ORBITAL DYNAMICS WITH A CONSTRAINED MINKOWSKI METRIC: TIME DILATATION AND RELATIVISTIC FORCE EQUATION.

by

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ABSTRACT

Orbital dynamics with a constrained Minkowski metric are used to calculate time dilatation without Einsteinian general relativity. The results are consistent with the observed precessional motion in the solar system. The lagrangian of special relativity is used to develop a relativistic force equation for orbital dynamics and to show that the origin of all orbits is spacetime torsion and curvature described correctly by Cartan geometry.

Keywords: ECE theory, constrained Minkowski method, time dilatation, lagrangian, force equation.

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1. INTRODUCTION

Recently in this series of papers {1 - 10} a method of describing orbital dynamics has been developed based directly on observation. This is the constrained Minkowski method or x theory. The method uses the observed orbit to constrain the infinitesimal line element or metric, and the constrained metric is used to construct the torsion and curvature elements of ECE theory. In its classical non-relativistic limit the x theory produces all observable orbits without the use of dark matter, which does not exist in nature and is a mathematical artifice as is well known. The x theory removes the multiple errors in Einsteinian general relativity (EGR), a theory that has been severely criticised for nearly a hundred years. In Section 2, the x theory is used to calculate time dilatation from an observed orbit of any kind, so this is the only self consistent method of calculating this phenomenon in cosmology. It cannot be calculated in any meaningful way with EGR, and any claim to have verified EGR experimentally is meaningless because the theory is incorrect in so many ways {1 - 10}. The lagrangian of the x theory is used to calculate the relativistic force equation of x theory, proving that orbital precession can be explained with a relativistic force equation. The concept of force is rejected in EGR, but that theory has in turn been rejected because of basic mathematical errors of many different kinds. In Section 3 a graphical analysis of the results is given.

2. TIME DILATATION AND FORCE EQUATION

Consider the unconstrained Minkowski line element {11}:

$$ds^2 = c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(1)$$

Here $d\tau$ is the infinitesimal of proper time, $g_{\mu\nu}$ is the metric, x^μ is the coordinate four vector, c is the assumed constant speed of light in vacuo. The line element is written using cylindrical polar coordinates (r , θ) in a plane. In this notation:

$$dx^0 = c\,dt, \quad dx^1 = dr, \quad dx^2 = r\,d\theta, \qquad -(2)$$
$$g_{00} = 1, \quad g_{11} = -1, \quad g_{22} = -1.$$

The orbit is defined by:

$$g = \frac{dr}{d\theta} \qquad -(3)$$

and:

$$ds^2 = c^2 dt^2 - \left(\frac{dr}{d\theta}\right)^2 d\theta^2 - r^2 d\theta^2. \qquad -(4)$$

So:

$$g = \frac{dr}{d\theta} = r\frac{dx^1}{dx^2} \;; \quad (dx^1)^2 = \left(\frac{g}{r}dx^2\right)^2. \qquad -(5)$$

Therefore:

$$ds^2 = g_{00}\,dx^0 dx^0 + g_{11}\left(\frac{g}{r}dx^2\right)^2 + g_{22}\left(dx^2\right)^2. \qquad -(6)$$

Let

$$f = \left(\frac{r}{g}\right)^2 \qquad -(7)$$

then:

$$ds^2 = g_{00}\,dx^0 dx^0 + \frac{g_{11}}{f}\,dx^2 dx^2 + g_{22}\,dx^2 dx^2 \qquad -(8)$$
$$= g_{00}\,dx^0 dx^0 + g_{22}'\,dx^2 dx^2.$$

Thus:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - \left(1 + \frac{1}{f}\right) r^2 d\theta^2$$

$$= c^2 dt^2 - v^2 dt^2 \qquad -(9)$$

because:

$$d\underline{r} \cdot d\underline{r} = v^2 dt^2 = \left(1 + \frac{1}{f}\right) r^2 d\theta^2 \qquad -(10)$$

Therefore:

$$\frac{d\tau}{dt} = \left(1 - \frac{1}{c^2}\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)\left(\frac{d\theta}{dt}\right)^2\right)^{1/2} \qquad -(11)$$

