CALCULATION OF THE PERIHELION PRECESSION WITH THE MINKOWSKI FORCE EQUATION.

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ABSTRACT

Planetary perihelion precession is calculated with the Minkowski force equation, a limit of the ECE theory. For elliptical orbits it is shown that it reduces to the Thomas precession. The Minkowski / Thomas precession differs greatly from the more well known planetary precession due to Einsteinian general relativity (EGR). For the outer planets EGR precession does not reduce to Minkowski precession as claimed by protagonists of the standard model. It is shown that the calculation of planetary precession in EGR is deeply flawed and self inconsistent.

Keywords: ECE theory, Minkowski equation, perihelion precession.

UFT 239

1. INTRODUCTION

It is well known that the orbits of planets in the solar system are precessing ellipses, ellipses whose perihelion moves. In recent work {1 - 10} the discovery has been made of fractal conical sections, which are generated by varying the precession constant x. In Section 2 the precession is calculated straightforwardly using the Lorentz factor of relativity. This factor is the essential element in the transformation of the Newton to Minkowski force equation. In the immediately preceding papers of this series the Minkowski equation has been developed for planar orbits of all kinds. It is found that the Minkowski precession reduces to the Thomas precession for elliptical orbits. The Minkowski / Thomas (MT) perihelion precession differs greatly from the well known but erroneous calculation of precession in Einsteinian general relativity (EGR). The EGR precession becomes orders of magnitude smaller than the MT precession for the outer planets, so that EGR precession does not reduce to MT precession. This result shows that general relativity does not reduce to special relativity. The EGR calculation of planetary precession is obviously incorrect because it is applied only to what is known in EGR theory as the anomalous precession. For the outer planets this is several orders of magnitude smaller than the experimentally observed precession. The procedure in standard physics is to calculate almost all the precession incorrectly and inconsistently using Newtonian theory, a procedure rooted in the history of astronomy and still adhered to inexplicably. The correct and self consistent method is obviously to calculate all the experimentally observed precession with the same force law. If EGR is chosen as a theory, the EGR force law must be used in the entire calculation, a highly non trivial N body problem of gravitation. Since EGR has been shown to be riddled with errors by many people for a hundred years {1 - 10} a novel method of calculating the Minkowski force law is developed at the end of Section 2 and graphed and discussed in Section 3.

2. CALCULATION OF LORENTZ / MINKOWSKI PRECESSION AND ITERATIVE PROCEDURE FOR THE MINKOWSKI FORCE LAW.

Consider the non relativistic angular momentum for orbital motion in a plane:

$$L_0 = mr^2 \frac{d\theta}{dt} . \quad -(1)$$

It follows that the angular velocity is:

$$\frac{d\theta}{dt} = \frac{L_0}{mr^2} . \quad -(2)$$

The relativistic angular momentum is {11}:

$$L = mr^2 \frac{d\theta}{d\tau} = \gamma L_0 \quad -(3)$$

where τ is the proper time and γ the Lorentz factor:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(4)$$

in which the velocity v is defined by the infinitesimal line element. In plane polar coordinates:

$$\begin{aligned} ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\underline{r} \cdot d\underline{r} &= (c^2 - v^2) dt^2 \quad -(5) \\ = c^2 dt^2 - dr^2 - r^2 d\theta^2 & \end{aligned}$$

so it follows that the velocity is:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 . \quad -(6)$$

The infinitesimal of time in the observer or laboratory frame is:

$$dt = \frac{mr^2}{L_0} d\theta \quad -(7)$$

and the infinitesimal of proper time is:

$$d\tau = \frac{mr^2}{L_0} \left(\frac{d\theta}{\gamma}\right) . \quad -(8)$$

It follows that the change:

$$dt \rightarrow d\tau \quad -(9)$$

is produced by:

$$d\theta \rightarrow \frac{d\theta}{\gamma} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} d\theta. \quad -(10)$$

For an orbital revolution of 2π radians:

$$\int_0^{2\pi} d\theta \rightarrow \int_0^{2\pi} \left(1 - \frac{v^2}{c^2}\right)^{1/2} d\theta \quad -(11)$$

i.e.:

$$2\pi \rightarrow \int_0^{2\pi} \left(1 - \frac{v^2}{c^2}\right)^{1/2} d\theta. \quad -(12)$$

If the orbit is an ellipse it can be shown that {1 - 10}:

$$v^2 = \left(\frac{L_0}{m\alpha}\right)^2 \left(1 + \epsilon^2 + 2\epsilon\cos\theta\right) \quad -(13)$$

where the half right latitude is defined by:

$$\alpha = (1+\epsilon)r_{min} = (1-\epsilon)r_{max} = (1-\epsilon^2)a = (1-\epsilon^2)^{1/2}b. \quad -(14)$$

Here ϵ is the orbital eccentricity, a and b the semi major and semi minor axes and r_{max} and r_{min} the maximum and minimum distances between an orbiting object of mass m and a mass M at one focus of the ellipse.

