ORBITAL EQUATION OF MOTION OF THE NEW RELATIVITY: WHIRLPOOL
GALAXIES AND ORBITAL PERTURBATIONS IN THE SOLAR SYSTEM.

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ABSTRACT

The constrained Minkowski method of ECE theory is used to develop the orbital
dynamics of whirlpool galaxies and the solar system in terms of a new equation of motion
that is able to analyse all observed orbits self consistently. The equation of motion shows the
presence of tiny perturbations in the orbit of a planet in the solar system, oscillatory
perturbations which are superimposed on the precessing ellipse. Features of whirlpool
galaxies are described by the new method.

Keywords: ECE theory, constrained Minkowski method, orbital modulations in the solar
system, whirlpool galaxies.
1. INTRODUCTION

Recently in this series of papers {1 - 10} a new theory of general relativity has been developed based on constraining the Minkowski metric with any observed orbit. This method leads along a natural path towards general relativity from special relativity because the Minkowski spacetime of special relativity is replaced with a four dimensional spacetime with non zero torsion and curvature elements generated by the existence of the orbit. The curvature and torsion elements are those of Riemann geometry, and are linked by the Evans identity {1 - 10} of geometry. The identity has been used to produce the field equations of ECE theory {1 - 10}. In Section 2 the identity is used to produce an orbital equation of general relativity valid for all orbits. It is exemplified in Section 3 in the solar system and in whirlpool galaxies. In the solar system it produces tiny oscillatory perturbations superimposed on the precessing elliptical orbit that is the main feature of planetary motion, and for whirlpool galaxies it rationalizes the motion of stars. In Section 4 the results are analysed numerically and graphically.

2. DEVELOPMENT OF THE EQUATION OF MOTION

In the preceding paper UFT207 on www.aias.us the equation of motion from the Evans identity was shown to be:

\[ \frac{d}{dt} \left( \frac{d\theta}{dt} \right) = 0 \quad - (1) \]

where:

\[ \theta = \left( \frac{r \, dr}{dx} \right)^2 \quad - (2) \]

The orbit in general is defined by:
\[ r(t) = f_1(\theta(t)) \]  

in which both \( r \) and \( \theta \) are functions of time \( t \). It was shown that:

\[ \frac{d\theta}{dt} = \frac{1}{2} \omega \frac{d\theta}{d\theta} \]  

where \( \omega \) is the orbital angular velocity:

\[ \omega = \frac{d\theta}{dt} \]  

Similarly:

\[ \frac{dF_i}{dt} = \frac{1}{2} \omega \frac{dF_i}{d\theta} \]  

where \( F_i \) is defined by:

\[ F_i = \frac{d\theta}{dt} \]  

Therefore:

\[ \frac{1}{2} \omega \frac{d}{d\theta} \left( \frac{1}{2} \omega \frac{dF_i}{d\theta} \right) = 0 \]  

and for non-zero angular velocity the orbital equation is:

\[ \frac{d}{d\theta} \left( \omega \frac{dF_i}{d\theta} \right) = 0 \]

Using the Leibniz rule:

\[ \frac{d\omega}{d\theta} \frac{dF_i}{d\theta} + \omega \frac{d^2F_i}{d\theta^2} = 0 \]

so the orbital equation becomes:
\[
\frac{d\omega}{d\theta} = -F(\theta)\omega, \quad -(11)
\]
\[
F(\theta) = \left( \frac{d^2 y}{d\theta^2} \right) / \left( \frac{dy}{d\theta} \right) \quad -(12)
\]
whose general solution is:
\[
\omega = \omega_0 \exp \left( -\int F(\theta) d\theta \right) = \omega_0 \exp \left( -\left( F_1(\theta) + C \right) \right) \quad -(13)
\]
where \( C \) is a constant of integration and where \( \omega_0 \) is needed for dimensional correctness. This equation will be applied to orbits of whirlpool galaxies and in the solar system in following sections. To conclude this section a second equation of motion is derived by constraining the Minkowski metric in a new way.