For the precessing conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \qquad -(12)$$

then:

$$\frac{dr}{d\theta} = \frac{x\epsilon}{\alpha} r^2 \sin(x\theta) \qquad -(13)$$

where:

$$\sin(x\theta) = \frac{1}{\epsilon r}\left(\epsilon^2 r^2 - (\alpha - r)^2\right)^{1/2} \qquad -(14)$$

So:

$$\frac{dr}{d\theta} = \frac{xr}{\alpha}\left(\epsilon^2 r^2 - (\alpha - r)^2\right)^{1/2} \qquad -(15)$$

From Eqs. (11) and (15):

$$\frac{d\tau}{dt} = \left(1 - \frac{1}{c^2}\left(r^2 + \left(\frac{xr}{\alpha}\right)^2\left(\epsilon^2 r^2 - (\alpha - r)^2\right)\right)\left(\frac{d\theta}{dt}\right)^2\right)^{1/2} \qquad -(16)$$

Finally:

$$L = \mu r^2 \, d\theta / dt \quad -(17)$$

so:

$$\frac{d\theta}{dt} = \frac{L}{\mu r^2} \quad -(18)$$

The final result for time dilatation is therefore:

$$\frac{d\tau}{dt} = \left(1 - \left(\frac{L}{c\mu r}\right)^2 \left(1 + \left(\frac{x}{d}\right)^2 \left(\epsilon^2 r^2 - (d - r)^2\right)\right)\right)^{1/2} \quad -(19)$$

and is graphed and discussed in Section 3. It is seen from Eq. (9) that in x theory:

$$d\tau = \left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2} dt \quad -(20)$$

so x theory is compatible automatically with special relativity. The conventional EGR theory is well known to be incorrect, and relies on an assumed infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2\left(1 - \frac{r_0}{r}\right) dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad -(21)$$

incorrectly attributed to Schwarzschild. It is well known {12} that Schwarzschild did not infer such a line element. Here:

$$r_0 = \frac{2MG}{c^2}, \quad d\mathbf{r} \cdot d\mathbf{r} = \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\theta^2, \quad -(22)$$

where m is an orbiting mass attracted to M, and where G is Newton's constant. From Eq. (22):

$$d\mathbf{r} \cdot d\mathbf{r} = v^2 dt^2 \quad -(23)$$

so:

$$ds^2 = c^2 d\tau^2 = \left(c^2\left(1 - \frac{r_0}{r}\right)^{-1} - v^2\right) dt^2 \quad -(24)$$

and the EGR time dilatation is:

$$\frac{d\tau}{dt} = \left(\left(1 - \frac{r_0}{r}\right)^{-1} - \frac{v^2}{c^2}\right)^{1/2}. \quad -(25)$$

This result is compatible with special relativity only in the limit:

$$r \rightarrow \infty \quad -(26)$$

which is meaningless, because in this limit there is no gravitational attraction. This is one of the many conceptual errors of EGR. The accompanying note 223(3) describes the basic error in EGR, its use of an incorrect force law, one that does not produce a precessing ellipse at all. The correct force law in x theory is obtained directly from observation and the same lagrangian dynamics.

In order to develop a consistent x theory in special relativity note that the lagrangian of the theory {11} is defined as follows. For a single, non-relativistic particle moving in a velocity independent potential the relativistic linear momentum is:

$$p = \frac{\partial \mathcal{L}}{\partial v} = \frac{\partial \mathcal{L}}{\partial \dot{x}} \quad -(27)$$

where $\mathcal{L}$ is the relativistic lagrangian. The relativistic momentum is:

$$p = \gamma m v \quad -(28)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad -(29)$$

Therefore:

$$\frac{\partial \mathcal{L}}{\partial v} = \gamma m v. \quad -(30)$$

The velocity independent part of the lagrangian is the potential energy U, and in constructing the lagrangian of special relativity this is assumed {11} to be the same as in classical dynamics.

Relativistic quantities are distinguished by * . The relativistic lagrangian is:

$$\mathcal{L} = T^* - U \quad -(31)$$

where:

$$T^* = T^*(v), \quad U = U(x). \quad -(32)$$

The Euler Lagrange equation gives:

$$\frac{\partial T^*}{\partial v} = \gamma m v \quad -(33)$$

so:

$$T^* = -\frac{mc^2}{\gamma} = -mc^2\left(1-\frac{v^2}{c^2}\right)^{1/2} \quad -(34)$$

and the lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U. \quad -(35)$$

Note carefully {11} that the kinetic energy T* is not the same as the quantity referred to as the relativistic kinetic energy:

$$T = (\gamma - 1)mc^2. \quad -(36)$$

This is why T* is distinguished from T. This is one of the obscurities of special relativity, and is accepted because the theory produces good agreement with data.

