de Broglie equation:

\[ \gamma' m_e c^2 = \frac{2 \omega'}{c} \]

and is \( c \) for all practical purposes for all scattering angles (Section 4.5). A photon as heavy as the electron does not conflict therefore with the results of the Michelson Morley experiment but on a cosmological scale a photon as heavy as this would easily account for any mass discrepancy claimed at present to be due to "dark matter". Photon mass physics differs fundamentally from standard physics as explained in comprehensive detail {1 - 10} in the five volumes of "The Enigmatic Photon" in the Omnia Opera of www.aias.us. A photon as heavy as the electron would mean that previous attempts at assessing photon mass would have to be re-assessed as discussed already in this chapter. The Yukawa potential would have to be abandoned or redeveloped.

However the theory of the photoelectric effect can be made compatible with a heavy photon as follows. Consider a heavy photon colliding with a static electron. The energy conservation equation is:

\[ \sqrt{m_0 c^2 + m_e c^2} = \gamma' m_0 c^2 + \gamma'' m_e c^2 \]

The de Broglie equation can be used as follows:

\[ \frac{2 \omega}{c} = \gamma' m_0 c^2 \]
\[ \frac{2 \omega''}{c} = \gamma'' m_e c^2 \]
If the photon is stopped by the collision then the conservation of energy equation is:
\[ h\omega + m_e c^2 = m_\gamma c^2 + h\omega'' \]  
(124)

where \( m_\gamma \) is the rest mass of the photon. This concept does not exist in the standard model because a massless photon is never at rest. So:
\[ m_\gamma = m_e + \frac{h}{c^2} \left( \omega - \omega'' \right) \]  
(125)

If for the sake of argument the masses of the photon and electron are the same, then:
\[ m_\gamma = m_e \]  
(126)

and:
\[ \omega = \omega'' \]  
(127)

i.e. all the energy of the photon is transferred to the electron.

If:
\[ \omega \neq \omega'' \]  
(128)

then:
\[ \frac{h}{c^2} (\omega - \omega'') = \bar{\varepsilon} + (m_\gamma - m_e) c^2 = \bar{\varepsilon} \]  
(129)

where \( \bar{\varepsilon} \) is the binding energy of the photoelectric effect. From Eq. (129):
\[ h\omega + m_e c^2 = m_\gamma c^2 + h\omega'' + \bar{\varepsilon} \]  
(130)
i.e.:  \[ \frac{1}{2} \omega = \frac{1}{2} \omega'' + \frac{1}{2} \phi = E + \phi - (131) \]

or:  \[ E = \frac{1}{2} \omega - \phi - (132) \]

which is the usual equation of the photoelectric effect, Q. E. D. The heavy photon does not disappear and transfers its energy to the electron, and the heavy photon is compatible with the photoelectric effect.

A major and fundamental problem for standard physics emerges from consideration of equal mass Compton scattering as described in UFT160 on www.aias.us. It can be argued as follows that equal mass Compton scattering violates conservation of energy. Consider a particle of mass \( m \) colliding with an initially static particle of mass \( m \). If the equations of conservation of energy and momentum are assumed to be true initially, they can be solved simultaneously to give:

\[ x^2 + (\omega^2 - x^2)^{1/2} (\omega^2 - x^2)^{1/2} \cos \theta = \omega' - (\omega - \omega')x - (133) \]

where:

\[ x = \omega_o = \frac{mc^2}{\phi} - (134) \]

is the rest frequency of the particle of mass \( m \), \( \omega' \) is the scattered frequency, and \( \omega \) the incoming frequency of particle \( m \) colliding with an initially static particle of mass \( m \). The scattering angle is \( \theta \) and from Eq. (133):
\[
\cos^2 \theta = \frac{\omega^2 + \omega (\omega - \omega')}{\omega^2 - \omega (\omega - \omega') - \omega'} - (135)
\]

In order that
\[
0 \leq \cos^2 \theta \leq 1 - (136)
\]
then:
\[
\omega < \omega' - (137)
\]

The de Broglie equation means that the collision can be described by:
\[
\omega + \omega_o = \omega' + \omega'' - (138)
\]
so:
\[
\omega + \omega_o = \omega' + \omega'' - (139)
\]
and:
\[
\omega - \omega' = \omega'' - \omega_o - (140)
\]