Consider the Minkowski metric in the plane:
\[
d\mathcal{L}^2 = 0 \quad -(14)
\]
i.e.
\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - dx^2 - r^2 d\theta^2 \quad -(15)
\]
Therefore:
\[
d\mathcal{L}^2 = c dt, \quad dx^1 = dx, \quad dx^2 = r d\theta, \quad -(16)
\]
\[
g_{00} = 1, \quad g_{11} = -1, \quad g_{22} = -1.
\]
The observed orbit is defined by:
\[
\theta = \frac{dx}{d\theta} \quad -(17)
\]
therefore Eq. (15) becomes:
\[
ds^2 = c^2 dt^2 - \left( \frac{dx}{d\theta} \right)^2 d\theta^2 - r^2 d\theta^2 \quad -(18)
\]
From Eq. (17):
\[ dx^2 = g^2 \, d\theta^2 \]  
so
\[ (dx')^2 = \left( \frac{g}{r} \, dx^2 \right)^2 \]  
and
\[ ds^2 = g_{00} \, dx^0 \, dx^0 + \frac{\delta_{11}}{g} \, dx^1 \, dx^2 + g_{22} \, dx^2 \, dx^3, \]
Define:
\[ g_{22}' = \frac{\delta_{11}}{g} + g_{22}, \quad g = \left( \frac{r}{\beta} \right)^2, \]
then:
\[ ds^2 = g_{00} \, dx^0 \, dx^0 + g_{22}' \, dx^2 \, dx^3. \]

The infinitesimal line element is that of a two dimensional space labelled 0 and 2. The metric matrix is:
\[ g_{\mu \nu} = \begin{bmatrix} 1 & 0 \\ 0 & -(1 + \frac{1}{g}) \end{bmatrix}. \]
The metric compatibility condition is:
\[ \partial_\mu g_{\mu \nu} = 0 \]
i.e.,
\[ \partial_\mu g_{00} = \partial_\mu g_{22}' = 0. \]
where:
\[ g_{00} = 1, \quad g_{22} = -\left(1 + \frac{1}{8}\right)^{-2} \]  - (27)

Therefore:
\[ \rho \left(1 + \frac{1}{8}\right) = 0. \]  - (28)

Eq. (25) can be expanded as:
\[ \frac{\partial}{\partial \rho} g_{\mu \nu} - \Gamma_{\mu \lambda}^{\lambda} g_{\lambda \nu} - \Gamma_{\nu \lambda}^{\lambda} g_{\mu \lambda} = 0. \]  - (29)

The only non trivial case is:
\[ \frac{\partial}{\partial \rho} g_{22} - \Gamma_{\rho 2}^{\lambda} g_{\lambda 2} - \Gamma_{\rho 2}^{\lambda} g_{2 \lambda} = 0. \]  - (30)

The metric is diagonal, so the only possibility is:
\[ \frac{\partial}{\partial \rho} g_{22} = 2 \Gamma_{\rho 2}^{2} g_{22}. \]  - (31)

The connection is antisymmetric \{1 - 10\} so the only possibility is:
\[ \Gamma_{02}^{2} = \frac{1}{2c g_{22}} \frac{dg_{22}}{d\xi} \]  - (32)

and the only torsion element is:
\[ T_{02}^{2} = - T_{20}^{2} = \frac{1}{c} \left(1 + \frac{1}{8}\right)^{-1} \frac{d}{d\xi} \left(1 + \frac{1}{8}\right). \]  - (33)

From this procedure a new equation of motion may be derived using computer algebra and the Evans identity, and this will be the subject of future work.
3. APPLICATION TO WHIRLPOOL GALAXIES AND THE SOLAR SYSTEM.

