TORSION AND CURVATURE ELEMENTS OF ANY ORBIT FROM ECE THEORY

by

M. W. Evans and H. Eckardt,

Civil List, AIAS and UPITEC,

www.webarchive.org.uk, www.aias.us, www.upitec.org, www.atomicprecision.com,

www.et3m.net

ABSTRACT

The Minkowski metric constrained by any orbit is shown to produce non-zero torsion and curvature elements in general for any orbit, or for any rotational motion in a plane. This is a new general relativity that describes all orbits self consistently in terms of Cartan's geometry, the basis for ECE theory. The antisymmetric Christoffel connection is computed from the metric elements using the metric compatibility theorem, and used to compute the torsion and curvature elements. The Evans identity is shown to be an exact identity, so this method amounts to a multiple cross check of the ECE theory and constrained Minkowski method.

Keywords: ECE theory, constrained Minkowski metric, torsion and curvature elements of any orbit.

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1. INTRODUCTION

In recent papers of this series {1 - 10} it has been demonstrated that the Einsteinian general relativity (EGR) is incorrect in several ways and is therefore obsolete. The basic idea of the EGR goes back to ancient times, and is encapsulated in the statement by Kepler: "Ubi materia ibi geometria", which means essentially that all matter is geometry. In the ECE series of papers it has been demonstrated that differential geometry must include the basic concepts of torsion and curvature. These are defined in the first and second Cartan structure equations {11} which define the torsion and curvature respectively for any mathematical space of any dimension using any coordinate system. Cartan also inferred an identity linking torsion and curvature {11}. Using Hodge duals the Evans identity has been shown to be an example of the Cartan identity {1 - 10} and the field equations of unified physics have been developed from the Cartan and Evans identities. The basic idea that physics is geometry is therefore retained, it is the basis of the philosophy of relativity. Einstein's ideas are now accepted to be incorrect because he developed them when torsion was unknown. He used the obsolete first and second Bianchi identities {1 - 10} in which torsion is missing. His field equation makes the obsolete second Bianchi identity proportional to the Noether Theorem through the Einstein constant k, so this field equation is incorrect and all deductions based on it are obsolete. It can now be shown in many ways that the EGR is incorrect and obsolete, some of them very simple {1 - 10}. Claims to have verified incorrect mathematics experimentally are obviously meaningless.

In Section 2 the new general relativity developed in this series of papers and monographs {1 - 10} is used to compute the torsion and curvature elements of any orbit observed in astronomy. It is no longer claimed that an orbit can be predicted, but it can be characterized incisively by geometry within the Keplerian philosophy: all physics is geometry. The geometry of the orbit is deduced by observation, and the orbit is characterized systematically by its torsion and curvature elements. Cartan's geometry is a development of Riemann's geometry and for the description of orbits, Riemann's geometry suffices by Ockham's Razor because it is a simpler description. The method used in this section is based on the Minkowski metric. In the unconstrained condition this is the metric of flat spacetime and of special relativity, and produces no torsion and no curvature. However, if it is constrained by an orbit, the spacetime becomes that of general relativity with non zero torsion and curvature in general. This method was introduced in UFT203 (www.aias.us). The metric elements of the constrained orbit are used to deduce the antisymmetric connections using metric compatibility. It is now well known and accepted {1 - 10} that the connection must be antisymmetric in its lower two indices as a very simple consequence of its basic definition using the commutator of covariant derivatives. Einstein used an incorrect symmetry for the connection, he used, arbitrarily, a connection that was symmetric in its lower two indices. A symmetric connection is however zero {1 - 11}, and produces no curvature and no torsion from its basic definition using the commutator. Having deduced the antisymmetric connection, computer algebra is used to deduce the non vanishing torsion and curvature elements, and to produce the various equations of the Evans identity. It is found that these methods are rigorously correct and self consistent.

In Section 3 tables of torsion and curvature elements are given for some orbits, notably the ellipse and precessing ellipse, and some spiral orbits. The code is written in general curvilinear coordinates, so any system of coordinates can be used, notably the cylindrical polar system, but if preferred the Cartesian system can be used, or spherical polar system. The method developed here considers planar orbits, but can be extended for use with three dimensional orbits. The method can be used with any orbit observed in astronomy, and is self consistent.

