

Calculation of the Anomalous Magnetic Moment of the Electron from the Evans-Unified Field Theory

Summary. The Evans Lemma is used to calculate the g factor of the electron to ten decimal places, in exact agreement with experimental data within contemporary experimental uncertainty. The calculation proves that the Evans unified field theory is a fully quantized and generally covariant field theory of all known radiated and matter fields. It therefore has clear advantages over quantum electrodynamics, which is a theory only of special relativity and only of photons and electrons without reference to gravitation. The Evans unified field theory has none of the well known weaknesses of quantum electrodynamics, for example unphysical infinities, renormalization and dimensional regularization. Quantum electrodynamics is also fundamentally self-contradictory, being at once acausal (sum over histories construction of the wave-function at a fundamental level, "the electron can do anything it likes") and causal (use of the Huygens Principle at a fundamental level, "the electron cannot do anything it likes"). The Evans theory is rigorously causal and based on general relativity.

Key words: Evans unified field theory; anomalous electron magnetic moment; anomalous electron g factor; criticism of quantum electrodynamics.

20.1 Introduction

The anomalous g factor of the electron can be explained only by a fully quantized field theory. Semi-classical field theories produce the result:

$$g = 2 \tag{20.1}$$

from the Dirac equation and minimal prescription with a classical electromagnetic potential which is zero in the vacuum. In semi-classical theories therefore the matter field (electron) is quantized with Dirac's equation (a limiting form of the Evans wave equation) but the electromagnetic field is still classical. In Section 20.2 the recently developed and generally covariant unified field theory of Evans [1]–[25] is used to calculate the observed g factor of the electron [26]:

$$g = 2.0023193048. \quad (20.2)$$

The theoretical result from the Evans theory agrees with the above experimental value to ten decimal places and within contemporary experimental uncertainty [26]. In Section 20.2 it is shown how this precise agreement between the Evans theory and experiment is obtained by quantizing both the matter field (the electron) and the radiated field (electromagnetic field) with the fundamental Evans Lemma. In Section 20.3 some criticisms of quantum electrodynamics are made, and it is argued that the Evans theory is to be preferred for several fundamental reasons.

20.2 Vacuum or Zero-point Energy in the Evans Theory

The famous anomaly in the g factor and magnetic moment of the electron originates in the existence of zero-point energy or vacuum energy in a photon ensemble regarded as a collection of harmonic oscillators. This ensemble of photons is described by the Evans Lemma [1]–[25]:

$$\square A^a{}_{\mu} = RA^a{}_{\mu} \quad (20.3)$$

where the potential field is the tetrad one-form within a \hat{C} negative coefficient $A^{(0)}$. The latter originates in the primordial magnetic fluxon \hbar/e (weber) where \hbar is the reduced Planck constant and where $-e$ is the charge on the electron. The well known transverse plane wave [27] is a solution of the Evans Lemma (20.3) with the scalar tetrad components:

$$\begin{aligned} A^{(1)}{}_X &= \frac{A^{(0)}}{\sqrt{2}} e^{i\phi}, & A^{(1)}{}_Y &= -i \frac{A^{(0)}}{\sqrt{2}} e^{i\phi} \\ A^{(2)}{}_X &= \frac{A^{(0)}}{\sqrt{2}} e^{-i\phi}, & A^{(1)}{}_Y &= i \frac{A^{(0)}}{\sqrt{2}} e^{-i\phi}. \end{aligned} \quad (20.4)$$

The following constant valued tetrad components:

$$A^{(0)}{}_0 = A^{(3)}{}_Z = A^{(0)} \quad (20.5)$$

are timelike and longitudinal solutions of the Evans Lemma. Here ϕ is the electromagnetic phase:

$$\phi = \omega t - \boldsymbol{\kappa} \mathbf{r} \quad (20.6)$$

a special case of the Evans phase law [1]–[25] of generally covariant unified field theory. Here ω is the angular frequency

$$\omega = \kappa c \quad (20.7)$$

at instant t and $\boldsymbol{\kappa}$ is the wavenumber at coordinate \mathbf{r} . In these equations c is the speed of light in vacuo. The transverse tetrad elements (20.4) are solutions of the Evans Lemma in the form:

$$(\square + \kappa^2) A^a_\mu = 0. \quad (20.8)$$

The photon ensemble is obtained by expanding the wavefunction:

$$\psi = e^{i\phi} \quad (20.9)$$

in a Fourier series [26, 27]. The energy levels of the photon ensemble are well known to be the energy levels of the harmonic oscillator problem:

$$En = \left(n + \frac{1}{2}\right) \hbar\omega \quad (20.10)$$

when there are no photons present ($n = 0$), there is a non-zero vacuum energy or zero-point energy:

$$En_0 = \frac{1}{2} \hbar\omega. \quad (20.11)$$

The photon ensemble is therefore regarded in the way that was originally inferred by Planck [26, 27], as a collection of harmonic oscillators. Atkins [27] describes the vacuum energy as being due to fluctuating electric and magnetic fields when there are no photons present. Therefore in the absence of photons the electron records the influence of these vacuum fields in its zitterbewegung or jitterbug motion. The result is that the g factor is changed from 2 of the Dirac equation to the experimental value in Eq. (20.2). This means that the electron wobbles about its equatorial axis as it spins [27].

