# ANIMATION OF PRECESSIONS DUE TO THE MINKOWSKI FORCE EQUATION. 

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#### Abstract

The Minkowski equation is used to show that an initially Newtonian orbit develops a precession due to the Lorentz factor. The precession can be evaluated analytically in the limit of nearly circular orbits, and a solution found for the polar angle as a function of time. An expression for precession may also be derived by considering the relativistic angular momentum, and put in a format suitable for animation. The search for new explanations of precession is made necessary by the many refutations of Einsteinian general relativity now available.


Keywords: Minkowski equation, precession of planar orbits, animation.

$$
\text { UFT } 241
$$

## 1. INTRODUCTION

During the course of development of ECE theory $\{1-10\}$ the Einsteinian general relativity (EGR) has been refuted in many ways. In recent papers a search has been initiated for a new explanation of precession in the solar system and in objects such as whirlpool galaxies. The most fundamental force equation of relativity is the Minkowski equation, which incorporates the Lorentz factor in to the Newton equation, and on the grounds of simplicity and Ockham's Razor is a good starting point for a new theory. In Section 2 the Minkowski equation derived in immediately preceding papers of this series is put in a format suitable for animation, and a related method developed on the grounds of the relativistic angular momentum. Some of the problems of the EGR are discussed. In Section 3 computation and animation is implemented to illustrate the Minkowski precessions of a planar orbit.

## 2. DEVELOPMENT OF THE MINKOWSKI FORCE EQUATION

Consider the format of the Minkowski force equation developed in UFT239 on www.aias.us:

$$
\underline{E}=m\left(\gamma^{4} \frac{d^{2} r}{d t^{2}}-\frac{\gamma^{2} L_{0}^{2}}{m^{2} r^{3}}\right) e_{r}+\frac{m \gamma^{4}}{c^{2}} \frac{d r}{d t} \frac{d^{2} r}{d t^{2}} \text { wren } \underline{e}_{\theta}-(1)
$$

where $\underline{E}^{\ell}$ and $\underline{e}_{\theta}$ are the radial and polar unit vectors, $m$ is the mass of an object orbiting in a plane, c is the vacuum speed of light, $\omega$ is the angular velocity, $r$ is the radial distance and $X$ the Lorentz factor:

$$
y=\left(1-\frac{v^{2}}{c^{2}} \cdot-1 / 2 \cdot-(2)\right.
$$

Here $L_{o}$ is the non relativistic angular momentum defined $\{11\}$ by:

$$
\begin{equation*}
L_{0}=m r^{2} \omega=m r^{2} \frac{d \theta}{d t} \tag{3}
\end{equation*}
$$

$$
\frac{d r}{d t}=-\frac{L_{0}}{m} \frac{d}{d \theta}\left(\frac{1}{r}\right), \frac{d^{2} r}{d t^{2}}=-\left(\frac{L_{0}}{m r}\right)^{2} \frac{d^{2}}{d \theta^{2}}\left(\frac{1}{r}\right),-(4)
$$

and the velocity of the Lorentz factor is defined by:

$$
v^{2}=\left(\frac{L_{0}}{m}\right)^{2}\left(\frac{1}{r^{2}}+\left(\frac{d}{d t}\left(\frac{1}{r}\right)\right)^{2}\right)-(5)
$$

Assume that the orbit is initially Newtonian:

$$
\frac{1}{r}=\frac{1}{\alpha}(1+t \cos \theta)-(6)
$$

where $\alpha$ is the half right latitude and $\in$ the eccentricity. If follows from Es. ( 1 , and ( 6 ) that

$$
\underline{F}=A \underline{e}_{r}+B \underline{e}_{\theta}-(7)
$$

- 

$$
A=-\frac{\gamma^{2} L_{0}^{2}}{m r^{2}}\left(\gamma^{2}+\frac{1}{r}\left(1-\gamma^{2}\right)\right)-(8)
$$

and:

$$
B=\frac{\gamma^{4} \cdot L_{0}^{4} \sin \theta \cos \theta}{m^{3} c^{2} r^{3} \alpha^{2}}-(9)
$$

in which the Lorentz factor is:

$$
\begin{aligned}
& \gamma=\left(1-v^{2} / c^{2}\right)^{-1 / 2}-(10) \\
& v^{2}=M G\left(\frac{2}{r}-\frac{1}{a}\right), a=\frac{d}{\epsilon^{2}-1} \cdot-(11)
\end{aligned}
$$

