19

The Derivation of O(3) Electrodynamics from The Evans Unified Field Theory

Summary. The theory of O(3) electrodynamics is derived from the generally covariant Evans unified field theory as a well defined particular case. The latter is described by spin connection components of the electromagnetic field after it has split from the gravitational field during the course of billions of years of evolution.

Key words: O(3) electrodynamics; generally covariant unified field theory; spin connection elements.

19.1 Introduction

Recently a generally covariant unified field theory of gravitation and electromagnetism has been developed [1]–[25] from a precursor gauge field theory - O(3) electrodynamics. The unified field theory is based on differential geometry, and completes Einstein's search [26] of 1925 to 1955 for a structure that is a logical extension of his earlier 1915 theory of gravitation. The latter is based on a Riemann geometry in which the torsion tensor is zero by construction. Einstein subsequently made attempts to develop a unified field theory in which electromagnetism, as well as gravitation, is thought of as a generally covariant property of spacetime. In so doing, spin or torsion has to be incorporated within general relativity. To obtain a unified field theory it is therefore insufficient to replace derivatives by covariant derivatives with Christoffel connections, as in the Einstein Maxwell equations. The reason is that the Christoffel connection is symmetric in its lower two indices, so that the torsion tensor is subsequently zero by construction:

$$T^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\mu\nu} - \Gamma^{\kappa}{}_{\nu\mu} = 0. \tag{19.1}$$

This means that spin is not self-consistently included in the Einstein Maxwell equations. Einstein realized this and pursued the correct geometry for thirty years. Evidently this has to be a geometry in which the torsion tensor is non-zero and this is differential geometry [27]. It has been shown recently [1]–[25]

that this geometry can be translated directly into a generally covariant unified field theory in which the basic building block is the vector-valued tetrad one-form $q^a{}_\mu$. The potential field of the electromagnetic sector is then defined by:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu \tag{19.2}$$

where $A^{(0)}$ is a C negative coefficient originating in the primordial fluxon $\hbar/e$. The electromagnetic sector of the unified field is defined by the covariant exterior derivative:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{19.3}$$

where $d\wedge$ denotes the exterior derivative and $\omega^a{}_b$ the spin-connection.

In Section 19.2 the structure of O(3) electrodynamics is obtained from Eq. (19.3), thus proving that O(3) electrodynamics is a well defined limit of the Evans unified field theory. A brief discussion of this result is given in Section 19.3.

19.2 The Limit of O(3) Electrodynamics

The spin connection in the unified field theory [1]–[25] must also obey the following cyclic equation:

$$\begin{aligned} d \wedge F^a &= A^{(0)} \left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \right) = \mu_0 j^a \\ &\sim 0. \end{aligned} \tag{19.4}$$

In vector notation Eq. (19.4) is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}^a = 0 \tag{19.5a}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E}^a + \frac{\partial \boldsymbol{B}^a}{\partial t} = 0 \tag{19.5b}$$

and for each index a Eq. (19.5a) is the Gauss law of magnetism and Eq.(19.5b) is the Faraday law of induction Eq.(19.4) is the experimental constraint:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \tag{19.6}$$

or free space condition. Using the Maurer-Cartan structure equations:

$$T^b = D \wedge q^b \tag{19.7}$$

$$R^a{}_b = D \wedge \omega^a{}_b \tag{19.8}$$

it is seen that a particular solution of Eq. (19.6) is:

$$\omega^a{}_b = -\frac{1}{2} \kappa \epsilon^a{}_{bc} q^c \tag{19.9}$$

where κ is a wavenumber and

$$\epsilon^a{}_{bc} = g^{ad}\epsilon_{dbc}. \tag{19.10}$$

Here

$$g^{ad} = \mathrm{diag}(1, -1, -1, -1) \tag{19.11}$$

is the metric of the orthonormal tangent spacetime and ϵ_{dbc} is the Levi-Civita symbol.

It follows that:

$$F^1 = d \wedge A^1 + gA^2 \wedge A^3 \tag{19.12}$$
$$F^2 = d \wedge A^2 + gA^3 \wedge A^1 \tag{19.13}$$
$$F^3 = d \wedge A^3 + gA^1 \wedge A^2 \tag{19.14}$$

where

$$g = \frac{\kappa}{A^{(0)}}. \tag{19.15}$$

In the complex circular basis [1]–[25], Eqs.(19.12)–(19.14) become:

$$F^{(1)*} = d \wedge A^{(1)*} - igA^{(2)} \wedge A^{(3)} \tag{19.16}$$
$$F^{(2)*} = d \wedge A^{(2)*} - igA^{(3)} \wedge A^{(1)} \tag{19.17}$$
$$F^{(3)*} = d \wedge A^{(3)*} - igA^{(1)} \wedge A^{(2)}, \tag{19.18}$$

and these are the fundamental definitions of the field tensors of O(3) electrodynamics [1]–[25].

The Evans spin field is:

$$B^{(3)*} = -igA^{(1)} \wedge A^{(2)} \tag{19.19}$$

and is observed experimentally in the magnetization of all materials by circular polarized electromagnetic radiation at any frequency, the inverse Faraday effec [1]–[25].

