PROOF OF CONNECTION ANTISYMMETRY BY CONSIDERATIONS OF ROTATION

by

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ABSTRACT

The Christoffel connection is proven to be the totally antisymmetric unit tensor in three dimensional rotation. In consequence the inhomogeneous part of the general coordinate transformation of the connection vanishes for each rotation generator. This proof is yet another fundamental counter example to the Einsteinian general relativity, which uses an incorrect symmetric symmetry for the connection.

Keywords: ECE theory, antisymmetric connection for rotation in three dimensions, counter example of Einsteinian general relativity.

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1. INTRODUCTION

In recent papers of this series {1 - 10} several inter-related proofs have been given of the fact that the Christoffel connection must be antisymmetric in its lower two indices. These proofs refute the Einsteinian general relativity (EGR) and all attempts at a unified field theory that still use EGR. The only valid unified field theory is Einstein Cartan Evans (ECE) theory, which uses an antisymmetric connection and non-zero torsion. Claims to have "verified" EGR experimentally cannot be accepted, because experimental data cannot verify a mathematically incorrect theory. This disaster for natural philosophy came about because of inadequate scholarship. In refuting EGR, ECE is strengthened conversely by every proof. Scholars know clearly that EGR has been refuted for nearly a hundred years, by the best minds of successive generations, and so EGR is an example of George Bernard Shaw's science turned superstition.

In Section 2, well known and straightforward considerations of rotation in three dimensions are used to deduce that the Christoffel connection is within a factor of proportionality the totally antisymmetric unit tensor in three dimensions, the Levi Civita tensor. The Christoffel connection is therefore antisymmetric in its lower two indices, a direct counter example to EGR. In Section 3, some consequences for this finding are discussed.

2. THE CHRISTOFFEL CONNECTION FOR THREE DIMENSIONAL ROTATION

Consider the rotation of a vector about the Z axis in the X Y plane of three dimensional Euclidean geometry. The active rotation is defined as the rotation clockwise of the vector, keeping the axes fixed. The passive rotation is defined as rotation of the axes counter clockwise keeping the vector fixed. The end result is the same. The Christoffel connection is zero for the active rotation because the axes are fixed. However, the connection is non zero for the passive rotation because the axes are rotated. The connection is defined as the rotation of axes and rotation is the simplest example of a Christoffel connection {11, 12}. It is often claimed that the connection is zero for Minkowski and Euclidean geometry, but this is true if an only if the axes are kept constant. The passive rotation is almost never considered because it is convenient to keep the axes fixed and to have a fixed reference frame.

The active rotation through an angle θ is defined by:

$$X' = X\cos\theta + Y\sin\theta \quad -(1)$$

$$Y' = -X\sin\theta + Y\cos\theta \quad -(2)$$

and in matrix format:

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad -(3).$$

Denoting:

$$R_Z = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad -(4)$$

the infinitesimal rotation generator is defined conventionally as {11}:

$$J_Z = \frac{1}{i}\left(\frac{dR_Z}{d\theta}\right)_{\theta=0} = -i\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \quad -(5)$$

Similarly, infinitesimal rotation generators in the X and Y axes can be defined similarly, and they are related by a cyclically symmetric commutator equation:

$$[J_X, J_Y] = i J_Z. \quad -(6)$$

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Within a factor $\hbar$ they are the angular momentum operators of quantum mechanics. The rotation generator J_Z is:

$$J_z = -i\,\epsilon_{ij} \quad -(7)$$

where ϵ_{ij} is the antisymmetric unit tensor. The latter is dual to the unit axial vector ϵ_k through the totally antisymmetric unit tensor ϵ_{ijk}:

$$\epsilon_{ij} = \epsilon_{ijk}\,\epsilon_k . \quad -(8)$$

The metric is the diagonal matrix:

$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(9)$$

and the k index can be raised as follows:

$$\epsilon^{k}{}_{ij} = g^{kl}\,\epsilon_{ijl} = \epsilon_{ijk} . \quad -(10)$$

Therefore:

$$\epsilon_{ij} = \epsilon^{k}{}_{ij}\,\epsilon_k . \quad -(11)$$

These considerations rest on the idea of an active rotation in three dimensional Euclidean geometry, so there is no Christoffel connection present.