There therefore exists a QUANTIZED VACUUM POTENTIAL:

$$A^a_\mu{}^{(vac)} = A^{(0)} q^a_\mu{}^{(vac)} \quad (20.12)$$

which is the eigenfunction of the Evans Lemma corresponding to its minimum (zero-point) eigenvalue R_0 in the harmonic oscillator problem:

$$\square A^a_\mu{}^{(vac)} = R_0 A^a_\mu{}^{(vac)}. \quad (20.13)$$

The minimum eigenvalue is a minimum or least curvature as required by the Evans Principle of Least Curvature (a generally covariant synthesis [1]–[25] of the Hamilton Principle of Least Action and the Fermat Principle of Least Time [27]). The vacuum or zero point tetrad $q^a_\mu{}^{(vac)}$ is this eigenfunction within a scalar factor $A^{(0)}$, indicating self-consistently that the vacuum is the non-Minkowskian spacetime of Einstein's general relativity extended by Evans [1]–[25] into a generally covariant and rigorously causal unified field theory.

The Dirac wave equation has been inferred [1]–[25] as a limiting form of the Evans Lemma

$$\square q^a_\mu = R q^a_\mu \quad (20.14)$$

where the minimum eigenvalue or least curvature is defined by:

$$R \rightarrow R_0 = -(mc/\hbar)^2. \quad (20.15)$$

Here

$$\lambda = \frac{\hbar}{mc} \quad (20.16)$$

is the Compton wavelength of the electron where m is its mass. The Dirac spinor is defined by transposition of four tetrad elements in $SU(2)$ representation space into a column vector:

$$q^a{}_\mu = \begin{bmatrix} q^R_1 & q^R_2 \\ q^L_1 & q^L_2 \end{bmatrix} \rightarrow \begin{bmatrix} q^R_1 \\ q^R_2 \\ q^L_1 \\ q^L_2 \end{bmatrix} \quad (20.17)$$

So the Dirac spinor consists of right and left Pauli spinors:

$$\psi = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix}, \phi^R = \begin{bmatrix} q^R_1 \\ q^R_2 \end{bmatrix}, \phi^L = \begin{bmatrix} q^L_1 \\ q^L_2 \end{bmatrix} \quad (20.18)$$

which are identified as elements of [1]–[25], of a two component column vector (i.e. spinor) in $SU(2)$ representation space. The two Pauli spinors are interconverted by parity inversion:

$$\hat{\mathbf{P}}(\phi^R) = \phi^L. \quad (20.19)$$

The electron therefore has right and left handed half integral spin as well as mass. In this description it is a fermion with g factor of exactly 2. This is a description of causal general relativity without yet taking note of the all important effect of the quantized vacuum potential, and in this description the Dirac spinor is a solidly defined geometrical object, not an abstract and unknowable probability as in the special relativistic and acausal Copenhagen interpretation. The introduction of an unknowable (i.e. acausal) probability into causal natural philosophy is one of many reasons why Einstein, Schrodinger, de Broglie and the Determinist School reject Copenhagen quantum mechanics as an incomplete theory. The electron CANNOT do anything it likes [26], it is governed by a causal wave equation of motion (20.14), a limiting case of the Evans wave equation [1]–[25]. Analogously a dynamical object cannot do anything it likes, it is governed by Newton's laws of motion.

The wave equation (20.14) can be factorized [1]–[26] into a first order differential equation of motion:

$$(\gamma^a p_a - mc) q^b{}_\mu = 0 \quad (20.20)$$

where γ^a is the Dirac matrix. The equation (20.20) is generally covariant, and can be written in the tangent bundle spacetime of Evans' theory as:

$$(\gamma^a p_a - mc) q^b = 0 \quad (20.21)$$

for all indices μ of the base manifold. The latter is a non-Minkowskian or curved spacetime. In general all equations of generally covariant physics can always be written in this way as equations of the orthonormal tangent bundle spacetime, with indices such as μ of the base manifold suppressed or implied. This is standard practice [28] in differential geometry and the Evans theory is the geometrization of all physics, not only gravitation. The great advantage of writing the equations of generally covariant physics in the tangent bundle spacetime is that the latter is orthonormal by construction, with Minkowskian metric η^{ab} :

$$\eta^{ab} = \gamma^a \gamma^b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (20.22)$$

Here γ^a are standard Dirac matrices [1]–[26]. Any suitable orthonormal basis set such as the Pauli matrices can be used in the tangent bundle spacetime and so a complex valued spinor or two-vector can also always be defined in the tangent bundle spacetime. The tetrad is the invertible matrix that links vector components and basis vectors [28] in the tangent bundle and base manifold, and in the Evans unified field theory [1]–[25] the tetrad is the gravitational potential field. Within a factor $A^{(0)}$ the tetrad is also the electromagnetic potential field.

