Reduction of the ECE Theory of Electromagnetism to the Maxwell-Heaviside Theory

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Abstract

It is shown here that for a single polarization, the ECE theory of electromagnetism reduces to that of Maxwell-Heaviside and that only one possible reduction to this form exists. A general gauge condition is introduced combining the antisymmetry relations of ECE, that offers the possibility of resonant solutions. It is also shown that the Lindstrom Constraint developed earlier is not of a general enough nature to be useable for most electromagnetic calculations.

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1. Introduction

It is common practice and almost always expected in physics, that when a new theory is developed which explains poorly or previously unexplained observations, that this new theory should reduce in some fashion to that which was already developed. By assuming torsion in the definition of space-time, and using Cartan geometry, a new physics results, one component of which is a new theory of electromagnetism, coined the ECE theory of electromagnetism [1].

A new theory in physics has to pass into existing theory in cases where well-known effects are described which do not require the new theory. Therefore the electromagnetic part of ECE theory has to become identical to Maxwell-Heaviside theory when effects of general relativity are not important or to be discarded. In earlier stages of ECE theory this was done by assuming the spin connections (representing curvature and torsion of space in general relativity) to be negligible. Then the ECE field equations reduce to the Maxwell-Heaviside equations directly. However, after discovery of additional conditions imposed by the underlying Cartan geometry, called the antisymmetry constraints [2-4], this simple procedure leads to a violation of these conditions. Therefore we investigated all possible ways of transition and found that only one method is consistent with the antisymmetry constraints. New insight into the principle nature of relativistic effects has been found. In chapter 2 we describe the correct transition and in chapter 3 the result is discussed.

The definition of electric intensity in ECE theory for a single polarization is [2, 3]

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_o \mathbf{A} + \boldsymbol{\omega}\phi \,. \tag{1}$$

As a result of fundamental antisymmetries in Cartan geometry, a new equation that constrains the definition of **E** in equation (1) is introduced. This electric antisymmetry equation is [4]

$$\frac{\partial \mathbf{A}}{\partial t} - \underline{\nabla}\phi + \omega_0 \mathbf{A} + \boldsymbol{\omega}\phi = 0 \, . \tag{2}$$

The ECE definition for **B** is [1]

$$\mathbf{B} = \underline{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \tag{3}$$

and correspondingly, the magnetic antisymmetry equation is [4]

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \omega_j A_k + \omega_k A_j = 0 \tag{4}$$

where the Einstein convention of summation over repeated indices is not implied. The Einstein convention will not be used anywhere in this paper unless so noted. **E** is the electric intensity, **B** is the magnetic induction, **A** is the magnetic vector potential, ϕ is the electric scalar potential, $\boldsymbol{\omega}$ is the vector spin connection and ω_o is the scalar spin connection. We will assume in this analysis that the active medium is a vacuum so that complications introduced when using a more complex medium is avoided.

Equations (1) through (4) define the fundamentals of ECE electromagnetic theory in vector form, for a single polarization.

2. Reduction of ECE EM Theory to that of Maxwell-Heaviside

For equation (1) to reduce to the Maxwell-Heaviside definition

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \, ,$$

one of four things must happen;

- $-\omega_o \mathbf{A} + \boldsymbol{\omega}\phi = 0$ or (5)
- $-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} = -\omega_o \mathbf{A} + \boldsymbol{\omega}\phi$ or (6)
- $\omega_o = 0$ and $\boldsymbol{\omega} = 0$ or (7)
- $\omega_o \mathbf{A} = \nabla\phi$ and $\frac{\partial \mathbf{A}}{\partial t} = -\boldsymbol{\omega}\phi$ (8)

For equation (3) to be compatible with Maxwell-Heaviside definition

$$\mathbf{B} = \underline{\nabla} \times \mathbf{A}$$

one of three things must happen;

- $\boldsymbol{\omega} = 0$ or (9)
- $\underline{\nabla} \times \mathbf{A} = -\boldsymbol{\omega} \times \mathbf{A}$ or (10)
- $\boldsymbol{\omega} \times \mathbf{A} = 0$ (11)

These options are summarized in the following table. The electric antisymmetry equation (2) has been applied to the calculations of **E** and **B**. There is not a requirement that the vector and scalar potentials be identical in both theories; they must be consistent throughout when going from the ECE representation to the Maxwell-Heaviside representation.