In the Newtonian limit:

$$
L_{0}^{2}=d m^{2} m G-(12)
$$

so A ind B can be expressed as follows:

$$
A=-\frac{\gamma^{4} m \frac{m t}{r^{2}}-\gamma^{2} \frac{d m b}{r^{3}}\left(1-\gamma^{2}\right) .}{}
$$

and

$$
B=m\left(\frac{m b}{c}\right)^{2} \frac{\sin \theta \cos \theta}{r^{3}}-(14)
$$

in which:

$$
\cos \theta=\frac{1}{\epsilon}\left(\frac{\alpha}{r}-1\right)-(15)
$$

and

$$
\sin ^{2} \theta+\cos ^{2} \theta=1 .-(16)
$$

In the limit of nearly circular orbits the apsidal angle is defined by:

$$
\psi=\pi\left(3+\frac{r}{F} \frac{d F}{d r}\right)^{-1 / 2}-(r)
$$

and in this limit:

$$
A \rightarrow-\gamma^{4} \frac{m \underline{M} G}{r^{2}},-(18)
$$

and:

$$
B \rightarrow 0, \quad-(19)
$$

so the force magnitude reduces to:

$$
F=-\frac{m m b}{r^{2}}\left(1+\frac{r_{0}}{r}\right)-(20)
$$

giving a precession:
where

$$
r_{0}=\frac{2 m b}{c^{2}}-(22)
$$

is the so called "Schwarzschild" radius.
From previous work the correct equation of the precessing ellipse is:

$$
r=\frac{d}{1+(\cos (x \theta)}-(23)
$$

whose force law can be worked out in the classical limit using Lagrangian methods, giving the result $\{1-10\}$ :

$$
F=\frac{-m M G x^{2}}{r^{2}}-\alpha\left(1-x^{2}\right) \frac{m M G}{r^{3}}-(24)
$$

Eqs. (20) and (24) are the same if:

$$
x \rightarrow 0 \quad-(25)
$$

and:

$$
d\left(1-x^{2}\right) m M G=2 m\left(\frac{M G}{c}\right)^{2}-(26)
$$

i. e.

$$
x^{2}=1-\frac{r_{0}}{\alpha}-(27)
$$

and

$$
d \sim r . \quad-(28)
$$

If

$$
r_{0} \ll r-(29)
$$

then an expression can be derived for x :

$$
x \sim 1-\frac{r_{0}}{2 r}-(30)
$$

Therefore:

$$
1-x \sim \frac{r_{0}}{2 r}-(31)
$$

and the precession of the perihelion is:

$$
2 \pi(1-x)=\pi \frac{r_{0}}{r}-(32)
$$

which is Eq. ( $2 \backslash$ ), Q. E. D.
In the Newtonian limit ( 12 ) the factor A can be written as:

$$
A=-\gamma^{4} \frac{m M G}{r^{2}}-\gamma^{2} \frac{\alpha m M}{r^{3}}\left(1-\gamma^{2}\right)-(33)
$$

and in the limit of nearly circular orbits the force is:

$$
\underline{F} \sim\left(-\gamma^{4} \frac{m m G}{r^{2}}-\gamma^{2} \frac{\left.\alpha m M G\left(1-\gamma^{2}\right)\right)}{r^{3}}\right) \frac{e^{r}}{}(34)
$$

Comparing Eqs. (24) and (34):

$$
x^{2}=\frac{r \gamma^{4}+\alpha \gamma^{2}\left(1-\gamma^{2}\right)-\alpha}{r-\alpha}-(35)
$$

where the Lorentz factor is defined by:

$$
\begin{equation*}
\gamma^{2}=\left(1-\frac{r}{r_{0}}\right)^{-1} \tag{36}
\end{equation*}
$$

In the limit:

$$
\begin{equation*}
\gamma \rightarrow 1 \tag{37}
\end{equation*}
$$

the precession factor x approaches unity.
Now calculate as a function of ra follows. First note that:

From Eq. ( 23 ):

$$
L_{0}=m r^{2} \frac{d \theta}{d t}=m r^{2} \frac{d \theta}{d r} \frac{d r}{d t}-(38)
$$

$$
\frac{d r}{d \theta}=\frac{-x r^{2}}{d} \sin (x \theta)-(39)
$$

with:

$$
\cos (x \theta)=\frac{1}{\epsilon}\left(\frac{\alpha}{r}-1\right)-(40)
$$

and

$$
\cos ^{2}(x \theta)+\sin ^{2}(x \theta)=1
$$

So

$$
\sin (x \theta)=\left(1-\frac{1}{\epsilon^{2}}\left(\frac{\alpha}{r}-1\right)^{2}\right)^{1 / 2}-(42)
$$