In whirlpool galaxies the stars are arranged in spirals recently analysed in detail in UFT198, in which the Cheshire lines were reported. In this section the equation of motion (11) is applied to the Archimedes and hyperbolic spirals. In general the Archimedes spiral is:

\[ r = a + b \theta \]  

where \( a \) and \( b \) are constants. So:

\[ \theta = \left( \frac{a + b \theta}{b} \right)^3 \]  

and:

\[ \frac{d\theta}{d\theta} = 2\theta (1 + \frac{a}{b}), \quad \frac{d^2\theta}{d\theta^2} = 2 \left( 1 + \frac{a}{b} \right) \]  

The equation of motion is therefore:

\[ \frac{d\omega}{d\theta} = -\frac{\omega \theta}{\omega} \]  

Considering for simplicity the case:

\[ a = 0, \quad r = b \theta \]  

the solution of Eq. (37) is found from:

\[ \int \frac{d\omega}{\omega} = -\int \theta \theta \]  

Assuming that the constant of integration is zero then the orbital angular frequency is:

\[ \omega = \omega_0 \exp \left( -\frac{r^2}{2b^2} \right) \]
The whirlpool galaxy is sketched in Fig. (1).

At the centre there is a large mass $M$ spinning with angular velocity $\omega_0$ at:

$$r = 0. \quad -(41)$$

So from Eq. (40):

$$\omega = \omega_0 \text{ (initially)} \quad -(42)$$

At the point $A$ the angular velocity has slowed to:

$$\omega = \omega_0 \exp \left( -\frac{R^2}{2} \right) \quad -(43)$$

where:

$$R = \int_0^R \left( 1 + \frac{(r)^2}{b^2} \right)^{1/2} \text{ d}r. \quad -(44)$$

The galactic torsion is:

$$T^{01} = \frac{1}{c(1+g)} \frac{\text{d}g}{\text{d}t} = \frac{\omega}{2c(1+g)} \frac{\text{d}g}{\theta} \quad -(45)$$

i.e.:

$$T^{01} = \frac{\omega_0}{c} \left( \frac{x}{1+x^2} \right) \exp \left( -x^3 \right) \quad -(46)$$
where:

\[ x_c = \frac{r}{b} \quad - (47) \]

The angular velocity and torsion dissipate to zero at the edges of the whirlpool. The orbital linear velocity of a star in the spiral is given by:

\[ v = \left( r^2 + \left( \frac{dr}{d\theta} \right)^2 \right)^{1/2} \omega \quad - (48) \]

so:

\[ v = (r^2 + b^2)^{1/2} \omega_0 \exp \left( -\frac{r^2}{b^2} \right) \quad - (49) \]

It is observed experimentally that:

\[ v \longrightarrow \text{constant} \quad - (50) \]

which is a condition that must be obtained by adjusting \( b \) and \( \omega_0 \).

The hyperbolic spiral is defined by:

\[ r = \frac{r_0}{\theta} \quad - (51) \]

Therefore:

\[ \frac{d}{d\theta} = \left( \frac{d\theta}{dr} \right)^2 = \theta^2 \quad - (52) \]

and the equation of motion is:

\[ \frac{d\omega}{d\theta} = -\frac{\omega}{\theta} \quad - (53) \]

a solution of which is:

\[ \theta = \frac{\omega_0}{\omega} \quad - (54) \]
The torsion of the hyperbolic spiral is:

\[ T^{01} = \frac{\omega_0}{c(1+\theta^2)} \quad -(55) \]

At the centre of the galaxy:

\[ \theta \to 0, \quad \omega \to \infty, \quad T^{01} \to \frac{\omega_0}{c} \quad -(56) \]

and at the edges:

\[ \theta \to \infty, \quad \omega \to 0, \quad T^{01} \to 0 \quad -(57) \]

The radial \( r \) is interpreted to mean that at the centre of the galaxy:

\[ r \to \infty \quad -(58) \]

and at the edges:

\[ r \to 0 \quad -(59) \]

so in the hyperbolic spiral the distance is measured from the edge inwards. In the Archimedes spiral it is measured from the centre outwards. The orbital velocity of the hyperbolic spiral is

\[ v = \frac{\omega_0 r_0}{\theta^2} \left(1 + \frac{1}{\theta^2}\right) \quad -(60) \]

so if the spiral is such that:

\[ \theta \to \text{constant} \quad -(61) \]

the velocity becomes constant as observed.