It is seen that the $\boldsymbol{B}^{(3)}$ spin field is the result of , and a fundamental property of, objective physics, i.e. the result of general relativity. Specially, $\boldsymbol{B}^{(3)}$ arises from the spinning frame which represents electromagnetism in objective physics, it is defined by the spin connection. The $\boldsymbol{B}^{(3)}$ spin field is not defined in the Maxwell-Heaviside field theory of special relativity because in that nineteenth century theory the electromagnetic field is an entity superimposed on a static frame in Minkowski spacetime("flat spacetime"). In the Minkowski spacetime there is no spin connection, and no $\boldsymbol{B}^{(3)}$ spin field. Therefore the Evans unified field theory of objective physics is preferred experimentally and on the fundamental basis of objectivity in physics.

There is no $\boldsymbol{E}^{(3)}$ field because:

$$\begin{aligned} F_{03}^{(3)*} &= \left(d \wedge A^{(3)}\right)^*_{03} - igA_0^{(1)} \wedge A_3^{(2)} \\ &= 0 \end{aligned} \tag{19.20}$$

as again observed experimentally. There is no electric analogue of the inverse Faraday effect. Analogously, the plane of polarization of light is rotated only by a static magnetic field (the Faraday effect), and is not rotated by a static electric field.

19.3 Discussion

The O(3) electrodynamic structure derived in Section 19.2 is a geometric structure in which the tangent bundle index a is a well defined property of differential geometry and therefore of generally covariant physics. The original O(3) electrodynamics [1]–[25] was developed as a gauge field theory in which the internal index a is an index of the fiber bundle imposed on a flat spacetime. The mathematical structure is the same in both cases, but the geometrical interpretation is preferred because it is generally covariant. All laws of physics must be generally covariant and objective in any frame of reference. The latter interpretation allows field unification through the first Bianchi identity of differential geometry, for example, producing the homogeneous field equation of the unified field:

$$D \wedge F^a = R^a{}_b \wedge A^b. \tag{19.21}$$

When the electromagnetic and gravitational fields split into separate entities, equation (19.1) splits into:

$$d \wedge F^a = 0 \tag{19.22}$$

$$R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b. \tag{19.23}$$

The Bianchi identity used in Einstein's gravitational theory of 1915 is also obeyed, in tensor notation it becomes the familiar:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\nu\rho\mu} + R_{\sigma\rho\mu\nu} = 0. \tag{19.24}$$

Eq. (19.24) is true if and only if Eq. (19.1) is true, i.e. if and only if the Christoffel connection is symmetric, in which case the torsion tensor vanishes, and with it the electromagnetic field. This is therefore a self-consistent argument, the electromagnetic field being the torsion tensor.

More generally the Bianchi identity (19.24) is not true, this means physically that there can be a tiny influence of gravitation on electromagnetism and vice versa. This influence must be tiny because it is known that the Gauss Law and Faraday Law of induction hold to great precision., i.e. it is known experimentally that Eq. (19.22) must be true to great precision. The familiar Maxwell-Heaviside structure is:

$$F = d \wedge A \tag{19.25}$$

$$d \wedge F = 0 \tag{19.26}$$

and is well known to be the archetypical theory of special relativity, invariant only upon Lorentz transformation, and not under general coordinate transformation [27]. The Evans unified field theory in contrast is generally covariant, and thus objective under any type of coordinate transformation. It is therefore preferred to the Maxwell-Heaviside field theory. Experimentally, the Evans field theory is preferred because it is capable of self-consistently generating non-linear optical phenomena such as magnetization by circularly polarized electromagnetic radiation (the inverse Faraday effect [1]–[25]. The latter is magnetization in any material and at any frequency by the Evans spin field:

$$\boldsymbol{B}^{(3)*} = -igA^{(1)} \wedge A^{(2)}, \tag{19.27}$$

a fundamental property of electromagnetic radiation at any frequency and in any state of polarization. The Evans spin field is a generally covariant property of nature, i.e a property and prediction of Einstein's theory of general relativity recently extended by Evans [1]–[25] to a unified field theory of all radiated and matter fields. The Evans spin field is a non-linear property of electromagnetic radiation and is an element of the torsion form. For these reasons it is not defined in the Maxwell-Heaviside field theory of electromagnetism, because the latter theory is linear and Lorentz covariant only. The Maxwell-Heaviside field theory does not consider the torsion form or spin connection of differential geometry because it is a flat spacetime theory in which the spin connection, torsion and Riemann forms all vanish. Thus, the homogeneous field equation of the Maxwell-Heaviside field theory is Eq. (19.26), in which the covariant exterior derivative $D\wedge$ is replaced by the exterior derivative $d\wedge$ and in which the spin connection vanishes. There being no spin connection, the internal index a loses meaning, so Eq. (19.22) becomes Eq. (19.26). In this way we may recover the Maxwell-Heaviside structure from the Evans unified field theory in the limit of flat spacetime where the spin connection approaches zero asymptotically. In this limit gravitation is obviously absent, because the spacetime is flat and the Riemann form has vanished.