2. TORSION AND CURVATURE OF THE CONSTRAINED MINKOWSKI METRIC

Consider a planar orbit in the cylindrical polar system of coordinates (r, θ).

The plane is defined by:

$$dZ^2 = 0. \quad -(1)$$

The orbit is defined in general by:

$$f(r, \theta, t) = r^2 \left(\frac{d\theta}{dr}\right)^2 \quad -(2)$$

in which both r and θ are functions of time t because the orbit defines a moving object of mass m orbiting a mass M. The unconstrained Minkowski metric in the plane (1) is characterized by the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2. \quad -(3)$$

Here c is the assumed constant speed of light in vacuo, τ is the proper time, and t is the observer time. Eqs. (2) and (3) give the constrained Minkowski line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - (1 + f) dr^2 \quad -(4)$$

from which the metric elements are:

$$g_{00} = 1, \; g_{11} = -(1 + f). \quad -(5)$$

The antisymmetric connection is obtained using Eq. (19) of UFT188:

$$\Gamma^{\alpha}_{\rho\alpha} = \frac{1}{2} g^{\alpha\alpha} \partial_{\rho} g_{\alpha\alpha} \quad -(6)$$

using the convention of that paper. In Eq. (6), summation over the repeated indices is not implied. Eq. (6) originates in metric compatibility. Computer algebra shows that the two non-zero connections of any orbit in a plane are:

$$\Gamma^1_{01} = -\Gamma^1_{10} = \frac{1}{c}\,\frac{\partial g/\partial t}{2(1+g)} \quad -(7)$$

and

$$\Gamma^1_{21} = -\Gamma^1_{12} = \frac{\partial g/\partial \theta}{2r(1+g)}. \quad -(8)$$

The two non-zero torsion elements of any orbit in a plane are twice these connections:

$$T^1_{01} = -T^1_{10} = \frac{1}{c}\,\frac{\partial g/\partial t}{(1+g)}, \quad -(9)$$

$$T^1_{21} = -T^1_{12} = \frac{1}{r}\,\frac{\partial g/\partial \theta}{(1+g)}. \quad -(10)$$

Computer algebra shows that the two Evans identities of any orbit in a plane are:

$$D_0 T^1_{01} := R^1_{001} \quad -(11)$$

and:

$$D_2 T^1_{21} := R^1_{221}. \quad -(12)$$

These identities provide new orbital equations for any orbit. The elements of the Riemann curvature that appear in Eqs. (11) and (12) are:

$$R^1_{001} = \frac{1}{c^2}\left(\frac{2(1+g)\,d^2g/dt^2 - (dg/dt)^2}{4(1+g)^2}\right) \quad -(13)$$

and $$R^{1}{}_{221} = \frac{1}{r^2}\left(\frac{2(1+f)\,d^2f/d\theta^2 - (df/d\theta)^2}{4(1+f)^2}\right). \quad -(14)$$

Computer algebra shows that Eqs. (11) and (12) reduce to:

$$6\frac{d^2f}{dt^2}(1+f) = 5\left(\frac{df}{dt}\right)^2 \quad -(15)$$

and

$$6\frac{d^2f}{d\theta^2}(1+f) = 5\left(\frac{df}{d\theta}\right)^2 \quad -(16)$$

which imply that the Evans identity reduces to:

$$\frac{d^2f}{dt^2} = \left(\frac{d\theta}{dt}\right)^2 \frac{d^2f}{d\theta^2}. \quad -(17)$$

Now use the chain rule of differentiation as follows:

$$g = \frac{df}{dt}, \quad \frac{dg}{dt} = \frac{d^2f}{dt^2} = \frac{dg}{d\theta}\frac{d\theta}{dt} = \frac{d^2f}{d\theta\, dt}\frac{d\theta}{dt}, \quad -(18)$$

and

$$h = \frac{df}{d\theta}, \quad \frac{dh}{d\theta} = \frac{d^2f}{d\theta^2} = \frac{dh}{dt}\frac{dt}{d\theta} = \frac{d^2f}{dt\, d\theta}\frac{dt}{d\theta}. \quad -(19)$$

By isotropy:

$$\frac{d^2f}{d\theta\, dt} = \frac{d^2f}{dt\, d\theta}. \quad -(20)$$

Divide Eq. (18) by Eq. (19) to give Eq. (17), Q.E.D. The Evans identity is a consequence of the chain rule, and is therefore proven. Inter alia, the chain rule of differentiation originates in the Evans identity, an example of the Cartan identity.