The relativistic effect of Eq. (9) is therefore to produce:

$$\Delta\theta = 2\pi(1 - x) \quad -(15)$$

where:

$$x = \frac{1}{2\pi}\int_0^{2\pi}\left(1-\left(\frac{L_0}{mc\alpha}\right)^2\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)^{1/2}d\theta \quad -(16)$$

is the precession factor, and the ellipse becomes a precessing ellipse. In the Newtonian theory {11}:

$$L_0^2 = m^2 \underline{M} G \alpha \quad -(17)$$

so the precession factor becomes:

$$x = \frac{1}{2\pi}\int_0^{2\pi}\left(1-\frac{\underline{M}G}{\alpha c^2}\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)^{1/2}d\theta \quad -(18)$$

In EGR the wrongly attributed Schwarzschild radius is:

$$r_0 = \frac{2\underline{M}G}{c^2} \quad -(19)$$

so:

$$x = \frac{1}{2\pi}\int_0^{2\pi}\left(1-\frac{r_0}{2\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)^{1/2}d\theta \quad -(20)$$

is the Lorentz / Minkowski precession of planets.

In the solar system:

$$\frac{r_0}{2\alpha} \ll 1 \quad -(21)$$

so:

$$x \sim \frac{1}{2\pi}\int_0^{2\pi}\left(1-\frac{r_0}{4\alpha}\left(1+\epsilon^2+2\epsilon\cos\theta\right)\right)d\theta \quad -(22)$$

The precession angle is therefore:

$$\Delta\theta = 2\pi(1-x) = \frac{\pi r_0 (1+\epsilon^2)}{2d} \quad -(23)$$

i.e.

$$\Delta\theta = \frac{\pi r_0 (1+\epsilon^2)}{2(1-\epsilon) r_{max}} = \frac{\pi r_0 (1+\epsilon^2)}{2(1+\epsilon) r_{min}} \quad -(24)$$

in terms of r_{max} and r_{min}.

For the earth sun system:

$$r_0 = 2{,}950 \text{ m}, \quad \epsilon = 0.0167, \quad -(25)$$
$$r_{max} = 1.521 \times 10^{11} \text{ m}, \quad r_{min} = 1.471 \times 10^{11} \text{ m}, \quad d = 1.496 \times 10^{11} \text{ m}$$

and

$$\Delta\theta = 0.64'' \text{ per century}. \quad -(26)$$

The observed precession for the earth is 11,450 arc seconds per century and the result of EGR is 3.8345 arc seconds per century.

Eq. (12) is very similar to the Thomas precession calculated in UFT110 on www.aias.us of this series:

$$\Delta\theta = 2\pi\left(1-\left(1-\frac{v^2}{c^2}\right)^{1/2}\right). \quad -(27)$$

So the Thomas precession of the Earth in its orbit is given by Eq. (12), i.e. :

$$\Delta\theta\,(\text{Thomas precession}) = 0.64'' \text{ per century} \quad -(28)$$

assuming that v is given by Eq. (13). The observed mean orbital velocity of the earth is:

$$v = 2.978 \times 10^4 \text{ ms}^{-1} \quad -(29)$$

and this is given accurately by Eq. (13). The latter can be approximated by:

$$v^2 \sim MG / r_{av} = 2.98 \times 10^4 \, m s^{-1} \quad -(30)$$

using:

$$r_{av} = 1.496 \times 10^{11} m, \quad MG = 1.33 \times 10^{20} m^3 s^{-2}. \quad -(31)$$

From Eq. (27):

$$\Delta\theta \sim \pi \left(\frac{v}{c}\right)^2 \quad -(32)$$

for:

$$v << c \quad -(33)$$

and using Eqs. (29) and (32) the earth's Thomas precession is:

$$\Delta\theta = 3.10 \times 10^{-8} \text{ radians per year} \quad -(34)$$

in excellent agreement with Eq. (28).

The perihelion precession of EGR theory is the well known approximation {11}:

$$\Delta\theta \sim \frac{6\pi GM}{ac^2(1-\epsilon^2)} . \quad -(35)$$

Table 1 is a compilation of the observed, EGR and MT precessions of the planets excluding Pluto.