In this way therefore it is always possible to introduce gravitation into any equation of physics through the tetrad. The symmetric metric of the base manifold, the object used by Einstein and Hilbert in 1915 to define gravitation, is the dot or scalar product of two tetrads:

$$g_{\mu\nu} = q^a{}_{\mu} q^b{}_{\nu} \eta_{ab} \quad (20.23)$$

and so it is always possible to express the symmetric metric of the base manifold in terms of Dirac matrices of the tangent bundle spacetime:

$$g_{\mu\nu} = q^a{}_{\mu} q^b{}_{\nu} \gamma_a \gamma_b. \quad (20.24)$$

Therefore it is possible in theory to measure the effect of gravitation on the Dirac equation, both in the absence and presence of the electromagnetic field. This is possible in the Evans unified field theory but not possible in the contemporary "standard model" or in quantum electrodynamics.

In the presence of the vacuum potential the Evans equation (20.21) becomes:

$$\left(\gamma^a \left(p_a + eA^{(vac)}{}_a \right) - mc \right) q^b = 0. \quad (20.25)$$

The magnitude of the vacuum potential is defined by the ratio:

$$\alpha := 4\pi \frac{eA^{(vac)}}{\hbar\kappa} = 4\pi c \frac{eA^{(vac)}}{\hbar\omega} \quad (20.26)$$

where α is the fine structure constant [1]–[27]:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad (20.27)$$

in S.I. units. Here ϵ_0 is the vacuum permittivity. In reduced units of

$$4\pi\epsilon_0\hbar c = 1 \quad (20.28)$$

it is seen that:

$$eA^{(vac)} = \frac{e^2}{4\pi}En^{(vac)}. \quad (20.29)$$

This is the most fundamental way of making $A^{(vac)}$ proportional to $E^{(vac)}$ through the fundamental proportionality constant e , and this is the fundamental meaning of the vacuum potential and the reason why the fine structure constant appears in our calculation. The covariant version of Eq. (20.29) is:

$$eA^{(vac)} = \frac{e^2}{4\pi}p^{a(vac)} = \frac{\alpha}{4\pi}p^{a(vac)} \quad (20.30)$$

and is written in the tangent bundle spacetime following our general rule that all equations of generally covariant physics may be written in the tangent bundle with indices of the base manifold suppressed or implied. Thereby we arrive at the inference that Eq (20.30) is the minimal prescription [1]–[27] or covariant derivative for the vacuum potential.

The Evans equation (20.25) now becomes:

$$\left(\gamma^a p_a (1 + \alpha') - mc\right) q^b = 0 \quad (20.31)$$

where:

$$\alpha' = \frac{\alpha}{4\pi}. \quad (20.32)$$

Eqn. (20.31) can be interpreted as an increase in the Dirac gamma matrix for constant p_a :

$$\gamma^a \rightarrow \gamma^a (1 + \alpha'), \quad (20.33)$$

so the existence of α' is seen clearly through Eq. (20.34) to be a property of spacetime itself, and thus self-consistently as a vacuum property. It is always possible to express the Minkowski metric as the anti-commutator of Dirac matrices:

$$[\gamma^a, \gamma^b] = \gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab} \quad (20.34)$$

and the factor 2 of Eq. (20.34) can be interpreted as the Dirac g factor of the electron. From Eq (20.33) the g factor is therefore increased by the vacuum potential to:

$$g = \left(1 + \alpha'\right)^2 + \left(1 + \alpha'\right)^2 = 2 \left(1 + \frac{\alpha}{2\pi} + \frac{\alpha^2}{16\pi^2}\right). \quad (20.35)$$

This theoretical result is in agreement with the experimental result of Eq (20.2) to ten decimal places, the greatest known precision of contemporary physics.

