THE CARTAN GEOMETRY OF THE PLANE POLAR COORDINATES: ROTATIONAL DYNAMICS IN TERMS OF THE CARTAN SPIN CONNECTION.

by

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ABSTRACT

Cartan geometry is applied to the plane polar coordinates to calculate the tetrad and spin connection elements from first principles of geometry. It is shown that the Cartan torsion is non zero for the plane polar coordinates, thus refuting Einsteinian general relativity. The latter assumes incorrectly that the torsion is zero. Simple calculations based on the plane polar coordinates show that the Cartan torsion is a special case of a more generally defined torsion, a special case in which the connections are equal and opposite in sign. These new mathematical techniques are applied to rotational dynamics, and it is shown that the angular velocity is a Cartan spin connection.

Keywords: ECE theory, Cartan geometry of the plane polar coordinates, Cartan geometry of rotational dynamics.

UFT235

1. INTRODUCTION

In this series of papers and books {1 - 10} the ECE generally covariant unified field theory has been developed on the basis of Cartan's well known geometry {11} in which the two structure equations are used to define torsion and curvature. It has been shown that Einsteinian general relativity (EGR) is incorrect because of its neglect of one of the fundamentals of geometry, the Cartan torsion. In Section 2 the Cartan torsion is calculated with the plane polar coordinates, and shown to be non-zero. This simple exercise refutes EGR because in any geometry, the Cartan torsion is in general non-zero. It was shown in the first papers of ECE theory that the Cartan tetrads can be defined by using any two coordinate systems in any mathematical space in any dimension. The original concept by Cartan {1 - 11} used a tangent spacetime at point P to a base manifold. The tangent spacetime in Cartan geometry is a Minkowski spacetime if four dimensional theory is being used. Different types of tangent spacetime can be used. By superimposing one coordinate system on another in the same mathematical space, tetrads can be defined the most simply. This is done in Section 2 by using points in the plane polar coordinate system and points in the Cartesian system. The analysis is reduced to the simplest possible level by considering a plane. A vector can be represented by the plane polar coordinates {12 - 14}. The tetrad elements are the Cartesian components of the vector in plane polar representation. The Cartan spin connection is defined by the fact that the axes of the plane polar system rotate with respect to the fixed axes of the Cartesian system. Having defined the tetrad and spin connection components the first and second Cartan structure equations are used to calculate the Cartan torsion and curvature of the plane polar coordinates in two dimensions. The most important result is obtained that the torsion is not zero. If the Cartan torsion is not zero on the simplest possible level, then it is not zero in any geometry. This spells disaster for standard physics because EGR is based on zero torsion.

This technique is used in Section 3 to show that the angular velocity is a Cartan spin connection. The latter is therefore fundamental to all the familiar concepts of rotational dynamics. As usual the notes that accompany this paper on www.aias.us give a lot of detail of the calculations, and should be read in conjunction with this paper, UFT235.

2. CALCULATION OF THE CARTAN TORSION

Consider the well known {12 -14} unit vectors of the plane polar coordinates:

$$\underline{e}_r = \cos\theta\, \underline{i} + \sin\theta\, \underline{j} \quad -(1)$$

$$\underline{e}_\theta = -\sin\theta\, \underline{i} + \cos\theta\, \underline{j}\,. \quad -(2)$$

The unit vectors depend on time {12} and rotate. The unit vectors $\underline{i}$ and $\underline{j}$ of the Cartesian system do not depend on time and are fixed or static. The four elements of the Cartan tetrad $q^a_{\ \mu}$ are defined by:

$$\begin{bmatrix} \underline{e}_r \\ \underline{e}_\theta \end{bmatrix} = \begin{bmatrix} q^{(1)}_{\ 1} & q^{(1)}_{\ 2} \\ q^{(2)}_{\ 1} & q^{(2)}_{\ 2} \end{bmatrix} \begin{bmatrix} \underline{i} \\ \underline{j} \end{bmatrix}. \quad -(3)$$

This is an example of the general definition {11}:

$$V^a = q^a_{\ \mu} V^\mu. \quad -(4)$$

The four tetrad components are:

$$q^{(1)}_{\ 1} = \cos\theta,\ q^{(1)}_{\ 2} = \sin\theta,$$
$$q^{(2)}_{\ 1} = -\sin\theta,\ q^{(2)}_{\ 2} = \cos\theta, \quad -(5)$$

and the tetrad matrix is:

$$q^a_{\ \mu} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \quad -(6)$$

Note carefully that this is also the rotation matrix about Z:

$$\begin{bmatrix} V_X' \\ V_Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_X \\ V_Y \end{bmatrix} \quad -(7)$$

for any vector $\underline{V}$. It follows from Eqs. (4) and (7) that:

$$\begin{bmatrix} V^{(1)} \\ V^{(2)} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} \quad -(8)$$

where:

$$V^{(1)} = V_X', \quad V^{(2)} = V_Y', \quad -(9)$$
$$V^1 = V_X, \quad V^2 = V_Y.$$