Therefore:
\[
\omega'' < \omega_o - (141)
\]

From Eqs. (137) and (141):
\[
\omega + \omega_o < \omega' + \omega'' - (142)
\]

However the initial conservation of energy equation is (139), so the theory violates conservation of energy and contradicts itself. This is a disaster for particle scattering theory because violation of conservation of energy occurs at the fundamental level. Quantum electrodynamics and string theory, or Higgs boson theory of particle scattering are invalidated.
If two particles of mass \( m_1 \) and \( m_2 \) collide and both are moving, the initial conservation of energy equation is:

\[
\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma_1' m_1 c^2 + \gamma_2' m_2 c^2 - (143)
\]
i.e.

\[
\gamma_1 c^2 + \gamma_2 m_2 c^2 = \gamma_1' c^2 + \gamma_2' c^2 - (144)
\]

Define

\[
x_2 = \frac{\gamma_2 m_2 c^2}{\gamma_1} - (145)
\]

then:

\[
x_2 = \omega_2 = \omega' + \omega'' - \omega - (146)
\]

The equation of conservation of momentum is:

\[
\vec{p} = \vec{p}_1 + \vec{p}_2 = \vec{p}' + \vec{p}'' - (147)
\]

Solving Eqs. (143) and (147) simultaneously leads to:

\[
x_2 (\omega - \omega') = \omega \omega' - \left( x_1^2 + (\omega^2 - x_1^2) \right)^{1/2} (\omega' - x_1) \left( \omega' - x_1 \right)^{1/2} \cos \theta - (148)
\]

For equal mass scattering:

\[
\gamma_2 \times (\omega - \omega') = \omega \omega' - \left( x_1^2 + (\omega^2 - x_1^2) \right)^{1/2} (\omega' - x_1) \left( \omega' - x_1 \right)^{1/2} \cos \theta - (149)
\]
where
\[ x = mc^2 / \beta^2 \quad - (150) \]

By definition:
\[ \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (151) \]

so:
\[
\left( \omega^2 - x^2 \right)^{1/2} \left( \omega' \, 2 - x^2 \right)^{1/2} \cos \theta = \omega \omega' - (\omega - \omega') \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} x - x^2
\quad - (152)
\]

For
\[ \sqrt{\gamma} < c \quad - (153) \]

then:
\[
\left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \sim 1 + \frac{1}{2} \frac{v^2}{c^2} \quad - (154)
\]

so Eq. (152) is approximated by:
\[
\left( \omega^2 - x^2 \right)^{1/2} \left( \omega' \, 2 - x^2 \right)^{1/2} \cos \theta = - \left( (\omega - \omega') (x + \omega) + \frac{1}{2} \frac{v^2}{c^2} x (\omega - \omega') \right)
\quad - (155)
\]

Therefore:
\[
\left( \omega - x \right) \left( \omega + x \right) \left( \omega' - x \right) \left( \omega' + x \right) \cos \theta
\]
\[
= (x - \omega)^2 (x + \omega)^2 + \frac{\sqrt{2}}{c^2} x (\omega - \omega') (x - \omega') (x + \omega) + \frac{1}{4} \frac{v^2}{c^4} (\omega - \omega')^2
\quad - (156)
\]

To order \( \sqrt{\gamma} / c^2 \):
\[
\cos^2 \theta = \frac{x^2 + x (\omega - \omega') (1 + \sqrt{2} / c^2) - \omega \omega'}{x^2 - x (\omega - \omega') - \omega \omega'} \quad - (157)
\]
However:

\[ 0 \leq \cos^2 \theta \leq 1 \quad - (158) \]

so:

\[ (\omega - \omega') \left( 1 + \frac{v^2}{c^2} \right) < - (\omega - \omega') \quad - (159) \]

i.e.:

\[ \omega < \omega' \quad - (160) \]

The conservation of energy equation (143) is:

\[ \omega + \omega_2 = \omega' + \omega'' \quad - (161) \]

so:

\[ \omega' - \omega = \omega_2 - \omega'' \quad - (162) \]

From Eqs. (160) and (162)

\[ \omega_2 > \omega'' \quad - (163) \]

Add Eqs. (160) and (163):

\[ \omega + \omega'' < \omega' + \omega_2 \quad - (164) \]

so conservation of energy is again violated at the fundamental level and the whole of particle scattering theory is refuted, including Higgs boson theory.