This agreement is therefore a very precise test of the Evans unified field theory and is due fundamentally to the least or minimum curvature produced by the vacuum energy:

$$En^{(vac)} = \frac{1}{2}\hbar\omega = \frac{1}{2}c\hbar\kappa = \frac{1}{2}c\hbar|R_0|^{1/2}. \quad (20.36)$$

20.3 Criticisms of Quantum Electrodynamics

In the received opinion at present the calculation of g is carried out with quantum electrodynamics (qed) by increasing the value of the Dirac matrix with the convergent vertex

$$\gamma_\mu \rightarrow \gamma_\mu + \Lambda_{\mu}^{(2)}. \quad (20.37)$$

This calculation is carried out in Minkowski or flat spacetime and so is not generally covariant as required by Einsteinian natural philosophy. The convergent vertex is defined by

$$\Lambda_{\mu}^{(2)} = \alpha' \left(p_\mu + p'_\mu \right) / (mc) \quad (20.38)$$

[1]–[26]. In qed dimensional regularization is used to remove primitive divergences of the path integral formalism [1]–[26], which has the effect:

$$e \rightarrow \mu^{2-d/2}e \quad (20.39)$$

in the lagrangian. Here μ is an arbitrary mass and d is the mass dimension of the lagrangian. This procedure leads to $\Lambda_{\mu}^{(2)}$ from the vertex graph:

$$-ie\Lambda_{\mu}(p, q, p + q). \quad (20.40)$$

The removal of infinities of this graph results in a change in the physical properties of the electron. The convergent $\Lambda_{\mu}^{(2)}$ is that part of the overall vertex with no k in the nominator of the integrand.

Quantum electrodynamics is not therefore a foundational theory of natural philosophy because it obtains the right result by arbitrary means: dimensional regularization, which changes e , and renormalization, which artificially removes infinities of the path integral method. Quantum electrodynamics is Lorentz covariant only (it is a theory of special relativity). Quantum electrodynamics uses the sum over histories description of the wavefunction. This is an acausal description in which the electron can do anything it likes [26], go backwards or forwards in time for example. This acausality or unknowability

is contradicted fundamentally and diametrically in qed by use [26] of the Huygens Principle, which expresses causality or knowability - the wavefunction is built up by superposition in causal historical sequence - an event is always preceded by a cause, and nothing goes backwards in time. For these and other reasons qed was rejected by Einstein, Schrodinger, de Broglie, Dirac and many others from its inception in the late forties.

As we have seen the much vaunted precision of qed is obtained much more simply in the generally covariant and causal unified field theory of Evans. By Okham's Razor and for several other fundamental reasons discussed already, the Evans theory is preferred to quantum electrodynamics. Similarly the Evans theory is preferred to quantum chromodynamics, which has the same fundamental flaws as quantum electrodynamics with the added problem of quark confinement. Quarks are postulated to exist but to be unobservable [26]. Something that is both unknowable and unobservable springs from theism and has no place in natural philosophy - the causal description of the natural world. If nature were unknowable and unobservable we could never know anything about nature and this is what the Copenhagen School would have us swallow. Add to this the many "dimensions" of string theory and whither physics?

20.4 Discussion

The interaction of the electron and photon ensemble may always be described in the Evans unified field theory by:

$$(\gamma^a (i\hbar\partial_a - eA_a) - mc) q^b = 0. \quad (20.41)$$

This equation can be developed into a wave equation as follows:

$$(\gamma^a (i\hbar\partial_a - eA_a) - mc) (\gamma^b (-i\hbar\partial_b - eA_b^*) - mc) q^b = 0. \quad (20.42)$$

Here the whole of the operator acts on the tetrad as follows:

$$\left(\square + \frac{m^2 c^2}{\hbar^2} + \frac{emc}{\hbar^2} (A_a + A_a^*) + \frac{e^2}{\hbar^2} A^a A_a^* \right) q^b = 0 \quad (20.43)$$

where we have used the result [26]:

$$\square = \partial^a \partial_a = g^{ab} \partial_a \partial_b = \gamma^a \gamma^b \partial_a \partial_b. \quad (20.44)$$

In Eq. (20.43) it becomes clear that the overall effect of the electromagnetic field on the electron is to always to add a scalar curvature [1]-[25]:

$$R_{em} = -\frac{emc}{\hbar^2} (A_a + A_a^*) - \frac{e^2}{\hbar^2} A^a A_a^*. \quad (20.45)$$

In the presence of gravitation (curvature of the base manifold):

$$\frac{m^2 c^2}{\hbar^2} \rightarrow R \quad (20.46)$$

so the Evans equation (20.43) can be used to calculate the effect of the gravitational field on the electromagnetic field interacting with an electron. More generally the calculation and computation can be extended to atoms and molecules in a gravitational field. This is twenty first century physics, i.e. not possible in the contemporary standard model, but very important to new technologies. Finally, the Evans theory is more precise than quantum electrodynamics because it is able to calculate the g factor of the electron to any order in the fine structure constant. In quantum electrodynamics such calculations involve tremendous complexity and are deeply flawed philosophically. All said and done, physics is natural philosophy, and if the philosophy is flawed fundamentally it must, as always in the development of human thought, be replaced by something better.

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