The hamiltonian of special relativity is calculated from:

$$H = vp - \mathcal{L} \quad -(37)$$

Therefore:

$$H = \frac{1}{\gamma mc^2}\left(p^2c^2 + m^2c^4\right) + U = \frac{E^2}{\gamma mc^2} + U \quad -(38)$$

where the total energy is:

$$E = \gamma mc^2. \quad -(39)$$

So the hamiltonian in special relativity is:

$$H = E + U = T + E_0 + U \quad -(40)$$

where the rest energy is:

$$E_0 = mc^2 \quad -(41)$$

and where the relativistic kinetic energy is:

$$T = (\gamma - 1)mc^2. \quad -(42)$$

Therefore for relativistic x theory the complete lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(43)$$

in which the potential energy is:

$$U = \frac{mMGx^2}{r} + \frac{(x^2-1)L^2}{2mr^2}. \quad -(44)$$

The classical limit of Eq. (43) is defined as:

$$v << c \quad -(45)$$

and in this limit:

$$\mathcal{L} \rightarrow -mc^2\left(1 - \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) + \frac{mMGx^2}{r} + \frac{(x^2-1)L^2}{2mr^2}$$

$$= \frac{1}{2}mv^2 + \frac{mMGx^2}{r} + \frac{(x^2-1)L^2}{2mr^2} \quad -(46)$$

which is the classical lagrangian of x theory used in recent papers {1 - 10}, Q.E.D.

In cylindrical polar coordinates the lagrangian (43) is:

$$\mathcal{L} = -mc^2\left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right)^{1/2}\right) - U(r) \quad -(47)$$

using:

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2. \quad -(48)$$

The two Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(49)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}}. \quad -(50)$$

Define:

$$f(\dot{\theta}) = \left(1 - \frac{1}{c^2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right)\right)^{1/2} \quad -(51)$$

then:

$$\frac{\partial \mathcal{L}}{\partial f} = -\frac{1}{2} mc^2 f(\dot{\theta})^{-1/2} = -\frac{1}{2}\gamma mc^2. \quad -(52)$$

Using:
$$\frac{\partial f}{\partial \dot{\theta}} = -\frac{2r^2}{c^2}\dot{\theta} \quad -(53)$$

it is found that:
$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{\partial \mathcal{L}}{\partial f}\frac{\partial f}{\partial \dot{\theta}} = \gamma m r^2 \dot{\theta} \quad -(54)$$

so the relativistic angular momentum is:
$$L^* = \gamma m r^2 \frac{d\theta}{dt}. \quad -(55)$$

This is the same result as that given in previous work {1 - 10} from the line element:
$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2. \quad -(56)$$

Similarly:
$$\frac{\partial f}{\partial \dot{r}} = -\frac{2\dot{r}}{c^2}, \quad \frac{\partial f}{\partial r} = -\frac{2r\dot{\theta}^2}{c^2}, \quad -(57)$$

so:
$$\frac{\partial \mathcal{L}}{\partial r} = \gamma m r \dot{\theta}^2 - \frac{\partial U}{\partial r} \quad -(58)$$

and
$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = \gamma m \dot{r}. \quad -(59)$$

Therefore the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) = \gamma m \ddot{r} \quad -(60)$$

gives:

$$\gamma m\left(\ddot{r} - r\dot{\theta}^2\right) = F(r), \quad -(61)$$

$$L^* = \gamma m r^2 \dot{\theta}.$$

These equations have the same structure as the non relativistic theory {11} with:

$$m \rightarrow \gamma m. \quad -(62)$$

It follows that the relativistic force equation {11} is:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\gamma m r^2}{L^{*2}} F(r). \quad -(63)$$

For example, the observed orbit in the solar system is:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon \cos(x\theta)\right) \quad -(64)$$

where α is the semi right latitude. Therefore the relativistic force from Eqs. (63) and (64) is:

$$F^* = \gamma F \quad -(65)$$

where:

$$F = -\frac{x^2 L^2}{m r^2 \alpha} - \frac{(1 - x^2) L^2}{m r^3}. \quad -(66)$$

Therefore:

$$F^*(r) = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} F(\text{classical}) \quad -(67)$$

where

$$F = \frac{F^*}{\gamma} \quad -(68)$$

is the classical force:

$$F = -\frac{mMGx^2}{r^2} + \frac{(x^2-1)L^2}{mr^3} . \quad -(69)$$

These results are incorporated into the complete x theory as shown in note 223(6) accompanying this paper.