4.5 PHOTON MASS AND LIGHT DEFLECTION DUE TO GRAVITATION.

In papers of 1923 and 1924 (L. de Broglie, Comptes Rendues, 77, 507 (1923) and
Phil. Mag., 47, 446 (1924) Louis de Broglie used the concept of photon mass to lock together the Planck theory of the photon as quantum of energy and the theory of special relativity. He derived equations which are referred to as the de Broglie Einstein equations in this book. He quantized the photon momentum, producing wave particle dualism, and these papers led directly to the inference of the Schrödinger equation. In UFT 150B and UFT 155 on www.aias.us, photon mass was shown to be responsible for light deflection and time change due to gravitation and the obsolete methods of calculating these phenomena were shown to be incorrect in many ways. This is an example of a pattern in which the ECE theory as it developed made the old physics entirely obsolete. Photon mass emerged as one of the main counter examples to standard physics - the Higgs boson does not exist because of finite photon mass, which also implies that there a cosmological red shift without an expanding universe. Therefore photon mass also refutes Big Bang, as does spacetime torsion \( \{1 - 10\} \). The red shift can be derived from the original 1924 de Broglie Einstein equations without any further assumption and the de Broglie Einstein equations can be derived from Cartan geometry (chapter one).

The existence of photon mass can be proven as in UFT 157 on www.aias.us with light deflection due to gravitation using the Planck distribution for one photon. The result is consistent with a photon mass of about \( \frac{10}{51} \) for a light beam heated to 2,500 K as it grazes the sun and this result is one of the ways of proving photon mass, inferred by the B(3) field. Prior to this result, estimates of photon mass had been given as less than an upper bound of about \( \frac{52}{10} \) and many methods assumed the validity of the Yukawa potential. These methods have been criticized earlier in this chapter. The Einsteinian theory of light deflection due to gravitation used zero photon mass and is riddled with errors as shown in UFT 150B and UFT155. Therefore the experimental data on light deflection due to gravitation were
thoroughly reinterpreted in UFT 157 to give a reasonable estimate of photon mass. Once photon mass is accepted it works its way through into all the experiments that originally signalled the onset of quantum theory in the late nineteenth century: black body radiation, specific heats, the photoelectric effect, atomic and molecular spectra, and in the nineteen twenties, Compton scattering. As already argued in the context of the Proca equation, photon mass indicates the existence of a vacuum potential, which can be amplified by spin connection resonance to produce energy from spacetime.

The de Broglie Einstein equations are valid in the classical limit of the Proca wave equation of special relativistic quantum mechanics. It has already been shown that the Proca equation is a limit of the ECE wave equation obtained from the tetrad postulate of Cartan geometry and the development of wave equations from the tetrad postulate provides the long sought for unification of gravitational theory and quantum mechanics. The ECE equation of quantum electrodynamics is:

\[(\Box + R)A^a_\mu = 0 \quad (165)\]

where \(R\) is a well defined scalar curvature and where:

\[A^a_\mu = A^{(o)}_a \gamma^a_\mu \quad (166)\]

Here \(A^{(o)}_a\) is the scalar potential magnitude and \(\gamma^a_\mu\) is the Cartan tetrad defined in chapter one. Eq. (165) reduces to the 1934 Proca equation in the limit:

\[R \to \left(\frac{mc}{\hbar}\right)^2 \quad (167)\]

where \(m\) is the mass of the photon, \(c\) is a universal constant, and \(\hbar\) is the reduced Planck constant. Note carefully that \(c\) is not the velocity of the photon of mass \(m\), and following
upon the Palermo memoir of Poincare, de Broglie interpreted $c$ as the maximum velocity available in special relativity.