Table 1 Options for Reducing ECE Electromagnetic Theory to that of Maxwell-Heaviside

Option		**E**	$\omega_0 \mathbf{A}$	$\boldsymbol{\omega}\phi$	**B**	$\boldsymbol{\omega}\times\mathbf{A}$
1	$\omega_o \mathbf{A} = \boldsymbol{\omega}\phi$	$-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$	$\frac{1}{2}\left(-\frac{\partial \mathbf{A}}{\partial t} + \underline{\nabla}\phi\right)$	$\frac{1}{2}\left(-\frac{\partial \mathbf{A}}{\partial t} + \underline{\nabla}\phi\right)$	$\underline{\nabla}\times\mathbf{A}$	0
2	$-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} =$ $-\omega_o \mathbf{A} + \boldsymbol{\omega}\phi$	$2\left(-\frac{\partial \mathbf{A}}{\partial t} - \omega_o \mathbf{A}\right)$	$\frac{\partial \mathbf{A}}{\partial t}$	-	$\underline{\nabla}\times\mathbf{A}$ *	$-\frac{1}{\phi}(\nabla\phi$ $+\frac{\partial \mathbf{A}}{\partial t})\times\mathbf{A}$
3	$\omega_o = 0$ $\boldsymbol{\omega} = 0$	$-2\frac{\partial \mathbf{A}}{\partial t}$	0	0	$\underline{\nabla}\times\mathbf{A}$	0
4	$\omega_o \mathbf{A} = \nabla\phi$ $\frac{\partial \mathbf{A}}{\partial t} = -\boldsymbol{\omega}\phi$	$2\left(-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}\right)$	$\nabla\phi$	$-\frac{\partial \mathbf{A}}{\partial t}$	$\underline{\nabla}\times\mathbf{A}$ *	$-\frac{1}{\phi}\frac{\partial \mathbf{A}}{\partial t}\times\mathbf{A}$
5	$\boldsymbol{\omega} = 0$	$-2\nabla\phi$	$-\frac{\partial \mathbf{A}}{\partial t} + \underline{\nabla}\phi$	0	$\underline{\nabla}\times\mathbf{A}$	0
6	$\underline{\nabla}\times\mathbf{A} = -\boldsymbol{\omega}\times\mathbf{A}$	$-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$ $-\omega_o \mathbf{A} + \boldsymbol{\omega}\phi$	-	-	$2\underline{\nabla}\times\mathbf{A}$	$-\nabla\times\mathbf{A}$
7	$\boldsymbol{\omega}\times\mathbf{A} = 0$	$-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$ $-\omega_o \mathbf{A} + \boldsymbol{\omega}\phi$	-	-	$\underline{\nabla}\times\mathbf{A}$	0

* for harmonic functions typically $\frac{1}{\phi}\frac{\partial \mathbf{A}}{\partial t}\times\mathbf{A} = 0$

As seen in the above table, option 1 and its variations, is a possible option for reducing ECE theory to that of Maxwell-Heaviside, and in fact is the only option.

Option (2) is not valid because of conflicting factor in the definition of **B.**

Option (3) is not valid because the definition of **E** is incorrect for Maxwell-Heaviside.

Option (4) is not valid for the same reasons as option (2).

Option (5) is not valid because the definition of **E** is incorrect for Maxwell-Heaviside.

Option (6) is not valid, since it is the Lindstrom Constraint [3], which is shown to be too restrictive for general Maxwell-Heaviside theory (see Appendix). In addition, the spin connections still appear in **E**.

Option (7) is not valid because the spin connections still appear in **E**.