The relativistic force (65) may be derived in another way as follows, providing a cross check on the result (65). Consider the velocity and acceleration in cylindrical polar coordinates:

$$\underline{v} = \dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{d}{dt}\left(r\,\underline{e}_r\right) \quad -(70)$$

and:

$$\underline{a} = \frac{d\underline{v}}{dt} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta . \quad -(71)$$

The relativistic angular momentum is:

$$L^* = \gamma m r^2 \dot{\theta} . \quad -(72)$$

Consider for example the static elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(73)$$

for which:

$$\frac{dr}{d\theta} = \frac{\epsilon}{\alpha} r^2 \sin\theta . \quad -(74)$$

Therefore:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{\epsilon L}{\gamma m \alpha}\sin\theta \quad -(75)$$

and: $$\dot{r} = \frac{1}{\gamma}\left(\frac{L^*\epsilon}{m\alpha}\right)\sin\theta, \quad \dot{\theta} = \frac{L^*}{\gamma m r^2}. \quad -(76)$$

It follows that: $$\ddot{r} = \left(\frac{L^*}{\gamma m r}\right)^2 \frac{\epsilon}{\alpha}\cos\theta \quad -(77)$$

and: $$\ddot{\theta} = -2\left(\frac{L^*}{\gamma m}\right)^2 \frac{\epsilon}{\alpha}\frac{1}{r^3}\sin\theta. \quad -(78)$$

Therefore: $$\ddot{r} - r\dot{\theta}^2 = \left(\frac{L^*}{\gamma m r}\right)^2\left(\frac{\epsilon}{\alpha}\cos\theta - \frac{1}{r}\right), \quad -(79)$$
$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.$$

The acceleration associated with the ellipse (73) is therefore:
$$\underline{a} = \left(\frac{L^*}{\gamma m r}\right)^2\left(\frac{\epsilon}{\alpha}\cos\theta - \frac{1}{r}\right)\underline{e}_r \quad -(80)$$

where: $$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(81)$$

so: $$\underline{a} = -\frac{1}{\alpha}\left(\frac{L^*}{\gamma m r}\right)^2\underline{e}_r. \quad -(82)$$

This is the relativistic acceleration due to Eq. (73).

In special relativity the relativistic force is defined {11} by:

$$\underline{F}^* = m \frac{d}{dt}\left(\gamma \underline{v}\right) \quad -(83)$$

so using the Leibnitz Theorem:

$$F^* = m\left(\frac{d\gamma}{dt} v + \gamma \frac{dv}{dt}\right) \quad -(84)$$

where:

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv}\frac{dv}{dt} = \left(\frac{d}{dv}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right)\frac{dv}{dt} = \frac{v}{c^2}\frac{dv}{dt}\gamma^3 . \quad -(85)$$

Therefore:

$$\begin{aligned} F^* &= m\left(\gamma^3 \frac{v^2}{c^2}\frac{dv}{dt} + \gamma \frac{dv}{dt}\right) \\ &= m\gamma\left(\frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-1} + 1\right) \quad -(86) \\ &= m\gamma^3 dv/dt \end{aligned}$$

In special relativity the Newton law is modified to Eq. (86).

From Eqs. (82) and (86):

$$F^* = \gamma F \quad -(87)$$

and this is the same as Eq. (65), Q. E. D.

