ECE THEORY OF SPECIFIC HEATS IN SOLIDS.

by

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ABSTRACT

The ECE theory of specific heats in solids is developed from the concept of curvature R in the ECE wave equation. The mean square photon mass is defined in terms of the quantized characteristics of the specific heat in the Einstein theory, and the mean square phonon mass similarly defined in the Debye theory. The curvature capacity and capcity of mean square photon mass are defined and the total energy density and number density of photons in black body radiation expressed in terms of mean square photon mass.

Keywords: ECE theory of specific heats in solids, mean square photon mass, mean square phonon mass, Einstein / Debye theory of specific heats, old quantum theory.

UFT 243

1. INTRODUCTION

During the course of development of ECE theory {1 - 10} the ECE wave equation has been derived from the fundamental tetrad postulate {11} of Cartan geometry and all the well known wave equations of physics derived therefrom. The concept of curvature R is an essential part of this derivation, and in previous work in papers UFT158 ff. has been applied to particle theory. The particle theory of standard physics was comprehensively refuted in UFT158 ff. at the foundational level and replaced by R theory based on the ECE wave equation. Einstein's theory of light deflection was thoroughly refuted in UFT150 ff., and it was shown that there were several basic errors in the Einstein theory of light deflection of a massless photon along a null geodesic. The correct theory of light deflection by gravitation requires identically non - zero photon mass. It was shown in 1992 (Omnia Opera of www.aias.us) that the B(3) field gives the first self consistent theory of the inverse Faraday effect, and shortly thereafter it was inferred that the B(3) field implies photon mass. In UFT158 ff. the concept of photon mass was used to refute the Einstein / de Broglie equations, and to refute the entire standard theory of particle physics at the basic level. The use of quantum electrodynamics and string theory does not answer this comprehensive refutation.

In Section 2, the concept of mean square photon mass is used to give a new explanation of the Einstein theory of specific heats, one of the cornerstones of the old quantum theory. The total energy density and number density of photons of black body radiation are expressed in terms of mean square photon mass. From that, the specific heat of a solid can also be expressed in terms of mean square photon mass in the Einstein theory and in terms of mean square phonon mass in the Debye theory. Mean square photon mass is directly related to the curvature R of the ECE wave equation. Finally in Section 3 the integrals needed in this theory are evaluated accurately by computer.

2. SPECIFIC HEATS AND MEAN SQUARE PHOTON AND PHONON MASS

In UFT158 ff on www.aias.us it was shown that the photon or quantum of energy is defined by:

$$E = \hbar\omega = \hbar c R^{1/2} \quad -(1)$$

where R is the curvature of the ECE wave equation {1 - 10}. Here $\hbar$ is the reduced Planck constant and c the velocity of light in the vacuum. The first three notes accompanying this paper (UFT243) on www.aias.us give full details. The mean curvature in the old quantum theory is therefore:

$$\langle R \rangle = \frac{1}{\hbar^2 c^2} \langle E^2 \rangle \quad -(2)$$

where the mean square energy $< E^2 >$ is defined by Boltzmann statistics as explained in full detail in the fourth note accompanying this paper. The mean square energy is defined by:

$$\langle E^2 \rangle = \frac{\sum_n E_n^2 \exp\left(-\frac{E_n}{kT}\right)}{\sum_n \exp\left(-\frac{E_n}{kT}\right)} \quad -(3)$$

and the square of energy is quantized by:

$$E^2 = 0,\ \hbar^2\omega^2,\ 2\hbar^2\omega^2,\ \ldots,\ n\hbar^2\omega^2. \quad -(4)$$

So the mean square energy of an ensemble of photons is:

$$\langle E^2 \rangle = \sum_n n\hbar^2\omega^2 x \Big/ \sum_n x \quad -(5)$$

where:

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(6)$$

Note that:

$$\sum_n x^n = (1-x)^{-1} = 1 + x + x^2 + \ldots \quad -(7)$$

and

$$\sum_n n x^n = x \frac{d}{dx} \sum_n x^n \quad -(8)$$

so the mean square energy is:

$$\langle E^2 \rangle = \hbar^2 \omega^2 x \frac{d}{dx} (1-x)^{-1} / (1-x)^{-1} = \hbar^2 \omega^2 \frac{x}{1-x} \quad -(9)$$

and the ensemble averaged or mean curvature is:

$$\langle R \rangle = \frac{\omega^2}{c^2} \left(\frac{x}{1-x} \right). \quad -(10)$$

The capacity of mean curvature is defined as:

$$C_v(R) = 3N \frac{d\langle R \rangle}{dT} = 3N \frac{\omega^2}{c^2} \frac{d}{dT} \left(\frac{x}{1-x} \right). \quad -(11)$$

The infinitesimal number density in photons per cubic metre is:

$$dN = \frac{\omega^2}{\pi^2 c^3} d\omega \quad -(12)$$

and the Planck distribution of the quantized square of energy is:

$$du^2 = \langle E^2 \rangle dN = \frac{\hbar^2 \omega^4}{\pi^2 c^3} \left(\frac{x}{1-x} \right) d\omega. \quad -(13)$$

The total energy squared of black body radiation is therefore:

$$u^2 = \int_0^\infty \frac{\hbar^2 \omega^4}{\pi^2 c^3} \left(\frac{x}{1-x} \right) d\omega. \quad -(14)$$

The mean energy in the Planck theory is:

$$\langle E \rangle = \hbar\omega \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1 \right)^{-1} \quad -(15)$$

so the mean energy and mean curvature are related by:

$$\langle E \rangle = \frac{\hbar c^2}{\omega} \langle R \rangle . \quad -(16)$$

From Eqs. (9) and (15):

$$\langle E^2 \rangle = \hbar\omega \langle E \rangle . \quad -(17)$$

The ECE wave equation {1 - 10} is:

$$(\square + R)\, q^a_\mu = 0 \quad -(18)$$

in which q^a_μ is the Cartan tetrad and in which the curvature is related to mass m by:

$$R = \left(\frac{mc}{\hbar}\right)^2 . \quad -(19)$$

Therefore the mean square photon mass is defined by:

$$\langle m^2 \rangle = \frac{\hbar\omega}{c^4} \langle E \rangle = \frac{1}{c^4} \left(\frac{\hbar^2 \omega^2}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \right) , \quad -(20)$$

In the older Einstein / de Broglie theory:

$$E = \hbar\omega = \gamma m c^2 \quad -(21)$$

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad -(22)$$

where γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(23)$$

and where the photon momentum $\underline{p}$ is:

$$\underline{p} = \hbar \underline{\kappa} \quad -(24)$$

with $\underline{\kappa}$ as the wavenumber. Eqs. (21) and (22) are refuted comprehensively in UFT160 ff. so must be replaced by concepts such as Eq. (20).

The density of states in the Planck Einstein theory is defined by:

$$\rho(\omega) = \frac{dU}{d\omega} = \frac{\hbar\omega^3}{\pi^2 c^3}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1} \quad -(25)$$

and is quantized as follows in terms of mean square photon mass:

$$\langle m^2 \rangle = \hbar \left(\frac{\rho(\omega)}{\omega c \pi^2}\right). \quad -(26)$$

From Eqs. (25) and (26):

$$\frac{dU}{d\omega} = \frac{\pi^2 \omega c}{\hbar}\langle m^2\rangle = \frac{\hbar\omega^3}{\pi^2 c^3}\left(e^{\hbar\omega/kT} - 1\right)^{-1}. \quad -(27)$$

The total energy density of black body radiation can therefore be expressed in terms of mean square photon mass as follows:

$$U = \frac{\pi^2 c}{\hbar}\int_0^\infty \omega \langle m^2\rangle \, d\omega. \quad -(28)$$

This integral is evaluated by computer in Section 3. Atkins {12} claims that:

$$U = \left(\frac{\pi^2 k^4}{15 c^3 \hbar^3} \right) T^4 \quad -(29)$$

but this result is not consistent with computer algebra as explained in the following Section 3. There appear to be several errors like this in ref. (12).

