SIMPLE PROOFS OF THE ANTISYMMETRY OF THE CHRISTOFFEL CONNECTION.

by

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ABSTRACT

It is proven straightforwardly in two ways that the Christoffel connection is antisymmetric in its lower two indices, thus directly refuting the Einsteinian general relativity. The correct derivation is given of the Newton equation from the geodesic equation, using a correctly antisymmetric Christoffel connection.

Keywords: ECE theory, antisymmetry of the Christoffel connection, derivation of Newton's equation from the geodesic equation.

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INTRODUCTION

The Christoffel connection was introduced into geometry in 1869 in such a way that it was defined to be symmetric in its lower two indices. There are symmetric Christoffel symbols of the first and second kind. The Christoffel connection augmented the earlier work of Riemann, who introduced the idea of the symmetric metric tensor. About thirty years later tensor analysis was developed and the concept of curvature introduced by Levi-Civita, Ricci, Bianchi and co workers. At that time, the symmetric Christoffel connection was the only definition available, and was used by Einstein in his development of general relativity as is well known. The Einstein field equation is correct if and only if the Christoffel connection is symmetric. In about 1923 however Cartan and his co workers introduced the concept of torsion, which exists if and only if the Christoffel connection is antisymmetric. So it became clear that the original 1869 definition by Christoffel was too restricted. Cartan communicated with Einstein as is well known but the antisymmetric part of the connection continued to be ignored. In the ECE series of papers {1-10} it has been proven in several ways that the Christoffel connection has no symmetric part, it is wholly antisymmetric. The entire era of Einsteinian general relativity (EGR) has been refuted.

In Section 2 a completely simple proof of the antisymmetry of the connection is given using the well known commutator method {11} of generating torsion and curvature simultaneously. The EGR theory incorrectly asserts a symmetric connection as axiomatic, so the torsion is incorrectly zero by axiom. The commutator method on the other hand isolates the connection, showing that its lower index symmetry is that of the commutator, i.e. antisymmetric. The null commutator is a commutator and is zero because it is symmetric. Therefore the symmetric connection is zero. Q.E.D. This finding is enough to refute EGR completely. EGR is a meaningless theory and claims to have verified it are misplaced entirely.

In Section 3 a further simple proof of the antisymmetry of the connection is given using the general coordinate transformation. Full details are given in the notes accompanying this paper on www.aias.us. Finally, in Section 4, it is shown very simply that the Einsteinian derivation of the Newton equation from the geodesic equation fails immediately because of its use of a symmetric Christoffel connection. An example of a correct derivation is given using an antisymmetric Christoffel connection.

2. COMMUTATOR PROOF OF CONNECTION ANTISYMMETRY

The commutator of covariant derivatives {11} in geometry is an operator that acts on any tensor in a space of any dimensions to produce the torsion and curvature tensors. Its action on a vector V^ρ produces the following result:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(1)$$

where the torsion tensor is defined by:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \quad -(2)$$

and where $R^\rho{}_{\sigma\mu\nu}$ is the curvature tensor. It is well known that both the torsion and curvature are antisymmetric:

$$T^\lambda{}_{\mu\nu} = -T^\lambda{}_{\nu\mu} \quad -(3)$$

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu} \quad -(4)$$

because the commutator is antisymmetric:

$$[D_\mu, D_\nu] V^\rho = -[D_\nu, D_\mu] V^\rho \quad -(5)$$

Let:

$$\Gamma^{\lambda}_{\mu\nu} = A, \quad \Gamma^{\lambda}_{\nu\mu} = B \quad -(6)$$

then

$$[D_\mu, D_\nu] V^\rho = (B - A) D_\lambda V^\rho + \ldots \quad -(7)$$

$$[D_\nu, D_\mu] V^\rho = (A - B) D_\lambda V^\rho + \ldots \quad (8)$$

Now let:

$$\mu = \nu \quad -(9)$$

and Eq. (7) gives:

$$B - A = 0 \quad -(10)$$

while Eq. (8) gives:

$$A - B = 0. \quad -(11)$$

So it follows that:

$$B - A = A - B = 0, \quad -(12)$$

$$2A = 2B = 0. \quad -(13)$$

The symmetric connection is zero Q. E. D. and the Christoffel connection is always antisymmetric:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}. \quad -(14)$$

This proof uses the fact that the null commutator is a commutator which is zero because it is symmetric:

$$[D_\mu, D_\nu]\, V^\rho = 0, \quad \mu = \nu \quad -(15)$$

The EGR theory uses a symmetric connection and is a meaningless theory.