Table 1: Planetary Precession (arcsec / century)

Planet	Minkowski	Einstein	Observed (x10)
Mercury	1.65	42.195	5,750
Venus	0.89	8.6186	2,040
Earth	0.64	3.8345	11,450
Mars	0.42	1.3502	16,280
Jupiter	0.30	0.0623	6,550
Saturn	0.165	0.0137	19,500
Uranus	0.03	0.0024	3,340
Neptune	0.02	0.0008	360

It is seen from this Table that the observed perihelion precession is much larger than the EGR result. This order of magnitude discrepancy between EGR and the experimental data is claimed to be due to other gravitational effects in standard physics. These effects are calculated however with Newtonian theory, whereas they should be calculated with EGR theory. This gross error was introduced by Einstein and has been adhered to dogmatically. It is claimed in standard physics that it is possible to calculate the gravitational effect of all planets and other objects on the perihelion precession of the orbit of a given planet. This is a highly non trivial n body problem of gravitation using perturbation theory and supercomputers. The gross error in this procedure is that it uses the Newtonian force law:

$$\underline{F} = -\frac{mMG}{r^2}\,\underline{e}_r \qquad -(36)$$

and produces a given percentage of the observed precession. The rest of the precession is known as the anomalous precession. Einstein chose to attribute the anomalous precession to general relativity, using his force law:

$$\underline{F} = -\left(\frac{mMG}{r^2} + \frac{3L_0^2MG}{mc^2r^4}\right)\underline{e}_r \qquad -(37)$$

but did not apply this universal force law to the other part of the precession. He should have applied Eq. (37) to all of the experimentally observed precession. This is obvious in retrospect but the unfortunate fact of history is that Einstein made a basic blunder.

For Saturn for example the observed precession is 1,950 arc seconds a century, but the Einstein theory gives 0.0137 arc seconds a century. The standard physics calculates 19,499.9863 arc seconds a century with Newtonian theory, without general relativity at all. It claims that general relativity applies only to 0.0137 arc seconds a century, whereas general relativity should apply to 1,950 arc seconds a century, with force law (37). In other words the n body gravitational problem must use force Eq. (37) under all circumstances, and not just for the anomalous precession. When this computational procedure is carried out correctly, it will be seen whether EGR is successful. It is well known that criticisms such as these have been made of EGR for a century by many scientists. For the planet Mercury the total observed perihelion precession is 575 arc seconds per century compared with the EGR claim of 42.195 arc seconds per century. So in this case the standard physics claims that its wholly incorrect n body perturbation theory produces precisely 5 707 805 arc seconds per century. If EGR is to be used self consistently then the force law of Eq. (37) should be used, and it will change the result 5 707 805 arc seconds because the Newtonian force law (36) used to produce this result will have been changed to the EGR force law (37). It follows that the EGR claim to reproduce the anomaly will also fail. This is simple to see, and illustrates how dogma can cloud judgement.

The force law (37) can be written as:

$$\underline{F} = -\frac{mMG}{r^2}\left(1+\frac{3}{2}\frac{d\,r_0}{r^2}\right)\underline{e}_r \quad -(38)$$

For the earth sun system:

$$r_0 = 2{,}950 \text{ m}$$
$$d = 1.496 \times 10^{11} \text{ m} \quad -(39)$$
$$r_{av} = 1.496 \times 10^{11} \text{ m}$$

so:

$$\underline{F} = -\frac{mMG}{r^2}\left(1 + 2.96 \times 10^{-8}\right)\underline{e}_r \quad -(40)$$

This force law is claimed in EGR theory to be universal, and as such must be used in all circumstances, and not carefully selected for use as in the calculation of the precession anomaly. The correction in Eq. (40) means that the mass of the sun, which is about 2 x 10^{30} kilograms, is increased by order 10^{22} kilograms. This should have consequences everywhere in the solar system. For example, if the earth sun distance is changed by about 10^{-8} the effect of EGR will be annulled.

So EGR is not a credible theory in any way {1 - 10}. The Minkowski force equation on the other hand is based on the most fundamental idea of relativity, the Lorentz transformation. The Lorentz Minkowski Thomas precession can be observed with a Foucault pendulum in a much cleaner way than planetary precession, which is probably the least suitable phenomenon with which to test a theory, being complicated with many contributory factors.

Finally in this Section the Minkowski force equation for planar orbits is developed in order to illustrate methods of solution. In view of the numerous errors in EGR the Minkowski equation can be used for all planar orbits self consistently, whereas EGR fails qualitatively for whirlpool galaxies and also in the solar system as just argued.