However, the active rotation is equivalent to the passive rotation, which is defined as:

$$\underline{i}' = \underline{i}\cos\theta - \underline{j}\sin\theta \quad -(12)$$

$$\underline{j}' = \underline{i}\sin\theta + \underline{j}\cos\theta \quad -(13)$$

where $\underline{i}$ and $\underline{j}$ are Cartesian unit vectors. In matrix format:

$$\begin{bmatrix} \underline{i}' \\ \underline{j}' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix} \quad -(14)$$

Denoting:

$$R_{Zp} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad -(15)$$

the infinitesimal passive rotation generator is:

$$J_{Zp} = -i \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} . \quad -(16)$$

The rotation generators for active and passive rotations are related by:

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad -(17)$$

and one is the inverse of the other. The following quantity is a constant of the rotation:

$$r = \left(X^2 + Y^2\right)^{1/2} = \left(X'^2 + Y'^2\right)^{1/2} . \quad -(18)$$

The Christoffel connection in general is defined by the covariant derivative in the general

space:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda \quad -(19)$$

The equivalence of active and passive rotations was defined in UFT199 by:

$$\underset{\text{(active)}}{\partial_\mu V^\nu} = \underset{\text{(passive)}}{\Gamma^\nu_{\mu\lambda} V^\lambda} . \quad -(20)$$

For a rotation about the Z axis in the XY plane:

$$\frac{\partial X'}{\partial Y} = \Gamma^1_{23} Z'. \quad -(21)$$

Here:

$$\frac{\partial X'}{\partial Y} = \sin\theta, \quad \Gamma^1_{23} = \frac{\epsilon^1_{23} \sin\theta}{Z'} \quad -(22)$$

and so:

$$\Gamma^1_{23} = -\Gamma^1_{32}. \quad -(23)$$

The connection is antisymmetric in its lower two indices QED.

From Eq. (21):

$$\left(\frac{d}{d\theta}\left(\frac{\partial X'}{\partial Y}\right)\right)_{\theta=0} = 1 = \epsilon^1{}_2 = \epsilon_{12} \quad -(24)$$

where

$$\epsilon_{ij} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad -(25)$$

and

$$\left(\frac{d}{d\theta}\left(\Gamma^1_{23} Z'\right)\right)_{\theta=0} = \epsilon^1{}_{23}\epsilon_3 = \epsilon^1{}_{23}\epsilon^3 \quad -(26)$$

with

$$\epsilon^1{}_{23} = 1, \quad \epsilon^3 = 1. \quad -(27)$$

So Eq. (21) is:

$$\epsilon^i{}_j = \epsilon^i{}_{jk}\epsilon^k. \quad -(28)$$

This is the well known equation defining the antisymmetric unit tensor in terms of the Levi Civita tensor and axial unit vector. The metric is:

$$g_{ij} = g^{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(29)$$

and it follows that:

$$\epsilon^{i}{}_{j} = g^{ik}\epsilon_{kj} = \epsilon_{ij} \; ; \; \epsilon^{i} = g^{ik}\epsilon_{k} = \epsilon_{i} \; ; \quad -(30)$$
$$\epsilon^{i}{}_{jk} = g^{il}\epsilon_{ljk} = \epsilon_{ijk} \, .$$

Here:

$$\epsilon_{123} = -\epsilon_{132} = \epsilon_{231} = -\epsilon_{213} = \epsilon_{312} = -\epsilon_{321} = 1, \quad -(31)$$
$$\epsilon_{ij} = \epsilon_{ijk}\epsilon_{k} \quad -(32)$$

Eqs. (28) and (32) are well known and are usually interpreted as the duality of an axial unit vector with the antisymmetric unit tensor. However, they can also be interpreted as the equivalence of the active and passive rotations.

3. GENERAL COORDINATE TRANSFORMATION OF THE CHRISTOFFEL CONNECTION.

In general it is well known that the Christoffel connection transforms as:

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial x^{\nu'}}{\partial x^{\nu}}\Gamma^{\nu}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}}\right) \quad -(33)$$

under the general coordinate transformation. In three space dimensions:

$$\Gamma^{k'}_{i'j'} = \frac{\partial x^{i}}{\partial x^{i'}}\frac{\partial x^{j}}{\partial x^{j'}}\frac{\partial x^{k'}}{\partial x^{k}}\Gamma^{k}_{ij} - \frac{\partial x^{i}}{\partial x^{i'}}\frac{\partial x^{j}}{\partial x^{j'}}\frac{\partial}{\partial x^{i}}\left(\frac{\partial x^{k'}}{\partial x^{j}}\right). \quad -(34)$$

For a rotation about the Z axis in the XY plane:

$$\frac{\partial x^{3'}}{\partial x^{2}} = \frac{\partial Z'}{\partial Y} = 0 \quad -(35)$$

so the inhomogeneous term is zero and the connection transforms as:

$$\frac{\partial x^{1}}{\partial x^{1'}}\frac{\partial x^{2}}{\partial x^{2'}}\frac{\partial}{\partial x^{1}}\left(\frac{\partial x^{3'}}{\partial x^{2}}\right)=0 \quad -(36)$$