An examination of option 1, the only viable option, has that as a result of $\boldsymbol{\omega}\times\mathbf{A}=0$, equation (4) for this option becomes a new constraint, given by

$$\frac{\partial A_k}{\partial x_j}+\frac{\partial A_j}{\partial x_k}+2\omega_j A_k=\frac{\partial A_k}{\partial x_j}+\frac{\partial A_j}{\partial x_k}+\frac{2\omega_0}{\phi}A_j A_k=0. \tag{12}$$

We note that

$$\boldsymbol{\omega}\phi=\omega_0\mathbf{A}=\frac{1}{2}\left(-\frac{\partial\mathbf{A}}{\partial t}+\underline{\nabla}\phi\right). \tag{13}$$

Given equation (13), $\boldsymbol{\omega}\times\mathbf{A}=0$, means that

$$\underline{\nabla}\phi\times\mathbf{A}-\frac{\partial\mathbf{A}}{\partial t}\times\mathbf{A}=0. \tag{14}$$

For functions that can be represented as harmonic functions,

$$\frac{\partial\mathbf{A}}{\partial t}\times\mathbf{A}=0$$

so that $\underline{\nabla}\phi$ is parallel to $\mathbf{A}$. Note that $\boldsymbol{\omega}$ is also parallel to **A.**

Equation (12) becomes upon substitution of equation (13)

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \frac{1}{\phi}\left(-\frac{\partial A_j}{\partial t} + \frac{\partial \phi}{\partial x_j}\right)A_k = 0 \tag{15}$$

or upon re-arranging to eliminate the singularity in ϕ,

$$\left(\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k}\right)\phi + \left(-\frac{\partial A_j}{\partial t} + \frac{\partial \phi}{\partial x_j}\right)A_k = 0\,. \tag{16}$$

This equation, the equivalent of both antisymmetry equations, can be written in vector notation as

$$\left(\underline{\nabla}\mathbf{A} + (\underline{\nabla}\mathbf{A})^T\right)\phi = \left(\frac{\partial \mathbf{A}}{\partial t} - \underline{\nabla}\phi\right) \otimes \mathbf{A} \tag{17}$$

where $\otimes$ is the symbol for the outer product (matrix multiplication), and

$$\underline{\nabla}\mathbf{A} = \begin{pmatrix} \frac{\partial A_1}{\partial x_1} & \frac{\partial A_2}{\partial x_1} & \frac{\partial A_3}{\partial x_1} \\ \frac{\partial A_1}{\partial x_2} & \frac{\partial A_2}{\partial x_2} & \frac{\partial A_3}{\partial x_2} \\ \frac{\partial A_1}{\partial x_3} & \frac{\partial A_2}{\partial x_3} & \frac{\partial A_3}{\partial x_3} \end{pmatrix}.$$

The superscript ^{T} refers to the transpose of the matrix. Because the indices j, k are parts of a permutation of (1, 2, 3), only three off-diagonal matrix elements of (17) are to be considered.

If we define, which we can do since the spin connection terms do not appear in this reduction of the ECE theory, in the following manner

$$\underline{\nabla}\boldsymbol{\omega} = \underline{\nabla}\mathbf{A} + (\underline{\nabla}\mathbf{A})^T\,, \tag{18}$$

Equation (12), the magnetic antisymmetry equation, becomes

$$\frac{\partial \omega_i}{\partial x_j} + 2\omega_i A_j = 0 \tag{19}$$

which is equivalent to equation (15). Since $\boldsymbol{\omega}$ is unspecified in this theory, this equation does not limit **A** in any way. Thus the magnetic antisymmetry equation is not violated in this restricted theory.

We note that from the definition of option (1), that with equation (2), the electric antisymmetry equation becomes

$$\underline{\nabla} \cdot (\omega_o \mathbf{A}) = \underline{\nabla} \cdot (\boldsymbol{\omega}\phi) = \frac{1}{2}\underline{\nabla} \cdot \left(-\frac{\partial \mathbf{A}}{\partial t} + \underline{\nabla}\phi \right) = \frac{1}{2}\left(-\frac{\partial \underline{\nabla} \cdot \mathbf{A}}{\partial t} + \nabla^2 \phi \right). \tag{20}$$

For a homogeneous wave in ϕ:

$$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = 0 . \tag{21}$$

Assuming a vanishing derivative of the spin connection terms in Equation (20),

$$\underline{\nabla} \cdot (\omega_o \mathbf{A}) = \underline{\nabla} \cdot (\boldsymbol{\omega}\phi) = 0 , \tag{22}$$

we obtain by means of (21) and integrating over time:

$$-\underline{\nabla} \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 . \tag{23}$$