This procedure is in effect a multiple cross check on the concepts being used in the new general relativity. In the next section, tables of torsion and curvature elements are given for any planar orbit. In general these elements are non-zero.

3. TABLES OF TORSION AND CURVATURE ELEMENTS

These tables are constructed using the general curvilinear coordinates {12}, some description of which is given as follows. Consider the position vector $\underline{r}$ of a point P in three dimensions:

$$\underline{r} = X\underline{i} + Y\underline{j} + Z\underline{k}. \quad -(21)$$

Consider the curvilinear coordinate system (u_1, u_2, u_3) {12}. Then:

$$\underline{r} = \underline{r}(u_1, u_2, u_3). \quad -(22)$$

The unit vectors of the curvilinear coordinate system are:

$$\underline{e}_i = \frac{1}{h_i}\frac{\partial \underline{r}}{\partial u_i}, \; i = 1, 2, 3 \quad -(23)$$

in which the scale factors are:

$$h_i = \left| \frac{\partial \underline{r}}{\partial u_i} \right|, \; i = 1, 2, 3. \quad -(24)$$

In Eq. (6), consideration must be given to the del operator of the curvilinear system:

$$\underline{\nabla} = \frac{\underline{e}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\underline{e}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\underline{e}_3}{h_3}\frac{\partial}{\partial u_3} \quad -(25)$$

in which the scaling factors appear in the denominator. The unconstrained Minkowski metric in the curvilinear system is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - h_1^2 du_1^2 - h_2^2 du_2^2 - h_3^2 du_3^2 \quad -(26)$$

Its metric tensor in the three space dimensions is:

$$g_{ij} = g_{ji} = \frac{\partial \underline{r}}{\partial u_i} \cdot \frac{\partial \underline{r}}{\partial u_j}. \quad -(27)$$

If:

$$g_{ij} = 0 \text{ for } i \neq j \quad -(28)$$

the coordinate system is orthogonal and:

$$g_{11} = h_1^2, \; g_{22} = h_2^2, \; g_{33} = h_3^2. \quad -(29)$$

A typical orbital constraint in the curvilinear system is:

$$\frac{du_1}{du_2} = f_1(u_1, u_2) \quad -(30)$$

and the gradient of a function in the curvilinear system is:

$$\underline{\nabla} F = F_1 \underline{e}_1 + F_2 \underline{e}_2 + F_3 \underline{e}_3. \quad -(31)$$

For the cylindrical polar system for example, the three scale factors are:

$$h_1 = 1, \; h_2 = r, \; h_3 = 1 \quad -(32)$$

so the gradient in this system of coordinates is:

$$\underline{\nabla} F = \frac{\partial F}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial F}{\partial \theta}\underline{e}_\theta + \frac{\partial F}{\partial Z}\underline{e}_Z . \quad -(33)$$

In general the planar orbit in the curvilinear system is:

$$\frac{du_1}{du_2} = f_c(u_1, u_2, t) \quad -(34)$$

so the constrained Minkowski line element in the curvilinear system is:

$$ds^2 = c^2 dt^2 - \left(h_1^2 + \frac{h_2^2}{f_c^2} \right) du_1^2 \quad -(35)$$

giving the two metric elements:

$$g_{00} = 1, \quad g_{11} = -\left(h_1^2 + \frac{h_2^2}{f_c^2} \right) \quad -(36)$$

With these definitions some tables of torsion and curvature elements are given below

(Here tables and analysis by Dr. Horst Eckardt)

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