As in immediately preceding papers of the UFT series on www.aias.us the Minkowski force equation for all planar orbits is:

$$\underline{F} = -\frac{\gamma^2 L_0^2}{mr^2}\left(\gamma^2\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r + \frac{\gamma^4 L_0^2}{m^3 r^3 c^2}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\underline{e}_\theta \quad -(41)$$

in plane polar coordinates. In the limit

$$v << c \quad -(42)$$

the equation can be approximated by:

$$\gamma^2\left(\gamma^2\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right)=-\frac{mr^2F}{L_0^2}. \quad -(43)$$

It is possible to solve this equation by iteration. For the sake of illustration assume that the initial solution of the iterative procedure is the Newtonian:

$$F \sim -\frac{mMG}{r^2} \quad -(44)$$

then it follows that:

$$\gamma^2\left(\gamma^2\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right)=\frac{1}{\alpha} \quad -(45)$$

where

$$\alpha = \frac{L_0^2}{m^2MG}. \quad -(46)$$

Eq. (45) becomes:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{1}{\alpha}\left(1-2\frac{v^2}{c^2}+\frac{v^4}{c^4}\right)+\frac{v^2}{c^2r}. \quad -(47)$$

The assumption (44) assumes that the initial solution of the iterative process is an elliptical orbit:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{1}{\alpha} \quad -(48)$$

so the initial solution (44) assumes that:

$$\gamma \rightarrow 1, \quad v \rightarrow 0. \quad -(49)$$

However, for the ellipse:

$$v^2 = \left(\frac{L_0}{m\alpha}\right)^2 \left(1+\epsilon^2+2\epsilon\cos\theta\right) \quad -(50)$$

where:

$$\epsilon\cos\theta = \frac{\alpha}{r} - 1 \quad -(51)$$

so for the ellipse:

$$v^2 = 2\left(\frac{L_0^2}{m^2\alpha r}\right) + (\epsilon^2-1)\left(\frac{L_0^2}{m^2\alpha^2}\right). \quad -(52)$$

The initial assumption is however an elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(53)$$

and this is corrected by Eqs. (47) and (52) to produce a new orbit whose differential equation is:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} - \frac{v^2}{c^2}\left(\frac{2}{\alpha} - \frac{1}{r}\right) + \frac{1}{\alpha}\left(\frac{v}{c}\right)^4 \quad -(54)$$

where:

$$v^2 = 2\left(\frac{L_0^2}{m^2\alpha r}\right) + (\epsilon^2-1)\left(\frac{L_0}{m\alpha}\right)^2 \quad -(55)$$

and:

$$v \ll c. \quad -(56)$$

In the approximation (56):

$$L_0^2 = m^2 MG\alpha \quad -(57)$$

so:

$$v^2 = \frac{2mG}{r} + (\epsilon^2 - 1)\left(\frac{mG}{\alpha}\right) \quad (58)$$

and in the approximation (56), Eq. (54) is approximately:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} - 2\left(\frac{(\epsilon^2-1)mG}{c^2\alpha^2}\right) - \frac{mG}{\alpha r c^2}\left(5-\epsilon^2\right) + \frac{2mG}{c^2 r^2} \quad (59)$$

which can be written as:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} + \frac{2mG}{c^2r^2} - \frac{mG(5-\epsilon^2)}{\alpha r c^2} - \frac{2(\epsilon^2-1)mG}{c^2\alpha^2} \quad (60)$$

which can be compared with the result of the EGR theory {1 - 10}:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} + \frac{3mG}{c^2r^2}. \quad (61)$$

If the Newtonian assumption (57) is not used then:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} + \frac{2L_0^2}{m^2c^2r^2\alpha} - \frac{L_0^2(5-\epsilon^2)}{\alpha^2m^2c^2r} - \frac{2(\epsilon^2-1)L_0^2}{\alpha^3m^2c^2}. \quad (62)$$

By integrating Eq. (62) a new orbit and force law can be found. This force law can be used as the starting point in the second stage of the iterative procedure and so on.

3. NUMERICAL SOLUTIONS OF THE MINKOWSKI FORCE EQUATION.

Section by Dr. Horst Eckardt

Calculation of the perihelion precession with the Minkowski force equation

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3 Numerical solutions of the Minkowski force equation

Aim of this section is to compare the relativistic effects for a precessing ellipse in the Minkowski and Einstein limit. The γ factor is given by Eq.(4):

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{63}$$

where the velocity v for a precessing ellipse is according to previous work:

$$v^2 = \left(\frac{L_0}{m\,r}\right)^2 \left(1 - x^2\epsilon^2\right) + 2\left(\frac{x\epsilon L_0}{m\,\alpha}\right)^2 \frac{\alpha}{r}. \tag{64}$$