3. TRANSFORMATION OF THE CHRISTOFFEL CONNECTION.

It is well known {1 - 10} that the Christoffel connection transforms as:

$$\Gamma^{\nu'}_{\mu'\lambda'} = q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} \left(q^{\nu'}_{\nu} \Gamma^{\nu}_{\mu\lambda} - \partial_\mu q^{\nu'}_{\lambda} \right) \quad -(16)$$

under general coordinate transformation. In this notation:

$$q^{\mu}_{\mu'} = \frac{\partial x^\mu}{\partial x^{\mu'}}, \quad q^{\lambda}_{\lambda'} = \frac{\partial x^\lambda}{\partial x^{\lambda'}}, \quad q^{\nu'}_{\nu} = \frac{\partial x^{\nu'}}{\partial x^\nu}. \quad -(17).$$

Consider:

$$q^{\nu'}_{\lambda} = \frac{\partial x^{\nu'}}{\partial x^\lambda} = \frac{\partial x^{\nu'}}{\partial x^\nu} \frac{\partial x^\nu}{\partial x^\lambda}. \quad -(18)$$

In the arbitrary manifold however (Eq. (2.15) of ref. (11), 1997 online notes):

$$\frac{\partial x^\nu}{\partial x^\lambda} = \delta^\nu_\lambda \quad -(19)$$

where the Kronecker delta function is defined by:

$$\delta^\nu_\lambda = 1, \quad \nu = \lambda \quad -(20)$$
$$= 0, \quad \nu \neq \lambda.$$

Therefore:

$$\frac{\partial x^\nu}{\partial x^\lambda} = 0 \quad -(21)$$

unless:

$$\nu = \lambda. \quad -(22)$$

If Eq. (22) is true then:

$$\frac{\partial x^{\nu'}}{\partial x^{\lambda}} = \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} = \frac{\partial x^{\lambda'}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial x^{\lambda}} \quad -(23)$$
$$= 0$$

unless:

$$\mu = \lambda. \quad -(24)$$

So:

$$\partial_\mu q^{\nu'}_{\lambda} = 0 \quad -(25)$$

unless:

$$\mu = \nu = \lambda. \quad -(26)$$

This is inconsistent with the fact that:

$$\mu \neq \nu \neq \lambda \quad -(27)$$

in general. Therefore:

$$\frac{\partial x^{\nu'}}{\partial x^{\lambda}} = 0 \quad -(28)$$

in general and the connection transforms as a tensor:

$$\Gamma^{\nu'}_{\mu'\lambda'} = q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} q^{\nu'}_{\nu} \Gamma^{\nu}_{\mu\lambda}. \quad -(29)$$

The accompanying notes show that this result can be proven in several other ways, and the reader is referred to these notes in the UFT section of www.aias.us.

The EGR used the idea of a local Lorentz frame {11} in which spacetime is locally flat and the connections vanish. From Eq. (16) the transformed connection vanishes if:

$$q^{\sim'}_{\sim}\,\Gamma^{\sim}_{\mu\lambda} = \partial_\mu q^{\sim'}_{\lambda} \qquad -(30)$$

i.e. if:

$$\Gamma^{\sim}_{\mu\lambda} = \frac{\partial x^{\sim}}{\partial x^{\sim'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\sim'}}{\partial x^{\lambda}}\right) \qquad -(31)$$

then:

$$\Gamma^{\sim'}_{\mu'\lambda'} = 0. \qquad -(32)$$

However the correct result is obtained from Eqs. (28) and (31), i.e. if

$$\Gamma^{\sim}_{\mu\lambda} = 0 \qquad -(33)$$

then

$$\Gamma^{\sim'}_{\mu'\lambda'} = 0. \qquad -(34)$$

If the connection vanishes in one frame it vanishes in all frames. The idea of Riemann normal coordinates cannot be used. In turn the Einstein equivalence principle cannot stand as defined in EGR, because it depends on the locally flat Lorentz frame.