Therefore the Principle of Equivalence is also modified in special relativity to:

$$F^* = \gamma F = m\gamma^3 dv/dt \quad -(88)$$

from the classical:

$$F = -\frac{mMG}{r^2} = m\frac{dv}{dt} . \quad -(89)$$

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3 Graphical illustration of time dilation

In Figs. 1 and 2 the time dilation according to Eq.(19) is shown for three different x values. The curves are plotted for a strong ellipticity of 0.9. For $\alpha = 1$ then the maximum and minimum radius are

$$r_{min} = \frac{\alpha}{\sqrt{1+\epsilon}} = 0.725, \tag{90}$$

$$r_{max} = \frac{\alpha}{\sqrt{1-\epsilon}} = 3.162. \tag{91}$$

This means that the strong deviations of the time dilation from unity in Fig. 1 do not lie in the range of the orbit. However for Fig. 2 there are extreme effects, and for $x = 3$ the time delay is even not defined in a broad range. According to Eq.(19) the squared time dilation is

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{L^2}{c^2\,\mu^2\,r^2}\left(\frac{x^2}{\alpha^2}\left(\epsilon^2\,r^2 - (\alpha - r)^2\right) + 1\right). \tag{92}$$

Setting this expression to zero delivers two radius values between which the time dilation is not defined because the right hand side of Eq.(92) is negative:

$$r_0 = \frac{\pm\alpha\,L\,\sqrt{\left(\epsilon^2\,x^4 + (1-\epsilon^2)\,x^2\right)L^2 - \alpha^2\,c^2\,\mu^2\,x^2 + \alpha^2\,c^2\,\mu^2} - \alpha\,x^2\,L^2}{(\epsilon^2 - 1)\,x^2\,L^2 - \alpha^2\,c^2\,\mu^2} \tag{93}$$

This effect is further depicted in Figs. 3 and 4. In Fig. 3 the orbits for the three x values with parameters the same as for Fig. 2 are shown. In Fig. 4 the

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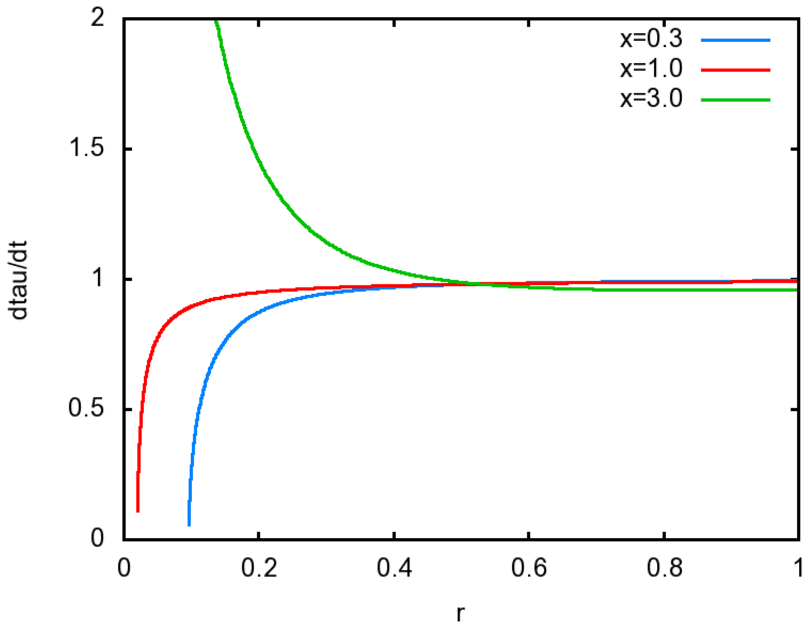

Figure 1: Time dilation $d\tau/dt$ for parameters $L = 1,\ c = 10,\ \mu = 1,\ \alpha = 1,\ \epsilon = .9$.

orbit for $x = 3$ with the two radius values of Eq.(93) is graphed. This means that the range between the red and blue circle is forbidden. If the particle is moving on the outer part of the orbit, it approaches the outer limiting radius asymptotically, since $d\tau/dt$ goes to zero here. This can be seen from Fig. 2, green curve. The limiting radius effectively defindes an event horizon. Thus this exotic spacetime behaviour could - at least theoretically - revive in the orbital dynamics of X Theory.