α evaluates from Eq.(17) to

$$\alpha = \frac{L_0^2}{m^2 MG} \tag{65}$$

and the wrongly attributed Schwarzschil radius (19),

$$r_0 = \frac{2MG}{c^2}, \tag{66}$$

can be rewritten by eliminating MG by Eq.(65):

$$r_0 = \frac{2L_0^2}{\alpha m^2 c^2}. \tag{67}$$

The precession factor x from Eq.(23) is

$$x = 1 - \frac{\left(\epsilon^2+1\right)\,{L_0}^2}{2\,\alpha^2\,m^2\,c^2}. \tag{68}$$

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From Eq.(41) follows for the radial component of the Minkowski force of a precessing ellipse

$$F_M = -\frac{\gamma^2 L_0^2}{m r^2 \alpha} \left(\frac{\alpha}{r} \left(1 - x^2\gamma^2\right) + x^2\gamma^2 \right). \tag{69}$$

In comparison, the Einstein result (37) is

$$F_E = -\frac{L_0^2}{\alpha m r^2} - \frac{3L_0^4}{\alpha m^3 c^2 r^4} \tag{70}$$

and the non-relativistic force is simply

$$F_{nr} = -\frac{L_0^2}{\alpha m r^2}. \tag{71}$$

Finally the precession factor x from Einstein theory is

$$x_E = 1 + \frac{3L_0^2}{\alpha^2\, m^2\, c^2} \tag{72}$$

for comparison. In all formulas the factor MG has been replaced by the geometrical quantities L_0 and α according to Eq.(17).

Fig. 1 shows the ratio v/c for four values of L_0 with $\epsilon = 0.3$, all other parameters set tu unity. L_0 is the determining parameter for given α and ϵ. All radial ranges are given in the range r_{min} to r_{max}. Obviously the L_0 values lead to a v/c ratio between roughly 0.1 and 0.7.

Fig. 2 presents $x(L_0)$ for the Minkowski and Einstein theory, given by Eqs.(68) and (72). For EGR, x grows quadratically with L_0 while it decreases quadratically with a smaller slope in the Minkowski case. This should provide a qualitative criterion to decide from experiments which theory describes precession correctly.

Fig. 3 shows the Minkowski, Einstein and Newton force law for a smaller $L_0 = 0.1$. The Einstein force deviates more from the Newtonian curve than the Minkowski force. In Fig. 4 the forces for the ultrarelativistic case are shown ($L_0 = 0.55$). Here the Einstein force again overestimates relativistic effects, compared to the Minkowski force.

Besides these analytic calculations, the obit $r(\theta)$ was calculated numerically in for different approximations.

non-relativistic force equation

The non-relativistic force equation (36), written with geometric quantities only, is

$$F_{nr} = -\frac{L_0^2}{\alpha\, m\, r^2} = -\frac{L_0^2}{\alpha\, m}\, u^2 \tag{73}$$

with $u = 1/r$. Lagrange theory leads to the differential equation

$$\frac{d^2 u}{d\,\theta^2} = -u + \frac{1}{\alpha} \tag{74}$$

to besolved (Eq.(48)). The numerical integration procedure gives the constant ellipse, see red curve in Fig. 5.

Einstein force equation

The force equation with Einstein correction (37),

$$F_E = -\frac{L_0^2}{\alpha\, m}\, u^2 - \frac{L_0^4}{\alpha\, m^3\, c^2} u^4, \tag{75}$$

leads to the differential equation

$$\frac{d^2\, u}{d\,\theta^2} = \frac{3\, {L_0}^2}{\alpha\, m^2\, c^2} u^2 - u + \frac{1}{\alpha}. \tag{76}$$

The results give a widely precessing ellipse for the same initial conditions as for the non-relativistic case, see green curve in Fig. 5. The parameters were chosen so that $v/c \approx 0.25$. Choosing higer v/c ratios leads to divergence of the Einstein method, showing again its weakness.

Minkowski force equation, approximated

In the last part of section 2, an approximation was developed for a precessing ellipse on base of the radial component of the Minkoski force equation

$$F_r = -\frac{L_0^2}{m} u^2 \gamma^2 \left(\gamma^2 \frac{d^2\, u}{d\,\theta^2} + u \right). \tag{77}$$

It is assumed in a first step that F_r in the above equation is approximately the Newtonian force (73) and the orbit is that of an ellipse. Then the orbital velocity v is known and the γ factor can be computed. Insering this into Eq.(77) leads to the equation

$$\frac{d^2 u}{d\,\theta^2} = \frac{2\, {L_0}^2}{\alpha\, m^2\, c^2} u^2 - \left(1 + \frac{\left(5 - \epsilon^2\right)\, {L_0}^2}{\alpha^2\, m^2\, c^2} \right) u - \frac{2\, \left(\epsilon^2 - 1\right)\, {L_0}^2}{\alpha^3\, m^2\, c^2} + \frac{1}{\alpha} \tag{78}$$

which is a new differential equation for determining the orbit $r(\theta)$. The result of umerical integration is shown in the blue curve of Fig. 5. The precession is much smaller compared to the Einstein result, in accordance with Fig. 4 where the Einstein force was shown to deviate more from the non-relativistic case than the Minkowski force.