Eq. (165) in the classical limit is the Einstein energy equation:

$$ p^\mu p_\mu = m^2 c^2 - (168) $$

where:

$$ p^\mu = \left( \frac{E}{c}, \mathbf{p} \right) - (169) $$

and where $m$ is the mass of the photon. Here $E$ is the relativistic energy:

$$ E = \gamma m c^2 - (170) $$

and $p$ is the relativistic momentum:

$$ p = \gamma m v_g - (171) $$

The factor $\gamma$ is the result of the Lorentz transformation and was denoted by de Broglie as:

$$ \gamma = \left( 1 - \frac{v_g^2}{c^2} \right)^{-1/2} - (172) $$

where $v_g$ is the group velocity:

$$ v_g = \frac{d\omega}{dk} \quad - (173) $$

The de Broglie Einstein equations are:

$$ p^\mu = \mathbf{E} k^\mu - (174) $$

where the four wavenumber is:

$$ k^\mu = \left( \frac{\omega}{c}, \mathbf{k} \right) - (175) $$
Eq. (174) is a logically inevitable consequence of the Planck theory of the energy quantum of light later called “the photon”, published in 1901, and the theory of special relativity. The standard model has attempted to reject the inexorable logic of Eq. (174) by rejecting m. Eq. (174) can be written out as:

\[ E = \hbar \omega = \gamma m c^2 \quad (176) \]

and:

\[ p = \hbar \kappa = \gamma m v _g \quad (177) \]

In his original papers of 1923 and 1924 de Broglie defined the velocity in the Lorentz transformation as the group velocity, which is the velocity of the envelope of two or more waves:

\[ v_g = \frac{\Delta \omega}{\Delta \kappa} = \frac{\omega_2 - \omega_1}{\kappa_2 - \kappa_1} \quad (178) \]

and for many waves Eq. (178) applies. The phase velocity \( v_p \) was defined by de Broglie as:

\[ v_p = \frac{E}{p} = \frac{\omega}{\kappa} \quad (179) \]

\[ v_g v_p = c, \quad (180) \]

which is an equation independent of the Lorentz factor \( \gamma \) and universally valid. The standard model makes the arbitrary and fundamentally erroneous assumptions:

\[ m = ? \cdot 0, \quad v_g = v_p = ? \cdot c \quad (180) \]

In physical optics the phase velocity is defined by:

\[ v_p = \frac{\omega}{\kappa} = \frac{c}{\nu} \quad (181) \]
where \( n(\omega) \) is the frequency dependent refractive index, in general a complex quantity (UFT 49, UFT 118 and OO 108 in the Omnia Opera on www.aias.us). The group velocity in physical optics is:

\[
v_g = c \left( n + \omega \frac{dn}{d\omega} \right)^{-1} \tag{182}
\]

and it follows that:

\[
v_p v_g = c^2 = \frac{c^2}{n \left( n + \omega \frac{dn}{d\omega} \right)} \tag{183}
\]

giving the differential equation:

\[
\frac{dn}{d\omega} = -\frac{n}{2\omega} \tag{184}
\]

A solution of this equation is

\[
n = \frac{D}{\omega^{1/2}} \tag{185}
\]

where \( D \) is a constant of integration with the units of angular frequency. So:

\[
n = \left( \frac{\omega_0}{\omega} \right)^{1/2} \tag{186}
\]

where \( \omega_0 \) is a characteristic angular frequency of the electromagnetic radiation. Eq (186) has been derived directly from the original papers of de Broglie \{1 - 10\} using only the equations (181) and (192) of physical optics or wave physics. The photon mass does not appear in the final Eq. (186) but the photon mass is basic to the meaning of the calculation. If \( \omega_0 \) is interpreted as the emitted angular frequency of light in a far distant star, then \( \omega_0 \) is the angular frequency of light reaching the observer. If:

\[
n > 1 \tag{187}
\]
then:
\[ \omega < \omega_0 \quad (188) \]

and the light has been red shifted, meaning that its observable angular frequency \( \omega \) is lower than its emitted angular frequency \( \omega_0 \), and this is due to photon mass, not an expanding universe. The refractive index \( n(\omega) \) is that of the spacetime between star and observer. Therefore in 1924 de Broglie effectively explained the cosmological red shift in terms of photon mass. "Big Bang" (a joke coined by Hoyle) is now known to be erroneous in many ways, and was the result of imposed and muddy pathology supplanting the clear science of de Broglie.
observer. Therefore in 1924 de Broglie effectively explained the cosmological red shift in terms of photon mass. "Big Bang" (a joke coined by Hoyle) is now known to be erroneous in many ways, and was the result of imposed and muddy pathology supplanting the clear science of de Broglie.