The original 1869 definition of the connection by Christoffel is similar to Eq. (31) an equation which implies a symmetric connection because:

$$\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\sim'}}{\partial x^{\lambda}}\right) = \frac{\partial}{\partial x^{\lambda}}\left(\frac{\partial x^{\sim'}}{\partial x^{\mu}}\right). \qquad -(35)$$

From Eq. (28) it is seen however that

$$\Gamma^{\sim}_{\mu\lambda} = \Gamma^{\sim}_{\lambda\mu} = 0 \qquad -(36)$$

i.e. the symmetric connection is zero Q. E.D. The antisymmetric connection is the only non zero connection and transforms as a tensor:

$$\Gamma^{\nu'}_{\mu'\lambda'} = q^{\mu}_{\mu'}\, q^{\lambda}_{\lambda'}\, q^{\nu'}_{\nu}\, \Gamma^{\nu}_{\mu\lambda} . \quad -(37)$$

The torsion transforms as a tensor:

$$T^{\nu'}_{\mu'\lambda'} = q^{\mu}_{\mu'}\, q^{\lambda}_{\lambda'}\, q^{\nu'}_{\nu}\, T^{\nu}_{\mu\lambda} \quad -(38)$$

and if it is zero in one frame it is zero in all frames.

4. A CORRECT DERIVATION OF THE NEWTON EQUATION FROM THE GEODESIC EQUATION.

The geodesic equation {1 - 11} is:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^{\mu}_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0 \quad -(39)$$

where the affine (or manifestly covariant) parameter has been chosen to be the proper time τ.

In standard EGR the reduction of this equation to the Newton equation consisted of assuming that in the space part of the equation with index i:

$$\frac{dx^i}{d\tau} \ll \frac{dt}{d\tau} \quad -(40)$$

where t is the time in the observer frame. This assumption reduces Eq. (39) to:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^{\mu}_{00} \left(\frac{dt}{d\tau}\right)^2 = 0 . \quad -(41)$$

However:

$$\Gamma^{\mu}_{00} = 0 \quad -(42)$$

by antisymmetry, so the EGR theory fails immediately.

The following is one correct way of reducing Eq. (39) to the format of Newtonian dynamics. Consider the spacelike part of Eq. (39):

$$\frac{d^2 x^i}{d\tau^2} = -\Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau}, \quad -(43)$$

using the method developed in UFT212 on www.aias.us:

$$\Gamma^i_{jk} = -\Gamma^i_{kj} = \frac{1}{r} \epsilon^i_{jk} \quad -(44)$$

where r is the magnitude of the radial vector. Multiply both sides of Eq. (43) by

$$x = \left(\frac{d\tau}{dt}\right)^2 \quad -(45)$$

to give:

$$\frac{d^2 x^i}{dt^2} = -\frac{1}{r} \epsilon^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt}. \quad -(46)$$

Newtonian dynamics are given by:

$$\frac{d^2 x^i}{dt^2} = -\frac{d\Phi}{dx^i}. \quad -(47)$$

In Eq. (47):

$$\frac{d\Phi}{dx^i} = \frac{d\Phi}{dt} \frac{dt}{dx^i} = \frac{1}{r} \epsilon^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} \quad -(48)$$

so:

$$\Phi = \frac{x^i}{r} \epsilon^i_{jk} v^j v^k = \frac{x^i}{r} v v^i \quad -(49)$$

Therefore Newtonian dynamics are defined by:

$$\Phi = \frac{v}{r} . x^i v_i \quad -(50)$$

with summation over repeated indices. The anti symmetric commutator gives Newtonian dynamics, a commutator defined by:

$$\Gamma^i_{jk} = \frac{1}{r} \epsilon^i_{jk} . \quad -(51)$$

More generally:

$$\frac{d^2 x^i}{dt^2} = -\Gamma^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} \quad -(52)$$

and most generally:

$$\frac{d^2 x^\mu}{d\tau^2} = -\Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} \quad -(53)$$

give non Newtonian dynamics. The EGR derivation uses a symmetric connection and is incorrect and meaningless.

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