To prove this result note that:

$$\underline{V} = V_X \underline{i} + V_Y \underline{j} = V^{(1)} \underline{e}_r + V^{(2)} \underline{e}_\theta$$
$$= V^{(1)}(\cos\theta\, \underline{i} + \sin\theta\, \underline{j}) + V^{(2)}(-\sin\theta\, \underline{i} + \cos\theta\, \underline{j}) = V^1 \underline{i} + V^2 \underline{j} \quad -(10)$$

so:

$$V^1 = V^{(1)}\cos\theta - V^{(2)}\sin\theta \quad -(11)$$
$$V^2 = V^{(1)}\sin\theta + V^{(2)}\cos\theta. \quad -(12)$$

Multiply Eq. (11) by $\cos\theta$ and Eq. (12) by $-\sin\theta$. It follows that:

$$V^{(1)} = V^1\cos\theta + V^2\sin\theta, \quad -(13)$$
$$V^{(2)} = -V^1\sin\theta + V^2\cos\theta \quad -(14)$$

which is Eq. (8), QED.

It has been proven that a rotation in the plane XY about Z defines the Cartan tetrad matrix and four elements of the Cartan tetrad.

Define the metric in the Cartesian system by $g_{\mu\nu}$ and the metric in the plane polar system by η_{ab}. By definition {11} the two metrics are related by:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad -(15)$$

The metrics are related to the infinitesimal line element by:

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu, \quad -(16)$$

$$ds^2 = \eta_{ab}\, dx^a dx^b, \quad -(17)$$

so {12 -14}:

$$ds^2 = dX^2 + dY^2 = dr^2 + r^2 d\theta^2, \quad -(18)$$

$$ds^2 = g_{11}\, dx^1 dx^1 + g_{22}\, dx^2 dx^2. \quad -(19)$$

In Cartesian coordinates:

$$dx^1 = dX, \; dx^2 = dY, \; g_{11} = g_{22} = 1, \quad -(20)$$

and in plane polar coordinates:

$$dx^1 = dr, \; dx^2 = r\,d\theta, \; g_{11} = g_{22} = 1. \quad -(21).$$

Eq. (15) means:

$$g_{11} = q^{(1)}_{\;1} q^{(1)}_{\;1} \eta_{(1)(1)} + q^{(2)}_{\;1} q^{(2)}_{\;1} \eta_{(2)(2)}, \quad -(22)$$

$$g_{22} = q^{(1)}_{\;2} q^{(1)}_{\;2} \eta_{(1)(1)} + q^{(2)}_{\;2} q^{(2)}_{\;2} \eta_{(2)(2)}. \quad -(23)$$

From Eqs. (5), (20) and (21), Eqs. (22) and (23) are correct and self sufficient. Eqs. (22) and (23) both give:

$$\cos^2\theta + \sin^2\theta = 1. \quad -(24)$$

From rotation generator theory {14}, if:

$$R_Z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad -(25)$$

then the operator known as the rotation generator is defined as:

$$J_Z = \frac{1}{i} \left. \frac{dR_Z}{d\theta} \right|_{\theta=0} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \quad -(26)$$

In three dimensions:

$$J_Z = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad J_Y = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix} \quad -(27)$$

and:

$$[J_X, J_Y] = i J_Z. \quad -(28)$$

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It follows that {14}:

$$R_Z(\theta) = \exp(i J_Z \theta). \quad -(29)$$

To prove this consider the Maclaurin series:

$$\exp(i J_Z \theta) = 1 + i J_Z \theta - \frac{1}{2!} J_Z^2 \theta^2 + \ldots$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \theta \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\theta^2}{2!} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad -(30)$$

Therefore the tetrad matrix is:

$$q^a_\mu = \exp(i J_Z \theta) \quad -(31)$$

where both sides are matrices in this notation.