In 1924 de Broglie also introduced the concept of least (or "rest") angular frequency:

$$\hbar \omega_0 = mc^2$$  \hspace{1cm} (189)

and kinetic angular frequency $\omega_K$. The latter can be defined in the non relativistic limit:

$$\hbar \omega = mc^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx mc^2 + \frac{1}{2} m v^2$$  \hspace{1cm} (190)

so:

$$\hbar \omega_K \approx \frac{1}{2} m v^2$$  \hspace{1cm} (191)

Similarly, in the non relativistic limit:

$$\hbar k = m v_0 + \frac{1}{2} m v_0^2 / c^2$$  \hspace{1cm} (192)

so the least wavenumber, $k_0$, is:

$$\hbar k_0 \approx m v_0$$  \hspace{1cm} (193)

and the kinetic wavenumber is:

$$\hbar k_K \approx \frac{1}{2} m v_0^3 / c^2$$  \hspace{1cm} (194)

The total angular frequency in this limit is:

$$\omega = \omega_0 + \omega_K$$  \hspace{1cm} (195)
and the total wavenumber is:

\[ K = K_0 + K_K \quad -(196) \]

The kinetic energy of the photon was defined by de Broglie by omitting the least (or “rest”) frequency:

\[ T = \frac{\hbar \omega_K}{2} \sim \frac{1}{2} m v_g^2 = \frac{p^2}{2m} \quad -(197) \]

where:

\[ p = m v_g \quad -(198) \]

Using Eqs. (189) and (193) it is found that:

\[ V_p = \frac{c^2}{\sqrt{v_g^2}} = \frac{\omega_o}{K_0} \quad -(199) \]

and using Eqs. (191) and (194)

\[ V_p = \frac{c^2}{\sqrt{g^2}} = \frac{\omega_K}{K_K} \quad -(200) \]

Therefore:

\[ V_p = \frac{\omega}{K} = \frac{\omega_0 + \omega_K}{K_0 + K_K} \quad -(201) \]

a possible solution of which is:

\[ \frac{\omega_K}{K_0} = V_p \quad -(202) \]

Using Eqs. (193) and (191):

\[ \frac{\omega_K}{K_0} = \frac{1}{2} v_g \quad -(203) \]

so it is found that in these limits:
The work of de Broglie has been extended in this chapter to give a simple derivation of the cosmological red shift due to the existence of photon mass, and conversely, the red shift is a cosmological proof of photon mass. In standard model texts, photon mass is rarely discussed, and the work of de Broglie is distorted and never cited properly. The current best estimate of photon mass is of the order of $10^{-52}$ kg. In UFT 150B and UFT 155 on www.aias.us the photon mass from light deflection was calculated as:

$$m = \frac{R_0}{c^2 a} \quad -(205)$$

using:

$$E = \hbar \omega. \quad -(206)$$

This gave the result:

$$m = 3.35 \times 10^{-41} \text{ kg} \quad -(207)$$

Here $R_0$ is the distance of closest approach, taken to be the radius of the sun:

$$R_0 = 6.955 \times 10^8 \text{ m} \quad -(208)$$

and $a$ is a distance parameter computed to high accuracy:

$$a = 3.3765447822 \times 10^9 \text{ m} \quad -(209)$$

In a more complete theory, given here, the photon in a light beam grazing the sun has a mean energy given by the Planck distribution $(1 - 10)$:

$$\langle E \rangle = \frac{\hbar \omega}{e^{\frac{\hbar \omega}{kT}} - 1} \quad -(210)$$
where \( k \) is Boltzmann's constant and \( T \) the temperature of the photon. It is found that a photon mass of:

\[
m = 9.14 \times 10^{-32} \text{ kg} - (2.11)
\]

is compatible with a temperature of 2,500 K. The temperature of the photosphere at the sun's surface is 5,778 K, while the temperature of the sun's corona is 1 - 3 million K. Using Eq. (17), it is found that:

\[
\sqrt{g} = 2.99757 \times 10^8 \text{ m s}^{-1} - (2.12)
\]

which is less than the maximum speed of relativity theory:

\[
c = 2.9979 \times 10^8 \text{ m s}^{-1} - (2.13)
\]