Minkowski force equation without orbital assumptions

In the described procedure we had to choose an eccentricity parameter for the initially assumed ellipse, $\epsilon = 0.6$. For a precessing ellipse this parameter is no more sharply defined, therefore it would be better to find an approximation which does not need this input. We modify the previous proceeding as follows. Instead of assuming an initial ellipse we use the Minkoswki force equation (77) directly, setting the force to the Newtonian force as a first step. We then arrive at the equation

$$\frac{d^2\, u}{d\,\theta^2} = \frac{1}{\alpha\, \gamma^4} - \frac{u}{\gamma^2} \tag{79}$$

where most constants have cancelled out. Since the orbit u is unknown a priori, we have to find a suitable expression for v which has to be inserted into

γ. We take the definition of the velocity in polar coordinates directly with instantaneous values of r and θ:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2. \tag{80}$$

Using

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt}, \tag{81}$$

$$\frac{du}{dr} = -\frac{1}{r^2} \tag{82}$$

and

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{L_0^2}{m^2 r^4} \tag{83}$$

the squared velocity can be written

$$\begin{aligned} v^2 &= \left(\frac{L_0}{mr^2}\right)^2 \left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right) \\ &= \left(\frac{L_0}{m}\right)^2 \left(u^2 + \left(\frac{du}{d\theta}\right)^2\right). \end{aligned} \tag{84}$$

Thus the orbit follows completely from the equations. The result is graphed as the brown curve in Fig. 5. It can be seen that the precession is even smaller than in the case of the preceding approximation (blue curve).

The described procedure is the first step of a possible iteration scheme, the force was assumed to be Newtonian and a new orbit was computed. In the next step a new force approximation has to be used. The force is obtained directly from (77) by inserting the obtained solution $u(\theta)$ at the right hand side. This numerically given $F_r(u)$ can be used to find a new solution. The iteration scheme is

$$F_n = -\frac{L_0^2}{m} {u_n}^2 \gamma_n^2 \left(\gamma_n^2 \frac{d^2\, u_n}{d\,\theta^2} + u_n\right), \tag{85}$$

$$\frac{d^2}{d\,\theta^2}\, u_{n+1} = -\frac{m\, F_n}{{L_0}^2\, u_{n+1}^2\, \gamma_{n+1}^4} - \frac{u_{n+1}}{\gamma_{n+1}^2}. \tag{86}$$

γ_n is obtained from $v_n(u_n)$. This iterative methods was implemented but came out to be unsuitable because the same basis equation is used for (85) and (86). This leads to the identical force F_{n+2} after having calculated u_{n+1}. A different scheme has to be found.

Finally we have compiled the values of minimum and maximum radius and eccentricities in Table 1. The extremal radii have been determined from the numerical u or r values, respektively. For an ellipse the eccentricity is defined by

$$\epsilon = -\frac{r_{min} - \alpha}{r_{min}} = \frac{r_{max} - \alpha}{r_{max}}. \tag{87}$$

Method	r_{min}	r_{max}	ϵ_1	ϵ_2
non-relativistic	0.625	2.500	0.600	0.600
Einstein	0.521	2.158	0.920	0.537
Minkowski, approx.	0.646	2.751	0.549	0.637
Minkowski, exact	0.643	2.755	0.555	0.637

Table 1: Comparison of extremal radii and eccentricities.

For precessing ellipses, both equations give different values, denoted by ϵ_1 and ϵ_2 in Table 1. In the non-relativistic case both are equal, in all other cases not. For the Einstein force calculation the difference is particlularly large, reflecting the excessive precession of the orbit. In both Minkowski cases the values are much closer to one another. Both Minkowski cases differ only marginally.