The general structure of Eq. (26) is that of the derivative of a tetrad, because the rotation matrix is a tetrad matrix. The tetrad postulate of Cartan geometry asserts {11} that the covariant derivative of the tetrad is zero:

$$D_\nu q^a_\mu = \partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu - \Gamma^\lambda_{\nu\mu} q^a_\lambda = 0 \quad -(32)$$

This equation can be expressed as:

$$\partial_\nu q^a_\mu = \zeta^a_{\nu\mu} := \Gamma^a_{\nu\mu} - \omega^a_{\nu\mu} \quad -(33)$$

and is a generalization of Eq. (26). Therefore the rotation generator is a special case of the zeta connection defined by:

$$\zeta^a_{\nu\mu} := \Gamma^a_{\nu\mu} - \omega^a_{\nu\mu} . \quad -(34)$$

The Cartan torsion associated with this process is defined to be {1 - 11}

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu = \Gamma^a_{\mu\nu} - \Gamma^a_{\nu\mu} . \quad -(35)$$

Any orbit in a plane is generated by a connection and torsion of geometry.

This is a new understanding of all orbits in a plane and originates in the fact that for any such orbit:

$$dr / d\theta \neq 0 . \quad -(36)$$

It follows that:

$$\frac{d\cos\theta}{dr} = -\frac{d\theta}{dr}\sin\theta , \quad -(37)$$

$$\frac{d\sin\theta}{dr} = \frac{d\theta}{dr}\cos\theta , \quad -(38)$$

so from Eq. (6):

$$\frac{dq^a_\mu}{dr} = \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \frac{d\theta}{dr} . \quad -(39)$$

From Eq. (33) the zeta connection matrix of any planar orbit is:

$$\zeta^{a}{}_{1\mu} = \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \frac{d\theta}{dr} \quad -(40)$$

Any planar orbit is proportional to the zeta connection, which is a rotation generator proportional to angular momentum and related to spacetime torsion. For example, for the elliptical orbit {1 - 11}:

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha} \sin\theta \quad -(41)$$

and the zeta connection elements are:

$$\zeta^{(1)}{}_{11} = \zeta^{(2)}{}_{12} = -\frac{\alpha}{\epsilon r^2}, \quad -(42)$$
$$\zeta^{(1)}{}_{12} = -\zeta^{(2)}{}_{11} = -\frac{\alpha}{\epsilon r^2} \operatorname{cotan}\theta .$$

The unit vectors of the plane polar system rotate and the Cartan spin connection defines the rotation. Denote the basis vectors by:

$$e_r = e^{(1)}, \quad e_\theta = e^{(2)} . \quad -(43)$$

By definition {11} the covariant derivative is defined by

$$D_\mu e^{(a)} = \partial_\mu e^{(a)} + \omega^{(a)}{}_{\mu(b)} e^{(b)} . \quad -(44)$$

However the ordinary four derivative vanishes because it is defined with a static frame of reference. The spin connection is defined to describe the rotating frame. For example:

$$\frac{De^{(a)}}{dt} = \frac{de^{(a)}}{dt} + \omega^{(a)}{}_{0(b)} e^{(b)} \quad -(45)$$

and

$$\frac{de^{(a)}}{dt} = 0 \quad -(46)$$

Therefore:

$$\frac{De^{(a)}}{dt} = \omega^{(a)}{}_{(b)}\, e^{(b)} \qquad (47)$$

In vector notation:

$$\frac{D\underline{e}_r}{dt} = \omega^{(1)}{}_{(2)}\, \underline{e}_\theta . \qquad (48)$$

This result is always denoted {12 - 14} by

$$\frac{d\underline{e}_r}{dt} = \omega \underline{e}_\theta = \frac{d\theta}{dt}\underline{e}_\theta \qquad (49)$$

but rigorously it should be:

$$\frac{D\underline{e}_r(t)}{dt} = \left(\frac{d\underline{e}_r}{dt}\right)_{static} + \omega \underline{e}_\theta(t) . \qquad (50)$$

It follows that

$$\omega^{(1)}{}_{(2)} = \omega \qquad (51)$$

a result of basic importance.