As discussed in Note 157(13) the mean energy \( < E > \) is related to the beam intensity \( I \) in joules per square metre by

\[
I = \frac{8\pi f^2}{c} < E > - (2.14)
\]

where \( f \) is the frequency of the beam in hertz. The intensity can be expressed as:

\[
I = \frac{8\pi f^2 m}{c^3} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - (2.15)
\]

The total energy density of the light beam in joules per cubic metre is:

\[
\mathcal{U} = \frac{1}{c} I - (2.16)
\]

and its power density in watts per square metre (joules per second per square metre) is:

\[
\mathcal{F} = c \mathcal{U} = \frac{g}{c} I = 8\pi f^3 m \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - (2.17)
\]
The power density is an easily measurable quantity, and implies finite photon mass through Eq. (217). In the standard model there is no photon mass, so there is no power density, an absurd result. The power density is related to the magnitude of the electric field strength (E) and the magnetic flux density (B) of the beam by:

\[ \Phi = \frac{\varepsilon_0 E^2}{\mu_0} \tag{218} \]

The units in S.I. are as follows:

\[ E = \text{volt m}^{-1} = \frac{J}{C m} \]
\[ B = \text{tesla} = \frac{Wb}{m^2} \]
\[ \varepsilon_0 = \frac{F}{J m} \]
\[ \mu_0 = \frac{H}{A m} \]

where \( \varepsilon_0 \) and \( \mu_0 \) are respectively the vacuum permittivity and permeability defined by:

\[ \varepsilon_0 \mu_0 = \frac{1}{c^2} \tag{220} \]

so:

\[ \Phi = \frac{8\pi}{3} \rho m \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \varepsilon_0 c E^2 = \frac{c B^2}{\mu_0} \tag{221} \]

4.6 DIFFICULTIES WITH THE EINSTEIN THEORY OF LIGHT DEFLECTION DUE TO GRAVITATION.

The famous Einstein theory of light deflection due to gravitation is based on the idea of zero photon mass because in 1905 Einstein inferred such an idea from the basics of special relativity, he conjectured that a particle can travel at c if and only if its mass is identically zero, and assumed that photons travelled at c. Poincare on the other hand realized that photons can travel at less than c if they have mass, and that c is the constant in the Lorentz transform. The Einsteinian calculation of light deflection due to gravitation was therefore based on the then new general relativity applied with a massless particle. In the
influential UFT 150B on www.aias.us it was shown that Einstein's method contains several fundamental errors. However precisely measured, such data cannot put right these errors, and the Einstein theory is completely refuted experimentally in whirlpool galaxies, so that it cannot be used anywhere in cosmology.

The Einstein method is based on the gravitational metric:

$$ds^2 = c^2 dt^2 - c^2 \left(1 - \frac{r_o}{r}\right)^{-2} dr^2 - \frac{r dr}{\left(1 - \frac{r_o}{r}\right)} - r^2 d\phi^2 \quad (222)$$

usually and incorrectly attributed to Schwarzschild. Here, cylindrical polar coordinates are used in the XY plane. In Eq. (220) $r_o$ is the so called Schwarzschild radius, the particle of mass $m$ orbits the mass $M$, for example the sun. The infinitesimal of proper time is $d\tau$.

The lagrangian for this calculation is:

$$\mathcal{L} = \frac{m}{2} \left(\left(\frac{dt}{d\tau}\right)^2 \left(1 - \frac{r_o}{r}\right)^{-2} \left(1 - \frac{r_o}{r}\right)^{-1} \left(\frac{dr}{d\tau}\right)^2 - \left(\frac{d\phi}{d\tau}\right)^2 \right) \quad (223)$$

and the total energy and momentum are given as the following constants of motion:

$$E = mc^2 \left(1 - \frac{r_o}{r}\right) \frac{dt}{d\tau}, \quad L = mr^2 \frac{d\phi}{d\tau} \quad (224)$$

Since $m \ll M$ the Schwarzschild radius is:

$$r_o = \frac{2MG}{c^2} \quad (225)$$

Therefore the calculation assumes that the mass $m$ is not zero. For light grazing the sun, this is the photon mass.