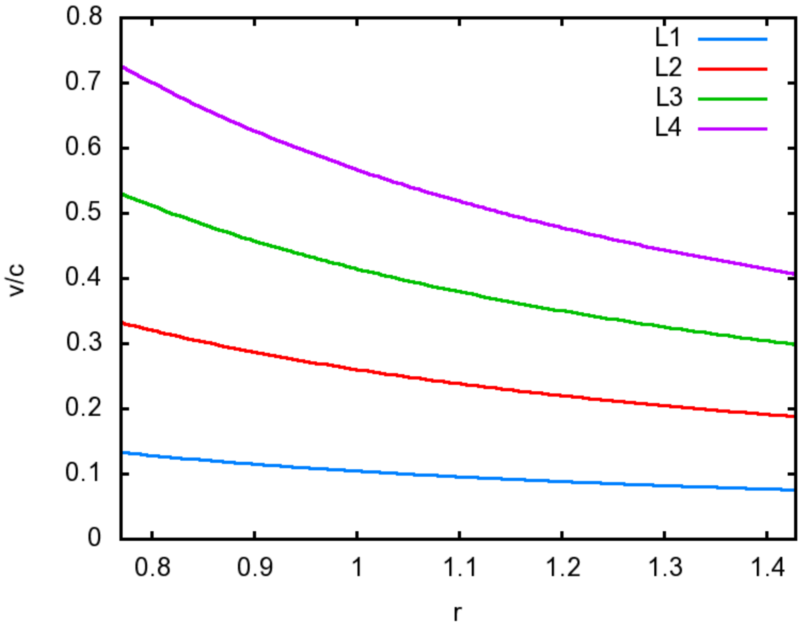

Figure 1: Ratio v/c for angular momenta $L1 = 0.1,\ L2 = 0.25,\ L3 = 0.4,\ L4 = 0.55$. Radial range is for $\epsilon = 0.3$.

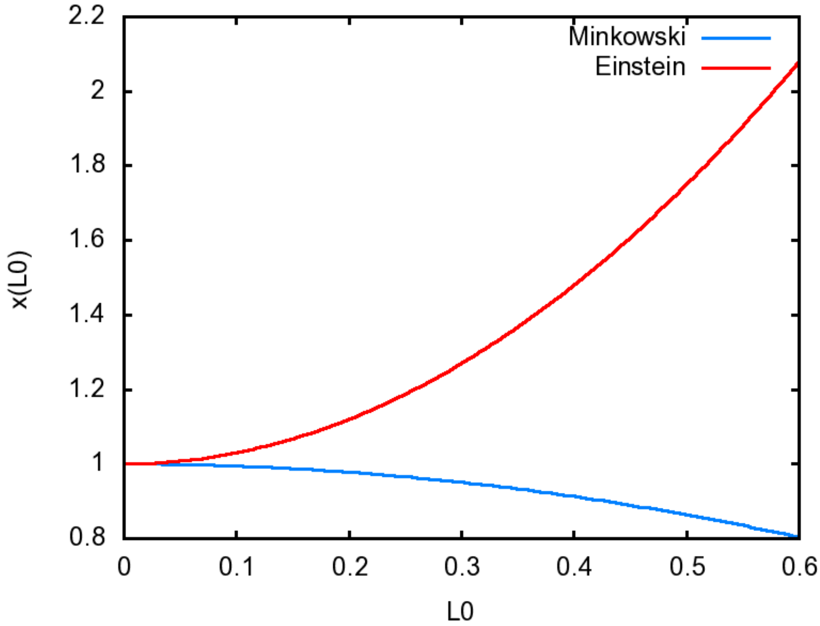

Figure 2: Dependence of precession factor $x(L_0)$.

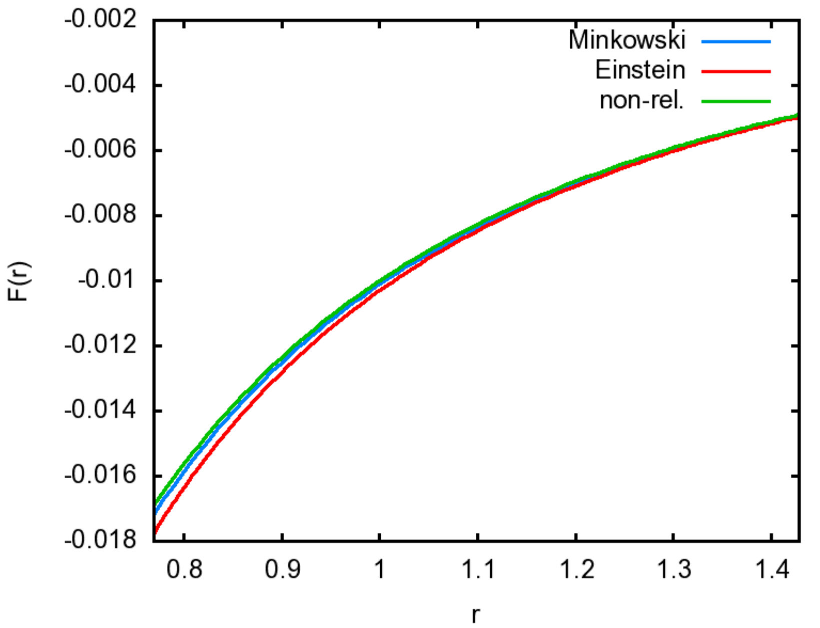

Figure 3: Minkowki, Einstein and non-relativistic force $F(r)$ within radial range for $\epsilon = 0.3$ with $L_0 = 0.1$.