With the definitions {12- 14}

$$\underline{\omega} = \omega \underline{k}, \qquad (52)$$

$$\underline{k} = \underline{e}_r \times \underline{e}_\theta, \qquad (53)$$

$$\underline{e}_\theta = \underline{k} \times \underline{e}_r, \qquad (54)$$

Eq. (50) can be expressed as:

$$\frac{D\underline{e}_r(t)}{dt} = \left(\frac{d\underline{e}_r}{dt}\right)_{static} + \underline{\omega} \times \underline{e}_r, \qquad (55)$$

a result of basic importance to rotational dynamics (Section 3). Similarly the basic kinematic result {12}:

$$\frac{d\underline{e}_\theta}{dt} = -\omega \underline{e}_r \quad -(56)$$

can be expressed as:

$$\frac{D e^{(2)}}{dt} = \omega^{(2)}{}_{0(1)} e^{(1)} . \quad -(57)$$

So the two spin connection elements of this type are:

$$\omega^{(1)}{}_{0(2)} = -\omega^{(2)}{}_{0(1)} = \omega . \quad -(58)$$

All the information needed to calculate the elements of the Cartan torsion is now available. For example:

$$T^{(1)}{}_{01} = \partial_0 q^{(1)}{}_1 - \partial_1 q^{(1)}{}_0 + \omega^{(1)}{}_{0(b)} q^{(b)}{}_1 - \omega^{(1)}{}_{1(b)} q^{(b)}{}_0 \quad -(59)$$

with summation over repeated indices (b). By definition:

$$q^{(0)}{}_0 = 1, \; q^{(1)}{}_0 = q^{(2)}{}_0 = 0, \quad -(60)$$

so the result reduces to:

$$T^{(1)}{}_{01} = \partial_0 q^{(1)}{}_1 + \omega^{(1)}{}_{0(2)} q^{(2)}{}_1 . \quad -(61)$$

Using the relevant tetrad and spin connection elements gives the result:

$$T^{(1)}{}_{01} = -2\omega \sin\theta = -2\frac{d\theta}{dt}\sin\theta . \quad -(62)$$

Proceeding similarly gives the torsion matrix:

$$T^{a}_{\mu\nu} = \begin{bmatrix} T^{(1)}_{01} & T^{(1)}_{02} \\ T^{(2)}_{01} & T^{(2)}_{02} \end{bmatrix} = 2\omega \begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix} \quad -(63)$$

which can be expressed as:

$$T^{a}_{\mu\nu} = 2\frac{d}{d\theta} q^{a}_{\mu} = 2\omega \frac{d}{d\theta} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad -(64)$$

Therefore:

$$\frac{1}{i} \begin{bmatrix} T^{(1)}_{01} & T^{(1)}_{02} \\ T^{(2)}_{01} & T^{(2)}_{02} \end{bmatrix}_{\theta = 0} = 2\omega \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = 2\omega J_Z \quad -(65)$$

where the infinitesimal rotation generator about Z is defined {14} as:

$$J_Z = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \quad -(66)$$

The final result is that the torsion matrix is proportional to the infinitesimal rotation generator:

$$\begin{bmatrix} T^{(1)}_{01} & T^{(1)}_{02} \\ T^{(2)}_{01} & T^{(2)}_{02} \end{bmatrix}_{\theta = 0} = 2i\omega J_Z. \quad -(67)$$

From the tetrad postulate (32):

$$\Gamma^{(a)}_{\mu\nu} = \partial_\mu q^{(a)}_{\nu} + \omega^{(a)}_{\mu(b)} q^{(b)}_{\nu} \quad -(68)$$

so there exist mixed index connections such as:

$$\Gamma^{(1)}_{01} = \partial_0 q^{(1)}_{1} + \omega^{(1)}_{0(2)} q^{(2)}_{1} = -2\omega\sin\theta. \quad -(69)$$

From Eq. (59):

$$\Gamma^{(1)}_{01} = -2\omega \sin\theta, \quad \Gamma^{(1)}_{10} = 0, \quad -(70)$$

so these mixed index connections are neither symmetric nor antisymmetric for the plane polar coordinates, for which the Cartan torsion is in general non zero. This result is alone enough to refute any theory of relativity based on zero torsion, notably EGR. The mixed index connection of type (68) can always be defined as the sum of symmetric (S) and antisymmetric (A) components by a basic theorem of matrices {15}. The Cartan torsion element (61) is defined {1 - 11} to be:

$$T^{(1)}_{01} = \Gamma^{(1)}_{01}(A) - \Gamma^{(1)}_{10}(A) \quad -(71)$$

and is always twice the antisymmetric connection $\Gamma^{(1)}_{01}(A)$ So:

$$T^{(1)}_{01} = 2\Gamma^{(1)}_{01}(A). \quad -(72)$$

More generally {11} any difference of connections such as:

$$T^{a}_{\mu\nu} = \Gamma^{a}_{\mu\nu} - \Lambda^{a}_{\nu\mu} \quad -(73)$$

is a torsion tensor. Clearly, the Cartan torsion (71) is a special case of the general definition in Eq. (73) {11}.