The mean energy of the Planck distribution is:

$$\langle E \rangle = \frac{c^4}{\hbar \omega} \langle m^2 \rangle \quad -(30)$$

so the heat capacity of the Einstein solid can be expressed in terms of mean square photon mass as follows:

$$C_v = 3N \frac{d\langle E \rangle}{dT} = \frac{3Nc^4}{\hbar \omega} \frac{d\langle m^2 \rangle}{dT} \quad -(31)$$

in which the capacity of mean square photon mass is:

$$\frac{d\langle m^2 \rangle}{dT} = \frac{(\hbar\omega)^3}{(kT)^2} \frac{k}{c^4} \frac{e^{\hbar\omega/kT}}{\left(1 - e^{\hbar\omega/kT}\right)^2} \quad -(32)$$

Consider the density of states, Eq. (25), and the infinitesimal of energy density in the Planck distribution:

$$dU = \langle E \rangle dN. \quad -(33)$$

From Eqs. (25) and (33):

$$dU = \rho(\omega) d\omega = \hbar\omega \, dN \quad -(34)$$

and

$$dN = \frac{\rho(\omega)}{\hbar\omega}\, d\omega \qquad -(35)$$

for one photon, in which

$$\langle E \rangle = \hbar\omega \qquad -(36)$$

It follows that:

$$dN = \frac{1}{\pi^2 c^3}\left(\frac{\omega^2}{e^{\hbar\omega/kT} - 1}\right) \qquad -(37)$$

and the number density of photons (number of photons per cubic metre) in black body radiation is:

$$N = \frac{1}{\pi^2 c^3}\int_0^\infty \left(\frac{\omega^2}{e^{\hbar\omega/kT} - 1}\right) d\omega \qquad -(38)$$

It follows that:

$$N = \frac{c}{\pi^2 \hbar^2}\int_0^\infty \langle m^2 \rangle\, d\omega. \qquad -(39)$$

The integral in Eq. (38) is evaluated by computer in Section 3, and again there is a discrepancy with the claim by Atkins {12}.

The Debye theory of specific heats in solids is described in detail in the eighth note accompanying this paper and is the solid state equivalent of the Planck law of black body radiation as is well known. It is often known as the "Debye correction" of the Einstein theory. The energy density of phonons in the Debye theory is:

$$\frac{U}{V} = \frac{\hbar F}{8\pi^3}\int_0^{\omega_m} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}\, d\omega \qquad -(40)$$

where ω_m is a maximum frequency chosen so that there are 3N vibrational states in the solid and where F is a coefficient of the theory. Therefore by direct analogy with the photon theory of this section, the mean square phonon mass is defined by:

$$\langle m^2 \rangle_{phonon} = \frac{\hbar^2}{c^4} \left(\frac{\omega^2}{e^{\hbar\omega/kT} - 1} \right) \quad -(41)$$

3. NUMERICAL EVALUATION OF INTEGRALS

Section by D. Horst Eckardt.

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3 Numerical evaluation of integrals

First we consider a numerical example of the temperature dependence of specific heat capacity C_v. This is given from statistical mechanics to be

$$C_v = \frac{3\,\hbar^2\,\omega^2\,N\,e^{\frac{\hbar\,\omega}{k\,T}}}{k\,T^2\left(e^{\frac{\hbar\,\omega}{k\,T}}-1\right)^2} \tag{42}$$

for an energy quantum

$$E = \hbar\,\omega. \tag{43}$$

When we replace the energy term by it's mass equivalent of de Broglie theory,

$$E = \gamma m c^2, \tag{44}$$

with the relativistic velocity factor γ defined by Eq.(23), we obtain an expression of C_v in dependence of the ratio v/c of the particle with mass m:

$$C_v = \frac{3\,c^4\,m^2\,N\,\gamma^2\,e^{\frac{\gamma\,m\,c^2}{k\,T}}}{k\,T^2\left(e^{\frac{\gamma\,m\,c^2}{k\,T}}-1\right)^2}. \tag{45}$$

This function is graphed in Fig. 1 for several ratios v/c with unity constants otherwise. The form of the resulting curves is known from the standard literature but parametrized by v/c here. One can see that curves deviating significantly from the non-relativistic case require a condition of about $v/c > 0.5$ up to the ultrarelativistic case.