From Eqs. ( 38 ) and ( 42 ):

$$
\frac{d r}{d \theta}=\frac{\left(-x r^{2}\right.}{\alpha}\left(1-\frac{1}{\epsilon^{2}}\left(\frac{d}{r}-1\right)^{2}\right)^{1 / 2}-(43)
$$

so:

$$
\frac{d \theta}{d r}=\frac{d}{\epsilon-x r^{2}}\left(1-\frac{1}{\epsilon^{2}}\left(\frac{d}{r}-1\right)^{2}\right)^{-1 / 2}-(44)
$$

and

$$
t=\frac{m d}{\epsilon L_{0}} \int \frac{1}{x}\left(1-\frac{1}{\epsilon^{2}}\left(\frac{d}{r}-1\right)^{2}\right)^{-1 / 2} d r-(45)
$$

in which x is given Eqs. $(35$ ) and ( 36 ).
Eq. $(45$ ) can be solved by computer algebra to give $r$ as a function of $t$ :

$$
r=f(t)-(46)
$$

in which the factor is a function of the other variables appearing in the analysis. Computer algebra must be used to produce this function and this was carried out by Dr. Horst Eckardt to give the definition:

$$
r=f\left(m, \alpha, \epsilon, L_{0}, t\right),-(47)
$$

The equation relating $r$ to $\theta$ is the orbital equation (23). So:

$$
\theta=\frac{1}{x} \cos ^{-1}\left(\frac{1}{\epsilon}\left(\frac{d}{r}-1\right)\right)-(48)
$$

and from Eqs. ( 47 ) and ( 48 ) the polar angle $\theta$ can be calculated as a function of time $t$. This was done by computer algebra and the result animated in Section 3 using a numerical method to produce a regular grid for animation.

As in UFT239 on www.aias.us it can be shown that the Lorentz factor produces a precession factor:

$$
\begin{aligned}
& \left.x=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left(1-\left(\frac{L_{0}}{m c \alpha}\right)^{2 / 1+\epsilon^{2}+2 \epsilon \cos \theta}\right)\right)_{-(4 a)}^{1 / 2} d \theta \\
& \text { commanding UFT238 on www wises she time is given by: }
\end{aligned}
$$

As in note 238(12) accompanying UFT238 on www.aias. us the time $t$ is given by:

$$
t=\frac{m \alpha^{2}}{L_{0}} \int \frac{1 \theta}{(1+\epsilon \cos (x \theta))^{2}}-\left(5_{0}\right)
$$

where $x$ is defined by Eq. (49). In Cartesian representation:

$$
x=\frac{d \cos \theta}{1+f \cos (x \theta)}, y=\frac{\alpha \sin \theta}{1+f \cos (x \theta)}
$$

$$
-(5)
$$

so $(X, Y)$ can be calculated as a function of $\theta$. and animated. For each $\theta$, the time $t$ can be calculated form Eq. ( 50 ), so (X,Y) can be calculated as a function of time $t$ and animated. For example:

$$
\left(x_{1}, y_{1}\right)=\left(\frac{\alpha \cos \theta_{1}}{1+\epsilon \cos \theta_{1}}, \frac{\alpha \sin \theta_{1}}{1+f \cos \theta_{1}}\right)-\left(s_{2}\right)
$$

where:

$$
t_{1}=\frac{m \alpha^{2}}{L_{0}} \int_{0}^{\theta_{1}} \frac{1 \theta}{(1+\epsilon \cos (x \theta))^{2}}-(53)
$$

$$
x=\frac{1}{2 \pi} \int_{0}^{2 \pi}\left(1-\frac{r_{0}}{2 \alpha}\left(1+\epsilon^{2}+2 \epsilon \cos \theta\right)\right)^{1 / 2} d \theta
$$

where:

$$
r_{0}=\frac{2 m G}{c^{2}} \quad-(55)
$$

As in note 239(7) accompanying UFT239 on www.aias.us, if:
then:

$$
\begin{aligned}
& \frac{r_{0}}{2 \alpha}<1-\left(<1-\frac{r_{0}\left(1+\epsilon^{2}\right)}{4 \alpha}-(57)\right. \\
& x \sim 1)
\end{aligned}
$$

and is constant, so the animation becomes simpler.
In order to illustrate the obsolescence of EGR consider the planets of the solar system and for each planet evaluate the EGR precession for nearly circular orbits using the methods developed in immediately preceding papers. For each planet the EGR force law is:

$$
F(r)=-\frac{m m b}{r^{2}}-\frac{3 m G L_{0}^{2}}{m c^{2} r^{4}}-(58)
$$

giving the precession:

$$
\Delta \theta=3 \pi \frac{r_{0}}{r} \quad-(59)
$$

for a nearly circular orbit. The total precession is:

$$
\sum \Delta \theta=3 \pi r_{0} \sum_{i=1}^{n} \frac{1}{r_{i}}-(60)
$$

and the relativistic correction is applied n times where n is the number of planets. For each planet it can be shown as in note 241(1) that:

$$
\sum \theta=2 \pi n+3 \pi r_{0} \sum_{i=1}^{n} \frac{1}{r_{i}}-(61) .
$$

so the total EGR correction is:

$$
\sum \Delta \theta=\sum \theta-2 \pi n .-(62)
$$

The standard physics claims the incorrect result:

$$
\Delta \theta=\theta-2 \pi-(63)
$$

Some further details of this argument are given on note 241(1) and in notes for UFT240. For this reason alone, EGR cannot be a correct theory.
3. COMPUTER ALGEBRA AND ANIMATION.

Section by H. Eckardt and B. Foltz

# Animation of precessions due to the Minkowski force equation 

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## 3 Computer algebra and animation

In this section we give some more details of the calculations in section 2 , compute the apsidal angle and the time dependence of the orbit. Finally we present a calculation scheme for animation. The radial and angular component of the Minkowski force (1) for an elliptic orbit (6) are given by Eqs.(8) and (9). These depend on the radial and angular coordinates, they can be expressed solely by radial coordinates via Eq.(6), leading to

$$
\begin{gather*}
A=\frac{\alpha^{2} c^{2} m L_{0}^{2}\left(\epsilon^{2} r L_{0}^{2}-r L_{0}^{2}+2 \alpha{L_{0}}^{2}-\alpha c^{2} m^{2} r^{2}\right)}{r^{2}\left(\epsilon^{2} r L_{0}^{2}-r L_{0}^{2}+2 \alpha L_{0}^{2}-\alpha^{2} c^{2} m^{2} r\right)^{2}}  \tag{64}\\
B=-\frac{\alpha^{2} c^{2} m(r-\alpha) \sqrt{\left(\epsilon^{2}-1\right) r^{2}+2 \alpha r-\alpha^{2}} L_{0}^{4}}{r^{3}\left(\epsilon^{2} r L_{0}^{2}-r L_{0}^{2}+2 \alpha L_{0}^{2}-\alpha^{2} c^{2} m^{2} r\right)^{2}} \tag{65}
\end{gather*}
$$

The apsidal angle in this approxiation is defined by Eq.(17). When $F(r)$ is approximated by the radial component only, the result is
$\psi=\frac{\pi \sqrt{\left(\left(\epsilon^{2}-1\right) r+2 \alpha\right) G M-\alpha c^{2} r} \sqrt{\left(\left(\epsilon^{2}-1\right) r+2 \alpha\right) G M-c^{2} r^{2}}}{\sqrt{\left(\left(2 \alpha \epsilon^{2}-2 \alpha\right) r+4 \alpha^{2}\right) G^{2} M^{2}+\left(\left(c^{2}-c^{2} \epsilon^{2}\right) r^{3}-6 \alpha c^{2} r^{2}+2 \alpha^{2} c^{2} r\right) G M+\alpha c^{4} r^{3}}}$.

This expression simplifies considerably, if we consider the radius $r=\alpha$ and replace the Newtonian parameters $M$ and $G$ by $L_{0}$ via Eq.(12):

$$
\begin{equation*}
\psi=\frac{\pi\left(\epsilon^{2}+1\right) L_{0}{ }^{2}-\pi \alpha^{2} c^{2} m^{2}}{\sqrt{2 L_{0}^{2}-\alpha^{2} c^{2} m^{2}} \sqrt{\left(\epsilon^{2}+1\right) L_{0}^{2}-\alpha^{2} c^{2} m^{2}}} \tag{67}
\end{equation*}
$$

[^0]The graph of Eq.(66) is shown in Fig. 1 for three values of $L_{0}$. Obviously there is a common crossing point for all $L_{0}$ values which is governed by $\epsilon^{2}-1$. The $\gamma$ factor can be calculated from Eqs.(10-12):

$$
\begin{equation*}
\gamma(r)=\frac{\alpha c m \sqrt{r}}{\sqrt{\left(\left(\epsilon^{2}-1\right) r-2 \alpha\right) L_{0}^{2}+\alpha^{2} c^{2} m^{2} r}} . \tag{68}
\end{equation*}
$$

It is graphed in Fig. 2 for the special case $r=\alpha$ to study its $L_{0}$ dependence. $\gamma$ rises asymptotically to infinity for $L_{0}$ values where $v$ comes near to $c$.