These elements of the torsion may be given the appellation "orbital torsion" {1 - 10} in analogy with orbital angular momentum. There also exist "spin torsion" elements such as:

$$T^{(1)}_{12} = \partial_1 q^{(1)}_2 - \partial_2 q^{(1)}_1 + \omega^{(1)}_{1(b)} q^{(b)}_2 - \omega^{(1)}_{2(b)} q^{(b)}_1. \quad -(74)$$

The evaluation of the spin torsion for the plane polar coordinates requires the evaluation of different spin connections from those used in orbital torsion. From the basic definitions of the unit vectors of the plane polar coordinates:

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta, \quad -(75)$$

$$\underline{e}_\theta = -\underline{i}\sin\theta + \underline{j}\cos\theta, \quad -(76)$$

it is seen that:

$$\frac{d\underline{e}_r}{dr} = \frac{d\theta}{dr}\underline{e}_\theta, \quad \frac{d\underline{e}_r}{d\theta} = \underline{e}_\theta. \quad -(77)$$

The infinitesimal line element is defined by:

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = g_{11}\, dx^1 dx^1 + g_{22}\, dx^2 dx^2 \quad -(78)$$
$$= dr^2 + r^2 d\theta^2$$

so

$$x^1 = r, \; x^2 = r\theta, \; \partial_1 = \frac{\partial}{\partial r}, \; \partial_2 = \frac{\partial}{\partial(r\theta)}. \quad -(79)$$

Defining:

$$y = r\theta \quad -(80)$$

then:

$$\frac{df}{dy} = \frac{df}{d\theta}\frac{d\theta}{dr}, \quad \frac{d\underline{e}_r}{d(r\theta)} = \frac{d\theta}{dr}\underline{e}_\theta. \quad -(81)$$

Eq. (77) may be written as:

$$\frac{de^{(1)}}{dx^1} = \omega^{(1)}{}_{1(2)}\, e^{(2)}, \quad \text{i.e.} \quad \frac{d\underline{e}_r}{dr} = \frac{d\theta}{dr}\underline{e}_\theta, \quad -(82)$$

so:

$$\omega^{(1)}{}_{1(2)} = \frac{d\theta}{dr}. \quad -(83)$$

Eq. (81) may be written as:

$$\frac{de^{(1)}}{dx^2} = \omega^{(1)}{}_{2(2)}\, e^{(2)} \quad -(84)$$

so

$$\omega^{(1)}{}_{1(2)} = \frac{d\theta}{dr}. \quad -(85)$$

Therefore the relevant spin connection components are

$$\omega^{(1)}{}_{1(2)} = \omega^{(1)}{}_{2(2)} = \frac{d\theta}{dr}. \quad -(86)$$

It follows that:

$$T^{(1)}{}_{12} = \frac{d\sin\theta}{dr} - \frac{d\cos\theta}{d(r\theta)} + \omega^{(1)}{}_{1(1)}\, q^{(1)}{}_{2} + \omega^{(1)}{}_{1(2)}\, q^{(2)}{}_{2} - \omega^{(1)}{}_{2(1)}\, q^{(1)}{}_{1} - \omega^{(1)}{}_{2(2)}\, q^{(2)}{}_{1} \quad -(87)$$

in which:

$$\omega^{(1)}{}_{1(1)} = \omega^{(1)}{}_{2(1)} = 0. \quad -(88)$$

So:

$$T^{(1)}{}_{12} = 2\frac{d\theta}{dr}\left(\cos\theta + \sin\theta\right) \quad -(89)$$

Similarly:

$$T^{(1)}{}_{21} = -2\frac{d\theta}{dr}\left(\cos\theta + \sin\theta\right) \quad -(90)$$

so the spin torsion element is antisymmetric:

$$T^{(1)}{}_{12} = -T^{(1)}{}_{21} \quad -(91)$$

as it must be by definition {1 - 11}, QED.