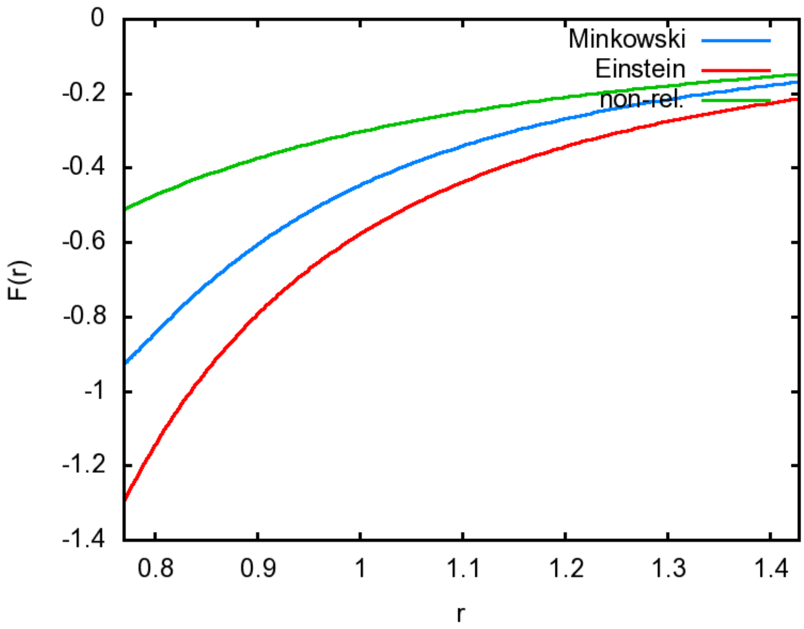

Figure 4: Minkowki, Einstein and non-relativistic force $F(r)$ within radial range for $\epsilon = 0.3$ with $L_0 = 0.55$.

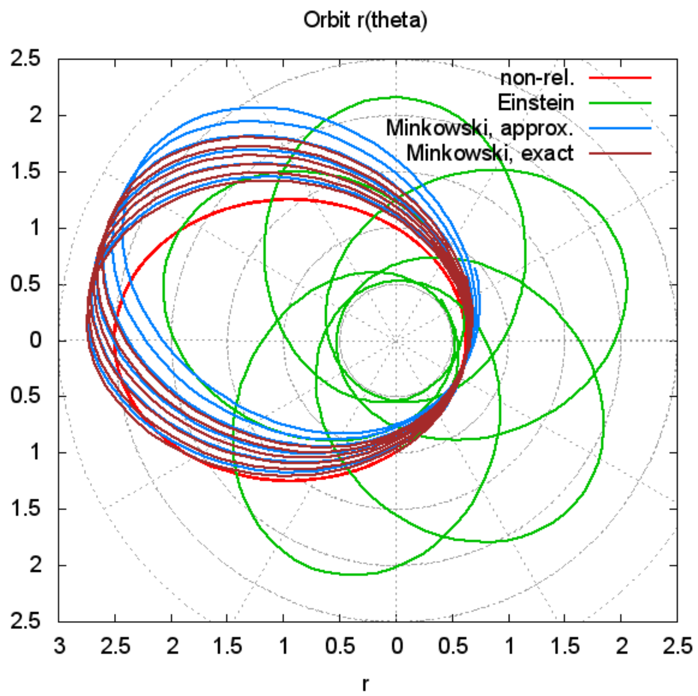

Figure 5: Orbits for Minkowki, Einstein and non-relativistic force laws.

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REFERENCES

{1} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity" (Cambridge International Science Publishing, CISP, www.cisp-publishing.com, 2012)

{2} M. W. Evans, Ed., J. Found. Phys. Chem., (CISP, 2011, six issues a year).

{3} M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP, 2011).

{4} M .W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis, Suffolk, 2005 - 2011).

{5} L .Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007).

{6} M. W. Evans, H. Eckardt and D. W. Lindstrom, papers and plenary papers on ECE theory published by journals of the Serbian Academy of Sciences, Found. Phys. Lett., Physica B, and Acta Phys. Polonica.

{7} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

{8} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley, 1992, 1993, 1997 and 2001), in two editions and six volumes.

{9} M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002), in ten volumes hardback and softback.

{10} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory"

(World Scientific, 1994).

{11} J. B. Marion and S. T. Thornton, "Classical Dynamics" (Harcourt, New York, 1988, third edition).