KEPLER'S THIRD LAW FOR A PRECESSING ORBIT.

by

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ABSTRACT

It is shown that Kepler's third law of 1619 develops into intricate detail for a precessing orbit in the solar system. The detail appears as the precession constant x is increased. The calculation is based on the new universal gravitational potential that is responsible for planetary precession, and for the precession of orbits in general. This calculation is carried out in the classical limit of ECE theory.

Keywords: Kepler's third law, precessing orbits, classical limit of ECE theory.

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1. INTRODUCTION

In recent papers of this series the precession of planetary orbits has been explained in terms of a new universal law of gravitation in the classical limit of ECE theory and without Einsteinian general relativity (EGR) {1 - 10}. In this paper the new universal law of gravitation is used to generalize Kepler's third law of planetary motion, inferred in 1619. In the solar system it is well known that the precession of the perihelion is very tiny, only a few arc seconds per century. This precessional angle is defined by $2\pi(x - 1)$ where x is the precession constant. Kepler's original third law was inferred for the orbit of Mars, which he found to be an ellipse (Kepler's first law). Kepler's second law of 1609 shows that the orbit sweeps out equal areas in equal times. Planetary precession was unknown to astronomers in Kepler's time and Newtonian dynamics applies only to orbits which are not precessing. Bernoulli was the first to show that these must be conic sections. Einstein made an attempt to explain precession using Riemann geometry but in recent papers of this series that theory, Einsteinian general relativity (EGR), has been shown to be erroneous in many ways. For example, EGR produces an incorrect force law for a precessing conical section. The correct force law is found by straightforward lagrangian dynamics as in recent UFT papers.

In section 2 the correct force law and gravitational potential is used to define correctly the precessing elliptical orbit, and Kepler's third law deduced from the precessing orbit. The result reduces to the law of 1619 when x is unity, but as x is increased it develops an intricate structure hitherto unknown to astronomy. Some of these structures are graphed in Section 3 and cannot be deduced from the erroneous EGR.

2. KEPLER'S THIRD LAW.

The precessing orbit of a mass m such as a planet around the sun of mass M is:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

in the plane cylindrical coordinate system (r, θ). Here α is the semi right latitude, ϵ is the ellipticity and x is the precession constant. As x is increased this orbit develops an intricate structure hitherto unknown to astronomy. It has been shown recently {1 - 10} that the format (1) can describe all known orbits. It has also been shown that the orbit can be derived from the lagrangian:

$$\mathcal{L} = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r) \quad -(2)$$

using the gravitational potential:

$$U(r) = -\frac{mMG}{r} - \frac{L^2}{2mr^2}\left(1 - x^2\right) \quad -(3)$$

where L is the constant total angular momentum defined by:

$$L = \mu r^2 \frac{d\theta}{dt}. \quad -(4)$$

The reduced mass is defined by:

$$\mu = \frac{mM}{m + M} \quad -(5)$$

and if M >> m the reduced mass is approximately m.

For any curve in two dimensions {11}:

$$dA = \frac{1}{2} r^2 d\theta, \quad -(6)$$

so the area of the precessing ellipse is:

$$A = \frac{1}{2}\int \left(\frac{\alpha}{1+\epsilon\cos(x\theta)}\right)^2 d\theta. \quad -(7)$$

Kepler's second law is therefore:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{L}{2\mu} \quad -(8)$$

and is the same for the precessing and static ellipses. From this law:

$$\int dt = \frac{2\mu}{L}\int dA. \quad -(9)$$

Denote:

$$\tau = \int dt, \quad A = \int dA \quad -(10)$$

to obtain:

$$\tau = \frac{2\mu}{L} A. \quad -(11)$$

The time τ taken to transcribe the angle θ can be measured to great accuracy in modern astronomy.

Now integrate Eq. (7) using the change of variable:

$$\beta = x\theta. \quad -(12)$$

The area is therefore:

$$A = \frac{\alpha^2}{2x}\int \frac{d\beta}{(1+\epsilon\cos\beta)^2}$$

$$= \frac{\alpha^2}{2x}\left[\frac{2}{(1-\epsilon^2)^{3/2}}\tan^{-1}\left((1-\epsilon^2)^{1/2}\tan\frac{\beta}{2}\right) - \frac{\epsilon\sin\beta}{(1-\epsilon^2)(1+\epsilon\cos\beta)}\right] \quad -(13)$$

and the time τ for a precessing orbit is:

$$\tau = \frac{2\mu}{L}A, \quad -(14)$$

$$A = \frac{\alpha^2}{2x}\left[\frac{2}{(1-\epsilon^2)^{3/2}}\tan^{-1}\left((1-\epsilon^2)^{1/2}\tan\left(\frac{x\theta}{2}\right)\right) - \frac{\epsilon\sin(x\theta)}{(1-\epsilon^2)(1+\epsilon\cos(x\theta))}\right].$$