In order to evaluate the spin torsion element:

$$T^{(2)}{}_{12} = -T^{(2)}{}_{21} \quad -(92)$$

new spin connection elements have to be defined. From Eqs. (75) and (76):

$$\frac{d\underline{e}_\theta}{dr} = -\frac{d\theta}{dr}\,\underline{e}_r \quad -(93)$$

and:

$$\frac{d\underline{e}_\theta}{d(\theta r)} = -\underline{e}_r\left(\frac{d\theta}{dr}\right) \quad -(94)$$

Eq. (93) is:

$$\frac{de^{(2)}}{dx^1} = \omega^{(2)}{}_{1(1)}\, e^{(1)} \quad -(95)$$

and Eq. (94) is:

$$\frac{de^{(2)}}{dx^2} = \omega^{(2)}{}_{2(1)}\, e^{(1)} \quad -(96)$$

so the required elements of the spin connection are:

$$\omega^{(2)}{}_{1(1)} = \omega^{(2)}{}_{2(1)} = -\frac{d\theta}{dr} \quad -(97)$$

Therefore:

$$T^{(2)}{}_{12} = -T^{(2)}{}_{21} = 2\frac{d\theta}{dr}\left(\cos\theta - \sin\theta\right) \quad -(98)$$

The relevant mixed index connections are:

$$\Gamma^{(1)}{}_{12} = 2\frac{d\theta}{dr}\cos\theta = \Gamma^{(2)}{}_{12}, \quad -(99)$$

$$\Gamma^{(1)}{}_{21} = -2\frac{d\theta}{dr}\sin\theta = -\Gamma^{(2)}{}_{21}, \quad -(100)$$

giving the matrix:

$$\begin{bmatrix} \Gamma^{(1)}_{12} & \Gamma^{(1)}_{21} \\ \Gamma^{(2)}_{12} & \Gamma^{(2)}_{21} \end{bmatrix} = 2\frac{d\theta}{dr}\begin{bmatrix} \cos\theta & -\sin\theta \\ \cos\theta & \sin\theta \end{bmatrix}. \quad -(101)$$

Therefore the spin torsion elements of the plane polar coordinates are non -zero, a result of basic importance that again refutes EGR.

In general the mixed index connections of type (101) are neither symmetric nor antisymmetric. Using the definition (71) the relevant antisymmetric connections are:

$$\Gamma^{(1)}_{12}(A) = 2\frac{d\theta}{dr}\left(\cos\theta + \sin\theta\right), \quad -(102)$$

$$\Gamma^{(2)}_{12}(A) = 2\frac{d\theta}{dr}\left(\cos\theta - \sin\theta\right). \quad -(103)$$

In vector format, using

$$T^{(1)}_{3} = \epsilon_{123}\, T^{(1)}_{12} \quad -(104)$$

the spin torsion is directly proportional to the vector angular velocity:

$$\underline{T}^{(1)} = 2\left(\cos\theta + \sin\theta\right)\frac{d\theta}{dr}\,\underline{k}. \quad -(105)$$

In a relativistic context the units of torsion are inverse metres, but in this non relativistic context the orbital torsion is defined to have the units of inverse seconds.

3. SPIN CONNECTION AND ANGULAR VELOCITY IN ROTATIONAL DYNAMICS

The result (55) of Section 2 for unit vectors is true for any vector $\underline{V}$, because any vector can be expressed in terms of unit vectors. Therefore:

$$\frac{D\underline{V}}{dt} = \frac{d\underline{V}}{dt} + \underline{\omega} \times \underline{V}. \quad -(106)$$

This is the familiar result of classical rotational dynamics {12}. In plane polar coordinates {12} the position vector r for example is:

$$\underline{r} = r\,\underline{e}_r \quad -(107)$$

so:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt} = \underline{v}_S + \omega r \underline{e}_\theta \quad -(108)$$

where $\underline{v}$ is the total velocity. It is the sum of the velocity in an inertial frame (frame with static coordinates):

$$\underline{v}_S = \frac{dr}{dt}\underline{e}_r \quad -(109)$$

and the orbital linear velocity {12}:

$$\underline{v}_o = \omega r \underline{e}_\theta = \underline{\omega} \times \underline{r} \quad -(110)$$

In component format:

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} = \dot{r}\underline{e}_r + r\dot{\underline{e}}_r \quad -(111)$$

We arrive at the important result that the orbital linear velocity is the result of the Cartan spin connection in plane polar coordinates.