The equation of motion is obtained from Eq. (222) by multiplying both sides by $\left(1 - \frac{r_o}{r}\right)$ to give:

$$m \left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - \left(1 - \frac{r_o}{r}\right) \left(mc^2 + \frac{L^2}{mr^2}\right) \quad (226)$$
The infinitesimal of proper time is eliminated as follows:

\[ \frac{d\tau}{d\tau} = \frac{d\phi}{d\tau} \frac{dx}{d\phi} = \left( \frac{L^2}{mc^2} \right) \frac{dx}{d\phi} \] 

\[ \frac{dx}{d\phi} \]

to give the orbital equation:

\[ \left( \frac{dx}{d\phi} \right)^2 = 4 \left( \frac{1}{b^2} - \left( 1 - \frac{r_0}{r} \right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right) \] 

\[ (228) \]

where the two constant lengths a and b are defined by:

\[ a = \frac{L}{mc}, \quad b = \frac{cL}{E} \] 

\[ (229) \]

The solution of Eq. \( (228) \) is:

\[ \phi = \int \frac{1}{r^2} \left( \frac{1}{b^2} - \left( 1 - \frac{r_0}{r} \right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-1/2} \, dr \] 

\[ (230) \]

and the light deflection due to gravitation is:

\[ \Delta \phi = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left( \frac{1}{b^2} - \left( 1 - \frac{r_0}{r} \right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-1/2} \, dr \] 

\[ (231) \]

where \( R_0 \) is the distance of closest approach, essentially the radius of the sun. Using:

\[ u = 1 / r, \quad du = - \frac{1}{r^2} \, dr \] 

\[ (232) \]

the integral may be rewritten as:

\[ \Delta \phi = 2 \int_{0}^{R_0} \left( \frac{1}{b^2} - \left( 1 - r_0 u \right) \left( \frac{1}{a^2} + u^2 \right) \right)^{-1/2} \, du \] 

\[ (233) \]

If we are to accept the gravitational metric for the sake of argument its correct use must be to assume an identically non zero photon mass and to integrate Eq. \( (233) \), producing an equation for the experimentally observed deflection \( \Delta \phi \) in terms of \( m, a \) and \( b \).

However, because of his conjecture of zero photon mass, Einstein used the null
geodesic condition:

\[ ds^2 = 0 \quad -(234) \]

which means that \( m \) is identically zero. This assumption means that:

\[ a = \infty \quad -(235) \]

However, the angular momentum \( L \) is a constant of motion, so Eq. \((235)\) means:

\[ m = 0, \quad \frac{d\phi}{d\tau} = \infty \quad -(236) \]

which in the obsolete physics of the standard model was known as the ultrarelativistic limit. In this Einsteinian light deflection theory Eq. \((223)\) is defined to be pure kinetic in nature, but at the same time the theory sets up an effective potential:

\[ V(r) = \frac{1}{2} m c^2 \left( -\frac{\alpha}{r} + \frac{a}{r^2} - \frac{\alpha a}{r^3} \right) \quad -(237) \]

and also assumes circular orbits:

\[ \frac{dr}{d\tau} = 0 \quad -(238) \]

However, this assumption means that:

\[ \frac{1}{b^2} = \left( 1 - \frac{\alpha}{r} \right) \left( \frac{1}{a} + \frac{1}{r^2} \right) \quad -(239) \]

and the denominator of Eq. \((230)\) becomes zero and the integral becomes infinite. In order to circumvent this difficulty Einstein assumed:

\[ \frac{\alpha}{r} \to 0 \quad -(240) \]
which must mean:
\[ r \to \infty \quad - (241) \]
and
\[ m \to 0, \quad \alpha \to \infty. \quad - (242) \]