In the solar system the main feature of a planetary orbit is the precessing ellipse:
\[ r = \frac{d}{1 + \varepsilon \cos(x \theta)} \]  

(62)

where \( d \) is the half right latitude, \( \varepsilon \) the eccentricity, and \( x \) the precession constant. In the Newtonian approximation \( \{11\} \):

\[ x = 1 \quad - (63) \]

and the angular velocity of a Newtonian orbit is:

\[ \omega_N = \frac{L}{mr^2} = \frac{L}{md^2} \left( 1 + \varepsilon \cos(x \theta) \right)^3 \]

(64)

where \( L \) is the constant total angular momentum, and \( m \) the mass of the planet. In the solar system:

\[ x - 1 \approx 10^{-6} \quad - (65) \]

so the angular velocity of a precessing elliptical planetary orbit is to an excellent approximation:

\[ \omega_p = \frac{L}{md^2} \left( 1 + \varepsilon \cos(x \theta) \right)^2 \]

(66)

The general solution is on the other hand given by eq. \( \{13\} \):

\[ \omega = \omega_0 \exp \left( - (F(x \theta) + C) \right) \]

(67)

The orbit is known experimentally to be a precessing ellipse to an excellent approximation, so:

\[ \omega - \omega_p = \omega_1 \]

(68)

where \( \omega_1 \) represents a tiny oscillatory perturbation of the precessing ellipse. Therefore:
\[
\frac{\omega_1}{\omega_0} = \exp(-C) \exp(-F_1(\theta)) - \frac{L}{m \omega_0 \omega_0^2} \left(1 + e \cos(\theta)\right)^2
\]

where:
\[
\left| \frac{\omega_1}{\omega_0} \right| \ll 1 \quad - (70)
\]

This condition is met by a large constant of integration C such that:
\[
C \gg F_1(\theta) \quad - (71)
\]

and by:
\[
\frac{L}{m \omega_0 \omega_0^2} \ll 1 \quad - (72)
\]

In the next section a numerical analysis is given of these perturbations, which mean that the orbit of a planet is not an exact precessing ellipse.

4. NUMERICAL ANALYSIS OF THE ORBITAL PERTURBATIONS.

Section by Dr Horst Eckardt.

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4 Numerical Analysis of the orbital perturbations

We start with a re-evaluation of expressions $F(\theta)$ and $F1(\theta)$ in Eqs.(12) and (13):

$$F(\theta) = \frac{d^2 f}{d\theta^2}, \quad (73)$$

$$F1(\theta) = \int F(\theta) \, d\theta. \quad (74)$$

The integral of Eq.(74) can be rewritten to

$$F1(\theta) = \log \left( \frac{df}{d\theta} \right) + C \quad (75)$$

with an integration constant $C$. The function $f$ is defined according to (2):

$$f = \left( \frac{r}{dr} \right)^2. \quad (76)$$

For the solar system we have

$$r(\theta) = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad (77)$$

which can be inverted to

$$\theta(r) = \frac{1}{x} \arccos \left( \frac{\alpha}{\epsilon r} - \frac{1}{\epsilon} \right). \quad (78)$$

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Evaluation of $F_1$ leads to

$$F_1(\theta) = \log \left( \frac{d}{d\theta} \left( -\frac{\epsilon^2 \cos(\theta x)^2 + 2\epsilon \cos(\theta x) + 1}{\epsilon^2 x^2 \left( \cos(\theta x)^2 - 1 \right)} \right) \right)$$

which enters Eq. (13)

$$\omega = \omega_0 \exp(-F_1(\theta) - C)$$

that has been plotted in Fig. 1 for some standard parameters. Obviously, for $C = 0$, $\omega$ is near to zero as expected but has some sharp infinities which do not appear in the approximated solution of section 3. These infinities are also there in the non-integrated function $F'(\theta)$, see Fig. 2. They arise from the zeros of the denominator in (79) appearing at

$$\theta = n \frac{\pi}{x}.$$
Figure 2: Function $F(\theta)$ with parameters $\omega_0 = 1, \epsilon = 0.1, \alpha = 1, x = 0.9, C = 0$. 
REFERENCES.


{4} Kerry Pendergast, “The Life of Myron Evans” (CISP, Spring 2011).