From Eq. (111) the acceleration is defined by:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt}\right)$$
$$= \frac{d^2 r}{dt^2}\underline{e}_r + \left(2\frac{dr}{dt}\frac{d\underline{e}_r}{dt} + r\frac{d^2\underline{e}_r}{dt^2}\right) \quad -(112)$$

The term in brackets is the acceleration due to the rotation of the frame of reference itself, so is due to the Cartan spin connection. From fundamentals {12}:

$$d\underline{e}_r / dt = \frac{d\theta}{dt}\underline{e}_\theta = \omega \underline{e}_\theta \quad -(113)$$

so:

$$\underline{a} = \frac{d^2 r}{dt^2}\,\underline{e}_r + \left(2\frac{dr}{dt}\,\omega\,\underline{e}_\theta + r\frac{d}{dt}\left(\omega\,\underline{e}_\theta\right)\right). \quad -(114)$$

In this equation:

$$\omega\,\underline{e}_\theta = \underline{\omega} \times \underline{e}_r, \quad \frac{d}{dt}\left(\omega\,\underline{e}_\theta\right) = \frac{d\omega}{dt}\,\underline{e}_\theta + \omega\frac{d\underline{e}_\theta}{dt} \quad -(115)$$

and the inertial or Newtonian velocity is:

$$\underline{v}_N = \frac{dr}{dt}\,\underline{e}_r. \quad -(116)$$

Therefore:

$$\underline{a} = \frac{d\underline{v}_N}{dt} + 2\underline{\omega} \times \underline{v}_N + \omega r\frac{d\underline{e}_\theta}{dt} + \frac{d\omega}{dt}\,\underline{e}_\theta. \quad -(117)$$

Using:

$$\frac{d\underline{e}_\theta}{dt} = -\omega\,\underline{e}_r \quad -(118)$$

the acceleration is:

$$\underline{a} = \frac{d\underline{v}_N}{dt} + 2\underline{\omega} \times \underline{v}_N + \frac{d\omega}{dt}\,\underline{e}_\theta - \omega^2 r\,\underline{e}_r \quad -(119)$$

in which

$$\underline{\omega} \times \underline{r} = \omega\,\underline{k} \times r\,\underline{e}_r = \omega r\,\underline{e}_\theta \quad -(120)$$

and:

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \omega^2 r\,\underline{k} \times \underline{e}_\theta = -\omega^2 r\,\underline{e}_r. \quad -(121)$$

The complete acceleration is therefore:

$$\underline{a} = \frac{d^2 r}{dt^2}\underline{e}_r + 2\underline{\omega}\times\underline{v}_N + \frac{d\underline{\omega}}{dt}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(122)$$

and in component format is:

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta . \quad -(123)$$

The Coriolis acceleration is:

$$\underline{a}_{Cor} = 2\underline{\omega}\times\underline{v}_N + \frac{d\underline{\omega}}{dt}\times\underline{r} = (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta , \quad -(124)$$

and the centrifugal acceleration is:

$$\underline{a}_{cent} = \underline{\omega}\times(\underline{\omega}\times\underline{r}) = -r\dot{\theta}^2\underline{e}_r . \quad -(125)$$

The inertial or Newtonian acceleration is:

$$\underline{a}_{Newton} = \frac{d^2 r}{dt^2}\underline{e}_r . \quad -(126)$$

We arrive at the important result that all these well known accelerations are due to the Cartan spin connection, which in vector format is the angular velocity $\underline{\omega}$.

In previous work {1 - 10} it was found that the Coriolis acceleration vanishes for all planar orbits:

$$\underline{a}_{Cor} = 2\underline{\omega}\times\underline{v}_N + \frac{d\underline{\omega}}{dt}\times\underline{r} = \underline{0} \quad -(127)$$

so the total acceleration for all planar orbits is:

$$\underline{a}\,(\text{orbit}) = \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}), \quad -(128)$$

and is the sum of the inertial and centrifugal accelerations. From previous work it was found that the inertial acceleration of the elliptical orbit is:

$$\underline{a} = \frac{d^2 r}{dt^2}\underline{e}_r = \left(\frac{L}{mr}\right)^2\left(\frac{1}{r} - \frac{1}{d}\right) \quad -(129)$$

where L is the conserved total angular momentum:

$$L = mr^2\omega. \quad -(130)$$

The elliptical orbit is defined by:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(131)$$

where α is the half right latitude and ϵ the eccentricity. For the circular orbit:

$$r = \alpha \quad -(132)$$

so the inertial acceleration of the circular orbit vanishes:

$$\underline{a} = \frac{d^2r}{dt^2}\,\underline{e}_r = \underline{0}. \quad -(133)$$

The inertial force is defined to be:

$$\underline{F}_N = m\frac{d^2r}{dt^2}\,\underline{e}_r \quad -(134)$$

and this is a general result valid for all planar orbits.