The effective potential was therefore defined as:
\[
\begin{align*}
\lim_{m \to 0, \alpha \to \infty, r \to \infty} V(r) &= mc^2 \left( \frac{q}{r} \right) \left( 1 - \frac{r_0}{r} \right) \quad - (243)
\end{align*}
\]
which is mathematically indeterminate. Einstein also assumed:
\[
mc^2 \to 0 \quad - (244)
\]

so the equation of motion (249) becomes:
\[
\frac{E^2}{2mc^2} = \frac{L^2}{mr^2} \left( \frac{1}{2} - \frac{m^2}{c^2 r} \right) \quad - (245)
\]
He used:
\[
\mathbf{r} = R_0 \quad - (246)
\]
in this equation, thus finding an expression for \( \mathbf{b}_0 \):
\[
\frac{1}{b_0^2} = \frac{1}{R_0^2} - \frac{r_0^3}{R_0^3} \quad - (247)
\]
Finally he used Eq. (247) in Eq. (233) with:
\[
a^2 \to \infty \quad - (248)\]
to obtain the integral:

\[
\Delta \varphi = 2 \int_0^{1/R_0} \left( \frac{R_0 - r_0}{R_0^3} - u^2 + r_0 u \right)^{-\frac{1}{2}} \, du.
\]

It was claimed by Einstein that this integral is:

\[
\Delta \varphi = \frac{4M_0G}{c^2 R_0} - (250) - (249)
\]

but this is doubtful for reasons described in UFT 150B, whose calculations were all carried out with computer algebra. The experimental result for light grazing the sun is given for example by NASA Cassini as

\[
\Delta \varphi = 1.75'' = 8.484 \times 10^{-6} \text{ rad}, - (251)
\]

but depends on the assumption of data such as:

- \( R_0 = 6.955 \times 10^8 \text{ m}, M = 1.9891 \times 10^{30} \text{ kg}, \)
- \( \mu = 6.67428 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \) - (252)

In fact only \( MG \) is known with precision experimentally, not \( M \) and \( G \) individually. The radius \( R_0 \) is subject to considerable uncertainty. If we accept the dubious gravitational metric for the sake of argument, the experimental data must be evaluated from Eq. (251) with finite photon mass, and independent methods used to evaluate \( a \) and \( b \).

Einstein’s formula (249) for light deflection depends on the radius parameters \( R_0 \) and \( r_0 \). \( R_0 \) represents the radius of the sun (6.955 \( \times 10^8 \) metres) while the so-called Schwarschild radius \( r_0 \) is 2,954 metres. So:

\[
r_0 \ll R_0 - (253)\]
which implies from Eq. (247) that:

\[ b_0 \approx R_0. - (254) \]

This gives the integral:

\[ \Delta \phi = 2 \int_0^{R_0} \left( \frac{R_0 - r}{R_0^3} \right) du - \pi \]

which has no analytical solution. Its numerical integration is also difficult, even with contemporary methods. The square root in the integral has zero crossings, leading to infinite values of the integrand and as discussed in Section 3 of UFT 150B there is a discrepancy between the experimental data, Einstein’s claim and the numerical evaluation of the integral.

The correct method of evaluating the light deflection is obviously to use a finite mass \( m \) in Eq. (231). In a first rough approximation, UFT 150B used:

\[ E = \frac{hc}{\lambda} - (256) \]

for one photon. More accurately a Planck distribution can be used. However Eq. (256) gives:

\[ \alpha = \frac{hc}{mc^2} b. - (257) \]

The parameter \( b \) is a constant of motion, and is determined by the need for zero deflection when the mass \( M \) of the sun is absent. This gives:

\[ \Delta \phi = 2 \int_0^{R_0} \left( \frac{1}{b^2} - u^2 \right)^{-1/2} du - \pi = 0 \]

and as described in UFT 150B this gives a photon mass of:

\[ m = 3.35 \times 10^{-41} \text{ kg} - (259) \]

which again a lot heavier than the estimates in the standard literature.
So in summary of these sections, the B(3) field implies a finite photon mass which can be estimated by Compton scattering and by light deflection due to gravitation. The photon mass is not zero, but an accurate estimate of its value needs refined calculations. These are simple first attempts only. There are multiple problems with the claim that light deflection by the sun is twice the Newtonian value, because the latter is itself heuristic, and because Einstein's methods are dubious, as described in UFT 150B and UFT 155. The entire Einstein method is refuted by its neglect of torsion, as explained in great detail in the two hundred and sixty UFT papers available to date.