As x is increased this develops the intricate structure graphed in Section 3. The static or non precessing ellipse is given by:

$$x = 1. \quad -(15)$$

The area of the static ellipse is:

$$A = \frac{1}{2}\int_0^{2\pi}\frac{d\theta}{(1+\cos\theta)^2} \quad -(16)$$

$$= \frac{\pi}{(1-\epsilon^2)^{3/2}} = \pi ab$$

where a and b are the semi major and minor axes. Therefore for a static ellipse:

$$\tau = \frac{2\pi\mu}{L}ab \quad -(17)$$

and it corresponds {11} to the Newtonian theory in which:

$$a = \frac{\alpha}{1-\epsilon^2} = \frac{k}{2|E|} \quad -(18)$$

and

$$b = \frac{\alpha}{(1-\epsilon^2)^{1/2}} = \frac{L}{(2\mu|E|)^{1/2}} \quad -(19)$$

Here E is the total energy and:

$$L^2 = \alpha\mu k = \alpha\mu m M G. \quad -(20)$$

Therefore:

$$\tau^2 = \frac{4\pi^2 \mu}{m M G} a^3 \quad -(21)$$

which is Kepler's third law for a non precessing orbit.

For a precessing orbit however Kepler's third law no longer holds, it is replaced by:

$$\tau = \frac{\mu \alpha^2}{x L} f(\theta) \quad -(22)$$

where:

$$f(\theta) = \frac{2}{(1-\epsilon^2)^{3/2}} \tan^{-1}\left((1-\epsilon^2)^{1/2} \tan\left(\frac{x\theta}{2}\right)\right) - \frac{\epsilon \sin(\theta x)}{(1-\epsilon^2)(1+\epsilon\cos(x\theta))}. \quad -(23)$$

From Eq. (20):

$$\tau = \frac{\left(a(1-\epsilon^2)\right)^{3/2}}{x m M G} f(\theta). \quad -(24)$$

The orbital observables are τ and ϵ and it is seen that the time taken to cover 2π radians is different for a precessing ellipse. As x is increased it becomes dramatically different as in Section 3.

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3 Graphical illustrations

The area of Kepler's third law is a complicated function of the angle θ and is graphed in Fig. 1 for three values of x. Because the inverse tangens function is defined in the interval $[-\pi/2, \pi/2]$, the result is defined modulo this range and jumps occur. It can be seen that the area function grows nearly linearly for the ellipticity of $\epsilon = 0.3$ chosen for the example.

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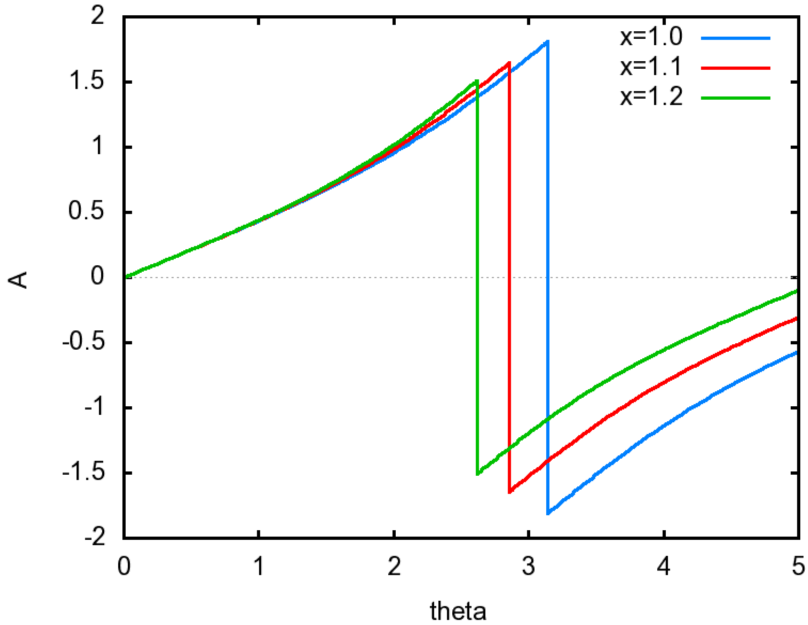

Figure 1: Area of Kepler's third law for parameters $\epsilon = 0.3, \alpha = 1$.

3. GRAPHICAL ILLUSTRATIONS

Section by Horst Eckardt.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and AIAS and others for many interesting discussions. Dave Burleigh is thanked for posting, Alex Hill, Robert Cheshire and Simon Clifford for translation and broadcasting.

REFERENCES

{1} M. W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity", (Cambridge International Science Publishing, CISP, www.cisp-publishing.com, 2011).

{2} M. W. Evans, Ed., J. Found. Phys. Chem., CISP 2011 onwards, six issues a year.

{3} M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP 2011).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis Academic, 2005 to 2011) in seven volumes.

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, 2007).

{6} K. Pendergast, "The Life of Myron Evans" (CISP, 2011).

{7} M. W. Evans and S. Kielich (Eds.), "Modern Nonlinear Optics" (Wiley, 1992, 1993, 1997 and 2001) in two editions and six volumes.

{8} M. W. Evans and L. B.Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

{9} M. W. Evans and J. -P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002) in ten volume hardback and softback.

{10} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific 1994).

{11} J. B. Marion and S. T. Thornton, "Classical Dynamics" (Harcourt, New York, 1988, 3rd. Ed.).