In the particular case of Newtonian dynamics {12}:

$$\underline{F}_N = \left(\frac{L^2}{mr^3} - \frac{L^2}{mr^2\alpha}\right)\underline{e}_r, \quad \alpha = \frac{L^2}{m^2MG} \quad -(135)$$

so the inertial force is:

$$\underline{F}_N = \left(\frac{L^2}{mr^3} - \frac{mMG}{r^2}\right)\underline{e}_r. \quad -(136)$$

In the received opinion {12} this result is interpreted as the force of attraction of the inverse square law:

$$\underline{F}_{att.} = -\frac{mMG}{r^2}\,\underline{e}_r \quad -(137)$$

added to a "pseudo-force" defined by:

$$\underline{F}_{pseudo} = -\frac{\partial U_c}{\partial r}\,\underline{e}_r = \frac{L^2}{mr^3}\,\underline{e}_r \,. \quad -(138)$$

This pseudo force is defined incorrectly as originating in an effective potential. However, the complete acceleration is:

$$\underline{a} = \left(\frac{L^2}{m^2 r^3} - \frac{L^2}{m^2 r^2 d}\right)\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(139)$$

where

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r\,\underline{e}_r = -\frac{L^2}{m^2 r^3}\,\underline{e}_r \quad -(140)$$

so the correct sum of accelerations in Eq. (139) consists of only one term:

$$\underline{a}\,(\text{sum}) = -\frac{L^2}{m^2 r^2 d}\,\underline{e}_r \quad -(141)$$

and this is a rigorously correct result that originates in the basic definition of acceleration. Using Eq. (130) for the angular momentum it is found that the total acceleration associated with the elliptical orbit is:

$$\underline{a} = \frac{r}{d}\,\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\frac{r^2 \omega^2}{d}\,\underline{e}_r \quad -(142)$$

and is due entirely to the spin connection, i.e. to the rotation of the axes and of space itself.

We arrive at the important conclusion that every planar orbit is due to the movement of space itself.

For a circular orbit:

$$\underline{a} = \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r \underline{e}_r \quad -(143)$$

so:

$$\frac{d^2 \underline{r}}{dt^2} = -\omega^2 \underline{r}. \quad -(144)$$

A solution of this equation is:

$$\underline{r} = \underline{r}(0) \exp(i\omega t) \quad -(145)$$

the real part of which is: $$\mathrm{Real}(\underline{r}) = \underline{r}(0) \cos \omega t \quad -(146)$$

so the vector $\underline{r}$ rotates in a circle with angular velocity ω, which is also the magnitude of the spin connection of Cartan. For an elliptical orbit:

$$\frac{d^2 \underline{r}}{dt^2} = -\left(\frac{r}{\alpha}\right) \omega^2 \underline{r}. \quad -(147)$$

We arrive at the important conclusion that the Newtonian interpretation is untenable because the correct total acceleration (141) is interpreted as a force of attraction. It is seen from Eq. (139) for example that the Newtonian procedure adds and subtracts a term:

$$\frac{L^2}{m^2 r^3} \underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{0} \quad -(148)$$

from the correct result (143). The Newtonian theory was devised for the elliptical orbit, in which case it gives the result (147). This is not a force of attraction, it is the equation of a rotating vector, Eq. (142). The planar orbit is not due to a balance of attraction and a pseudo force, it is due entirely to the motion of spacetime itself and to the Cartan spin connection. The total acceleration of the elliptical orbit is always:

$$\mathbf{a} = \left(\frac{L}{mr}\right)^2 \left(\frac{1}{r} - \frac{1}{\alpha}\right) \mathbf{e}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad -(149)$$

which is a special case of the general result for any planar orbit:

$$\mathbf{a} = \frac{d^2 r}{dt^2} \mathbf{e}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}). \quad -(150)$$

When the angular velocity vanishes, the acceleration reduces to the inertial:

$$\mathbf{a} = \frac{d^2 r}{dt^2} \mathbf{e}_r = -\frac{MG}{r^2} \mathbf{e}_r \quad -(151)$$

in which case an object of mass m is attracted to another object of mass M without movement of the axes of the frame of reference.

The traditional Newtonian viewpoint of an orbit is:

$$E = \frac{1}{2} m v^2 - mMG, \quad -(152)$$

$$V(r) = -\frac{mMG}{r} + \frac{L^2}{2mr^2}, \quad -(153)$$

$$\mathbf{F} = \left(-\frac{mMG}{r^2} + \frac{L^2}{m^2 r^3}\right) \mathbf{e}_r, \quad -(154)$$

and originates in the inertial term. It completely omits the spin connection. In the Newtonian viewpoint the half right latitude is chosen to give an ellipse provided that

$$\alpha = \frac{L^2}{m^2 MG}. \quad -(155)$$

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