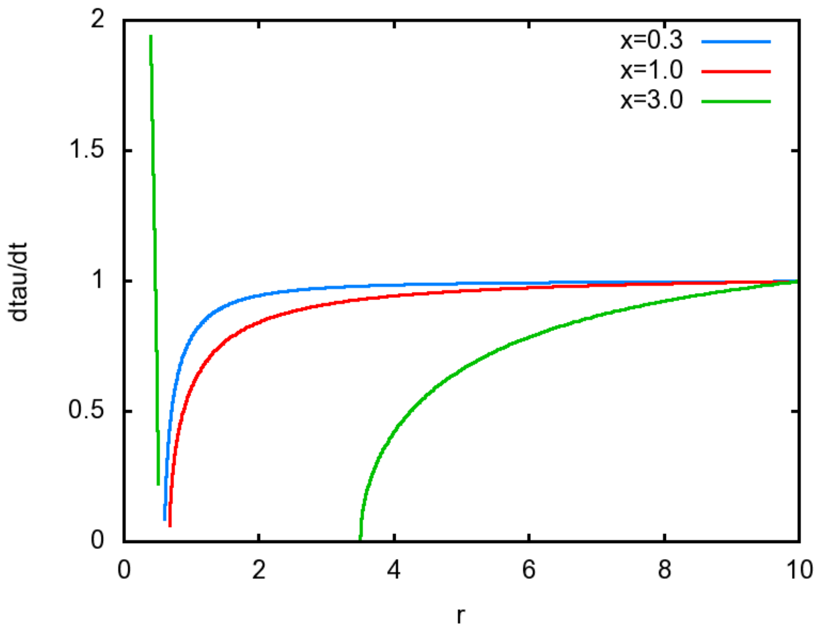

Figure 2: Time dilation $d\tau/dt$ for parameters $L = 3,\ c = 5,\ \mu = 1,\ \alpha = 1,\ \epsilon = .9$.

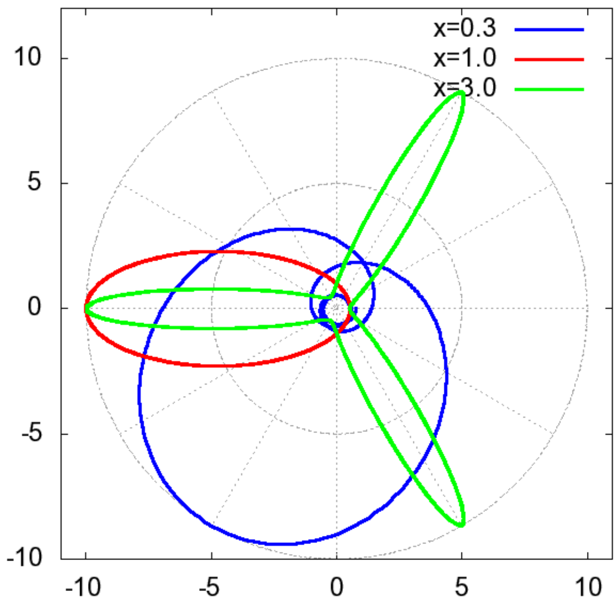

Figure 3: Orbits for parameters $\alpha = 1,\ \epsilon = 0.9$.

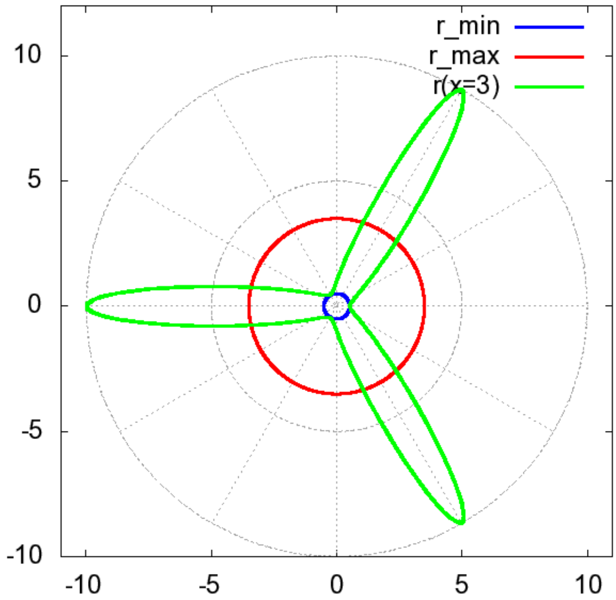

Figure 4: Orbit for $x = 3$ of Fig. 3 with minimum and maximum radius indicated.

SECTION 3: GRAPHICAL ILLUSTRATION OF TIME DILATATION.

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