This equation is very similar to the Lorenz Condition, often used as a gauge in Maxwell-Heaviside theory. The difference is the sign of the divergence term. If we assume the original Lorenz condition

$$\underline{\nabla} \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0 \tag{24}$$

to be valid instead of (23), we obtain by time integration and adding $\underline{\nabla} \cdot \mathbf{A}$ on both sides of Equation (20):

$$\int \underline{\nabla} \cdot (\omega_o \mathbf{A}) dt = \int \underline{\nabla} \cdot (\boldsymbol{\omega} \phi) dt = -\underline{\nabla} \cdot \mathbf{A} \tag{25}$$

or in differential form:

$$\underline{\nabla} \cdot (\omega_o \mathbf{A}) = \underline{\nabla} \cdot (\boldsymbol{\omega} \phi) = -\frac{\partial}{\partial t} \underline{\nabla} \cdot \mathbf{A}. \tag{26}$$

Both formulations (23) and (24) satisfy the electric as well as the magnetic antisymmetry equation as long as the additional assumptions for the spin connection terms (22) and (26) are valid.

3. Conclusions

We have thus demonstrated that if we assume

$$\omega_o \mathbf{A} = \boldsymbol{\omega} \phi$$

in the ECE theory of electromagnetism, then this theory reduces to the Maxwell-Heaviside theory. We have also noted in passing that the Lindstrom Constraint, which is a limited form of the magnetic antisymmetry equation, generates a solution that is equivalent, for the magnetic vector, to the ECE vacuum. This is not general enough for most electromagnetic applications.

We note that if we assume option 1 as expressed in equation (13) then equation (16) becomes a gauge condition. It is the most general gauge condition that is compatible with ECE theory[1] when reduced to the classical limit. From equation (16), for example we can see that a certain kind of "resonance" is possible: If the electric potential ϕ is near to zero, the derivatives of the vector potential **A** can take huge values and vice versa. This means that an electrical device driven by pulsed magnetic or electric signals could show anomalous effects.

Another more principle result is that, by equating both types of spin connections in option 1, we do not assume that both connections vanish. This means that effects of general relativity are always present even if they do not lead to measurable effects. This is different from the conventional view that one only has to "switch on" effects of relativity, when discrepancies between experiment and non-relativistic theory appear. Another example of this in classical physics is fluid dynamics. It has recently been shown [6] that a velocity field is generally described by Cartan geometry where the spin connections are equivalent to turbulence effects of the fluid. Nobody had assigned these effects to general relativity before.

[1] It should be noticed that in ECE theory there is no gauge because all potentials are defined as absolute values.

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Appendix: Limitations Imposed When Using the Lindstrom Constraint

The magnetic antisymmetry equation for a single polarization is

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \omega_j A_k + \omega_k A_j = 0\,. \tag{A-1}$$

The Lindstrom Constraint [4], a limited form of the magnetic antisymmetry equation, is given by

$$\frac{\partial A_k}{\partial x_j} - \frac{\partial A_j}{\partial x_k} + \omega_k A_j - \omega_j A_k = 0\,. \tag{A-2}$$

For equation (A-2) to be a sub-set of equation (A-1), we need

$$\frac{\partial A_j}{\partial x_k} + \omega_j A_k = 0 \tag{A-3}$$

which upon substitution into equation (A-2) requires that

$$\frac{\partial A_k}{\partial x_j} + \omega_k A_j = 0\,. \tag{A-4}$$

Equations (A-3) and (A-4) solve to give

$$\omega_1 = -\frac{1}{A_2}\frac{\partial A_1}{\partial x_2} = -\frac{1}{A_3}\frac{\partial A_1}{\partial x_3}\,, \tag{A-5}$$

$$\omega_2 = -\frac{1}{A_3}\frac{\partial A_2}{\partial x_3} = -\frac{1}{A_1}\frac{\partial A_2}{\partial x_1}\,, \tag{A-6}$$

$$\omega_3 = -\frac{1}{A_1}\frac{\partial A_3}{\partial x_1} = -\frac{1}{A_2}\frac{\partial A_3}{\partial x_2}\,. \tag{A-7}$$

These equations form part of the equation set that defines the ECE vacuum [5] and has solution for **A** given by

$$\mathbf{A} = \sum \mathbf{k} D_n Tanh(\mathbf{k}\cdot\mathbf{r} - \beta t)^n\,. \tag{A-8}$$

This solution is too limiting for a general electromagnetic problem, the limitation arising from not having three perpendicular waves with independent amplitudes.