However, this procedure loses a great deal of information, all of non-linear optics is thrown away, and the basic axiom of Einsteinian natural philosophy is thrown away: general covariance. So we reject this procedure despite the fact that it gives the Maxwell-Heaviside structure straightforwardly as an asymptotic limit of the Evans field theory. Much preferable is the O(3) electrodynamics procedure represented by:

$$F^a = D \wedge A^a \tag{19.28}$$

$$d \wedge F^a = 0 \tag{19.29}$$

and well tested experimentally as summarized in the literature [1]–[25]. Eq. (19.29) allows us to recover the well tested Gauss Law and Faraday Law of induction without losing general covariance and without losing the inverse Faraday effect and the Evans spin field. More generally, O(3) electrodynamics can be developed into a theory capable of describing any type of non-linear optical effect by choosing the spin connection to satisfy the experimental requirements of non-linear optics while retaining general covariance.

The Evans unified field theory also unifies causal general relativity with wave mechanics by developing the well known tetrad postulate of differential geometry:

$$D_\nu q^a{}_\mu = 0 \tag{19.30}$$

into the Evans Lemma:

$$\Box q^a{}_\mu = R q^a{}_\mu . \tag{19.31}$$

The lemma (19.31) is a subsidiary geometrical proposition which shows that spacetime is quantized. The wave-function is the tetrad $q^a{}_\mu$. In the electrodynamic sector the Lemma becomes:

$$\Box A^a{}_\mu = R A^a{}_\mu , \tag{19.32}$$

an equation which shows that the electromagnetic field is quantized in a causal manner. We therefore reject the acausal assertions of the Copenhagen School in favour of the causal axioms of the Determinist School. Finally a general form of the famous Einstein field equation:

$$R = -kT \tag{19.33}$$

is used to translate the Lemma into the Evans wave equation [1]–[25]:

$$(\Box + kT)\, q^a{}_\mu = 0. \tag{19.34}$$

Here k is the Einstein constant, T a contracted energy-momentum tensor and R a scalar curvature. Note carefully however that the quantities R and T are no longer confined to gravitation, they are properties of the Evans unified field. In the latter the Ricci tensor $R_{\mu\nu}$ and the canonical energy-momentum tensor $T_{\mu\nu}$ are asymmetric for the unified Evans field, and so both tensors must always be a sum of symmetric and anti-symmetric components from a basic theorem of matrices [28]. The metric $g_{\mu\nu}$ is also asymmetric in the Evans unified field, i.e. is a tensor product of tetrads:

$$g^{ab}{}_{\mu\nu} = q^a{}_\mu q^b{}_\nu . \tag{19.35}$$

However, we may still contract in the same way as Einstein:

$$R = g^{ab}{}_{\mu\nu} R^{\mu\nu}{}_{ab} ; T = g^{ab}{}_{\mu\nu} T^{\mu\nu}{}_{ab} \,; g^{ab}{}_{\mu\nu} g^{\mu\nu}{}_{ab} = 4, \tag{19.36}$$

and thus recover Eq. (19.35) from the famous Einstein wave equation:

$$R^{ab}{}_{\mu\nu} - \frac{1}{2} R g^{ab}{}_{\mu\nu} = k T^{ab}{}_{\mu\nu} . \tag{19.37}$$

Evidently, the latter also becomes an equation in asymmetric tensors in the Evans unified field theory and not just symmetric tensors as in the original 1915 gravitational theory of Einstein.

The Evans wave equation has been tested [1]–[25] in well known limits and shown to produce all the main equations of physics such as Dirac's equation. It is possible to recover the latter equation because the tangent bundle index a of the tetrad can be used in any representation space [27]. Having obtained Dirac's equation we have inferred fermions and elementary anti-particles from general relativity. Similarly the structure of nuclear weak and strong field theory can be inferred using appropriate representation spaces for the tangent bundle of the tetrad. A lagrangian can always be defined from which the field equation is recovered by variation and Euler Lagrange equation. The covariant derivative can always be chosen to build any type of generally covariant unified field theory of elementary particles. Having already obtained the Maxwell-Heaviside limit we can if desired automatically obtain the electro-weak field of Glashow, Weinberg and Salaam (GWS) as a limit of the Evans unified field theory. However we reject this procedure because it has the same inherent difficulties as described already: the Maxwell-Heaviside field throws away non-linear optics and general covariance, and therefore so does GWS theory. Far preferable is to develop O(3) electrodynamics into an electro-weak field theory and some attempts have already been made in this direction [1]–[25]. More generally the Evans unified field theory should be developed rigorously into a generally covariant electro-weak theory by suitable choice of representation space and geometry. The overall structure of quantum strong field theory (quantum chromodynamics) has also been recovered from the Evans unified field theory [1]–[25] using the required SU(3) representation space. Therefore if quarks exist as the most elementary particles they can if desired be described in this way from the Evans unified field theory. However quarks are postulated to be unobservable (confined) and in natural philosophy that which is unobservable is not physical. Finally the Evans spin field is the archetypical string [1]–[25] of the electromagnetic field and the Evans field theory could be developed with string theory. However the latter is often regarded as a mathematical construct rather than a theory of physics.

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