The next results are for a precessing ellipse:

$$
\begin{equation*}
r=\frac{\alpha}{1+\epsilon \cos (x \theta)} \tag{69}
\end{equation*}
$$

From Eq.(45) the time for taking a certain radius value, the inverse function of $r(t)$, can be found by integration:

$$
\begin{equation*}
t=\frac{\alpha m\left(\alpha \sin ^{-1}\left(\frac{\left(\epsilon^{2}-1\right) r+\alpha}{\alpha \epsilon}\right)+\sqrt{(1-\epsilon)(1+\epsilon)} \sqrt{(\epsilon-1) r+\alpha} \sqrt{(\epsilon+1) r-\alpha}\right)}{\sqrt{1-\epsilon}(\epsilon-1)(\epsilon+1)^{\frac{3}{2}} x L_{0}} \tag{70}
\end{equation*}
$$

which is graphed in Fig. 3. The plot defines the $r$ range of the precessing ellipse as expected. The curve flattens when x is increased (not shown). The exact value of $x(\theta)$, Eq.(49), cannot be computed analytically.

The inverse time dependence of $\theta(t)$ is defined by Eq.(50):

$$
\begin{equation*}
t=\frac{m \alpha}{L_{0}} \int \frac{d \theta}{(1+\epsilon \cos (x \theta))^{2}} \tag{71}
\end{equation*}
$$

The integral can be solved analytically, giving

$$
\begin{equation*}
t=\frac{\alpha^{2} m}{x L_{0}\left(1-\epsilon^{2}\right)}\left(\frac{2}{\sqrt{1-\epsilon^{2}}} \tan ^{-1}\left(\frac{(1-\epsilon) \sin (x \theta)}{\sqrt{1-\epsilon^{2}}(1+\cos (x \theta))}\right)-\frac{\epsilon \sin (x \theta)}{1+\epsilon \cos (x \theta)}\right) . \tag{72}
\end{equation*}
$$

When the precession factor $x$ is used from the approximation (57), we obtain

$$
\begin{equation*}
x=1-\frac{\left(\epsilon^{2}+1\right) L_{0}^{2}}{2 \alpha^{2} c^{2} m^{2}} \tag{73}
\end{equation*}
$$

which can be inserted into (72). The result is shown in Fig. 4. Because $x$ depends on $L_{0}$, the angular periods increase when $L_{0}$ goes up. To make the effects visible we chose values for $L_{0}$ in the ultrarelativistic range.

The animation requires knowlegde of $\theta(t)$ and $r(t)$. To avoid the effort for calculating the inverse functions out of $t(\theta)$ and $t(r)$ we used a simple recursion scheme where $t$ can be chosen from a regular grid so that no interpolations are required. From Eq.(38) the scheme

$$
\begin{array}{r}
\theta_{1}=0 \\
r_{i}=\frac{\alpha}{1+\epsilon \cos \left(x \theta_{i}\right)}, \\
\theta_{i+1}=\frac{L_{0}}{m r_{i}^{2}} t_{i} \tag{76}
\end{array}
$$

is obtained. The animation can be downloaded from the AIAS web site. Some example graphics are presented in Figs. 5 to 7 . What not can be seen in the printed version is how the velocity of the orbiting mass changes with the distance to the center.


Figure 1: Apsidal angle $\psi$ for angular momenta $L 1=0.01, L 2=0.03, L 3=$ 0.05. Other parameters: $\alpha=c=m=1, \epsilon=0.3$.


Figure 2: Relativistic factor $\gamma\left(L_{0}\right)$ with parameters $\alpha=c=m=1, \epsilon=0.3$.


Figure 3: Time dependence $t(r)$ with parameters $\alpha=c=m=x=1, \epsilon=0.3$ for different $L_{0}$ 's.


Figure 4: Time dependence $t(\theta)$ with parameters $\alpha=c=m=1, \epsilon=0.3$ for different $L_{0}$ 's.


Figure 5: Animation example: closed orbit.


Figure 6: Animation example: open orbit.


Figure 7: Animation example: exotic orbit with $x=0.25$.

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