Next we investigate evaluation of the intergral (28), the total energy density of black body radiation. From Eqs.(25-28) we have to solve the integral

$$U = \int_0^\infty \frac{\hbar\,\omega^3}{c^3\,\pi^2\,\left(e^{\frac{\hbar\,\omega}{k\,T}}-1\right)} d\omega. \tag{46}$$

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The computeralgebra program Maxima gives the following result for the indefinite integral:

$$U_{ind} = \frac{6\,k^4\,T^4}{c^3\pi^2\hbar^3}\left(li_4\left(e^{\frac{\hbar\,\omega}{k\,T}}\right) - \frac{\hbar\,\omega\,li_3\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{k\,T} + \frac{\hbar^2\,\omega^2\,li_2\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{2\,k^2\,T^2} + \frac{\hbar^3\,\omega^3\log\left(1-e^{\frac{\hbar\,\omega}{k\,T}}\right)}{6\,k^3\,T^3}\right) - \frac{\hbar\omega^4}{4c^3\pi^2}. \tag{47}$$

The function $li_n(z)$ is the polylogarithm [13] which should not be confused with polylogarithmic functions nor with the offset logarithmic integral which has a similar notation. The polylogarithm can be defined by a series expansion

$$li_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}. \tag{48}$$

For $n = 1$ it takes the simple form

$$li_1(z) = -\log(1-z). \tag{49}$$

z may be a complex number, and n can be any positive or negative integer. There is a number of theorems concerning this function [13] but there is no closed representation for $n > 1$. Some examples for polylogarithms are graphed in Fig. 2.

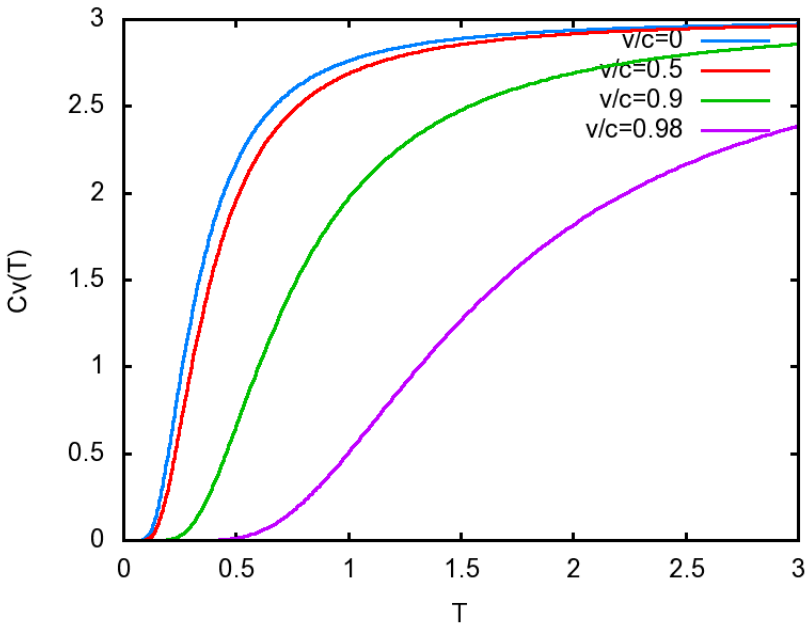

Figure 1: Function $C_v(T)$ for different ratios v/c.