The transformed connection is therefore:

$$\Gamma^{3'}_{1'2'}=\frac{\partial x^{1}}{\partial x^{1'}}\frac{\partial x^{2}}{\partial x^{2'}}\frac{\partial x^{3'}}{\partial x^{3}}\Gamma^{3}_{12} \quad -(37)$$

and is non-zero if and only if:

$$x^{3'}=x^{3}. \quad -(38)$$

This condition is true because:

$$Z'=Z. \quad -(39)$$

Similar considerations hold for rotations about the Y and X axes, and it is well known that any rotation can be described with three rotation generators. Almost always these are considered to be active rotations, but as shown in Section 2 the active rotations are equivalent to passive rotations, each with its Christoffel connection.

Recent papers of the UFT series have shown in several ways that the Christoffel connection must be antisymmetric in its lower two indices in general. The inhomogeneous term in the transformation of the connection must therefore vanish, because it is symmetric:

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}}\right)=\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial}{\partial x^{\lambda}}\left(\frac{\partial x^{\nu'}}{\partial x^{\mu}}\right)=0. \quad -(40)$$

Therefore the simple considerations of rotation given in this paper can be generalized to any space in any dimensions and in any frame of reference. These conclusions dispel the dogma of a century, one that incorrectly asserted that the connection is symmetric. From Eq. (40)

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \Gamma^{\nu}_{\mu\lambda} \quad -(41)$$

and as proven in Section 2,

$$\frac{\partial x^{\nu'}}{\partial x^{\lambda}} = 0 \quad -(42)$$

for any passive rotation in three dimensional Euclidean space. In order to prove this result in general, Cartan geometry is introduced. The base manifold is supplemented by a space at point P, a space defined by the Minkowski metric. The Cartan tetrad is defined for two vectors V^{a} and V^{μ} by:

$$V^{a} = q^{a}_{\mu} V^{\mu}. \quad -(43)$$

It follows that:

$$\frac{\partial x^{\nu'}}{\partial x^{\lambda}} = \frac{\partial x^{\nu'}}{\partial x^{a}} \frac{\partial x^{a}}{\partial x^{\lambda}} = q^{\nu'}_{a} q^{a}_{\lambda} \quad -(44)$$

and by definition in Cartan geometry:

$$q^{\nu'}_{a} q^{a}_{\lambda} = \delta^{\nu'}_{\lambda}. \quad -(45)$$

Therefore:

$$\frac{\partial x^{\nu'}}{\partial x^{\lambda}} = 0 \quad -(46)$$

unless

$$\nu' = \lambda. \quad -(47)$$

If:

$$x^{\nu'} = x^{\lambda} \quad -(48)$$

then

$$\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}}\right) = 0 \quad -(49)$$

and the inhomogeneous term vanishes for all spaces in any number of dimensions, Q. E. D.

This proof rests on the well known definition of the Cartan tetrad as a matrix that links the tangent spacetime with Latin indices to the base manifold with Greek indices. The mixed index antisymmetric connection therefore transforms as a tensor:

$$\Gamma^{a'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{a'}}{\partial x^{a}} \Gamma^{a}_{\mu\lambda} \quad -(50)$$

and so does the antisymmetric Christoffel connection. In Eq. (50):

$$\frac{\partial x^{a'}}{\partial x^{a}} \neq 0 \quad -(51)$$

in general in the tangent space. For example, if a rotation takes place in the tangent space then:

$$\frac{\partial x^{a'}}{\partial x^{a}} = \frac{\partial X'}{\partial X} = \frac{\partial X'}{\partial Y}\frac{\partial Y}{\partial X} = q^{a'}_{b} q^{b}_{a} \quad -(52)$$

In this rotation:

$$\frac{\partial X'}{\partial X} = \cos\theta, \quad \frac{\partial X'}{\partial Y} = \sin\theta, \quad \frac{\partial Y}{\partial X} = \tan\theta, \quad -(53)$$

so Eq. (52) is not zero. The definition (45) therefore applies only to tetrads linking the tangent space to the base manifold.

It is concluded that the Christoffel connection is always antisymmetric and always transforms as a tensor. In the incorrect dogma the Christoffel connection is symmetric and does not transform as a tensor. Einsteinian general relativity is therefore refuted in many ways, and ECE theory strengthened by many proofs.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and many others are thanked for many interesting discussions. Dave Burleigh is thanked for posting, and Alex Hill, Robert Cheshire and Simon Clifford for broadcasts and translation. The AIAS is established as part of the Newlands Family Trust (2012).

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