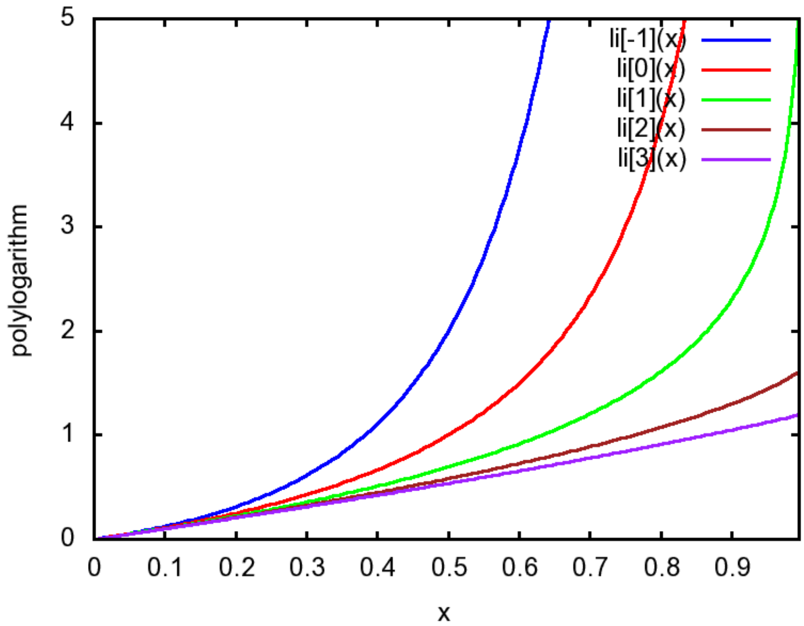

Figure 2: Some examples for the polylogarithm with real valued argument.

The result sought for U is the definite integral

$$U = \frac{\hbar}{c^3\pi^2} \lim_{\omega\to\infty} \left(\frac{6\,k^4\,T^4\,li_4\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar^4} - \frac{6\,k^3\,\omega\,T^3\,li_3\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar^3} \right.$$
$$\left. + \frac{3\,k^2\,\omega^2\,T^2\,li_2\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar^2} + \frac{k\,\omega^3\,T\log\left(1-e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar} - \frac{\omega^4}{4} - \frac{\pi^4\,k^4\,T^4}{15\,\hbar^4} \right). \tag{50}$$

This seems to reach the computational limits of Maxima. In particular the second to last term $\omega^4/4$ diverges at the upper limit. The argument of the polylogarithm should be smaller than unity which can only be assured by negative "binding energies"

$$\hbar\,\omega \to -\hbar\,\omega. \tag{51}$$

The last term of U is identical to that given for this integral in the standard literature, for example in [12]:

$$U = \frac{\pi^2\,k^4}{15\,c^3\,\hbar^3} T^4. \tag{52}$$

However it remains unclear how the other terms of (50) can be assumed to vanish.

A similar problem arises for the number density of photons, Eq.(38):

$$N = \frac{1}{c^3\pi^2} \int_0^\infty \frac{\omega^2}{e^{\frac{\hbar\,\omega}{k\,T}} - 1} d\omega. \tag{53}$$

The definite integral is according to Maxima

$$N = \frac{1}{c^3 \pi^2} \lim_{\omega \to \infty} \left(-\frac{2\,k^3\,T^3\,li_3\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar^3} + \frac{2\,k^2\,\omega\,T^2\,li_2\left(e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar^2} + \frac{k\,\omega^2\,T\log\left(1-e^{\frac{\hbar\,\omega}{k\,T}}\right)}{\hbar} - \frac{\omega^3}{3} + \frac{2\,\zeta\left(3\right)\,k^3\,T^3}{\hbar^3} \right) \tag{54}$$

where $\zeta(x)$ is the Riemann zeta function with $\zeta(3) \approx 1.202$. If the above approximations for the energy density can be applied here as well, the result will be

$$N = \frac{2\,\zeta\left(3\right)\,k^3\,T^3}{\pi^2\,c^3\,\hbar^3}. \tag{55}$$

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