VECTORIAL ANALYSIS OF THE CARTAN AND EVANS IDENTITIES AND THE ECE FIELD EQUATIONS.

by

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ABSTRACT

The Cartan and Evans identities of differential geometry are developed in vectorial format and a Hodge dual analysis applied to the result. It is shown that the vector curl in four dimensions is the curl in three dimensions and its Hodge dual. The antisymmetrized tensor product of a one form and a two form is expressed as a vector equation that greatly clarifies its meaning. The duality transformation of electrodynamics is used to show that the electric field strength can be expressed as the Hodge dual of the vector curl of potential. These results are extended to ECE theory and it is confirmed that the ECE equations give spin connection resonance.

Keywords: ECE theory, Cartan and Evans identities as vector identities, Hodge dual analysis, duality transformation of the field equations.

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1. INTRODUCTION

In this series of two hundred and fifty four papers to date {1 - 10} it has been shown that a unified field theory can be developed straightforwardly on the basis of general relativity with torsion. The geometry chosen is standard Cartan geometry {11}, defined by the two Cartan Maurer structure equations and the Cartan identity. The Evans identity has been shown to be the Cartan identity for Hodge duals of two forms in the four dimensions of spacetime. The structure equations define the relation between field and potential, and the Cartan and Evans identities define respectively the homogeneous and inhomogeneous field equations both of electromagnetism and of gravitation. These results are described in detail in the ECE Engineering Model on www.aias.us. Major discoveries of ECE theory include the fact that omission of torsion refutes the entire era of general relativity known as Einsteinian general relativity, so for example there are no black holes and there was no big bang. These conclusions are obvious from the geometry once it is realized that torsion is as fundamental as curvature. Another major conclusion of ECE theory is that exists a gravitomagnetic field.

These conclusions have been accepted by a vast readership of ECE theory that has been monitored systematically for nearly a decade. The standard model of physics has been discarded as essentially meaningless with the exception of a few of its equations. This change of thought is known as the post Einsteinian paradigm shift. The Higgs boson has been shown to be meaningless, and its application to electroweak theory shown to be wildly erroneous. Therefore physics has split into two entirely different subjects, the mathematically correct ECE theory, and the mathematically incorrect and obsolete standard physics. Since ECE theory is based directly on Cartan geometry, the former cannot be refuted mathematically simply because Cartan geometry is precise and elegant, and of course, unrefuted within its definitions, the structure equations and identities.

In Section 2 it is shown that the Cartan structure equation can be simplified and greatly clarified by use of Heaviside Gibbs vector analysis. The Cartan equations are elegant and precise, but are too abstract for immediate application to physics and engineering. For practical applications they must be developed in tensor notation, which is not used very much by engineers or chemists, and then developed in vector notation. Engineers use vector notation almost exclusively. Tensor notation is used by some chemists and physicists. The ECE equations in vector notation are the most useful, notably for spin connection resonance {1 - 10}, and are given in the ECE engineering model on www.aias.us. In section 2 it is shown that vector notation greatly clarifies the differential form notation. It is shown for example that the vector curl in four dimensions is the vector curl in three dimensions together with its Hodge dual. The Cartan identity is shown to be a well known vector identity, and it is shown that this identity gives self consistent results. The complexity of the antisymmetrized tensor products each side of the Cartan identity is greatly simplified and clarified by use of vector notation. Finally the Hodge dual method is applied to the vector analysis and is developed with the duality transformation of electromagnetism.

In Section 3 some other aspects of the vectorial development of Cartan geometry and ECE theory are discussed by Douglas L. Lindstrom and Horst Eckardt.

2. DEVELOPMENT OF THE VECTOR ANALYSIS

The first Cartan Maurer structure equation can be written {1 - 10} in its most concise and abstract format as follows:

$$T = D \wedge q = d \wedge q + \omega \wedge q \qquad -(1)$$

which is short hand for the usual form notation {11}:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \qquad -(2)$$

In tensor notation Eq. (2) is

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(3)$$

which gives two vector equations (see ECE Engineering Model) defining respectively the spin torsion:

$$\underline{T}^a_{spin} = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \quad -(4)$$

and the orbital torsion:

$$\underline{T}^a_{orbital} = -\underline{\nabla} q^a_0 - \frac{1}{c}\frac{\partial \underline{q}^a}{\partial t} - \omega_0{}^a{}_b \underline{q}^b + \underline{\omega}^a{}_b q^b_0 . \quad -(5)$$

Here T denotes the torsion, q denotes the Cartan tetrad, ω denotes the spin connection, D ^ denotes the covariant wedge product and d ^ the wedge product of differential geometry. This notation is explained in comprehensive detail in the ECE literature.

Similarly the second Cartan Maurer structure equation is:

$$R = D \wedge \omega = d \wedge \omega + \omega \wedge \omega \quad -(6)$$

which is short hand notation for:

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \quad -(7)$$

In tensor notation Eq. (7) becomes:

$$R^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{\nu b} - \partial_\nu \omega^a{}_{\mu b} + \omega^a{}_{\mu c}\omega^c{}_{\nu b} - \omega^a{}_{\nu c}\omega^c{}_{\mu b} \quad -(8)$$

which again denotes two vector equations, for the spin curvature:

$$\underline{R}^a{}_{b\,spin} = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(9)$$

and the orbital curvature:

$$\underline{R}^a{}_{b\,\text{orbital}} = -\underline{\nabla}\, \omega^a{}_{0b} - \frac{1}{c}\frac{\partial \underline{\omega}^a{}_b}{\partial t} - \omega^a{}_{0c}\, \underline{\omega}^c{}_b + \underline{\omega}^a{}_c\, \omega^c{}_{0b}. \quad -(10)$$

The Cartan identity is in shorthand notation:

$$D \wedge T := R \wedge q \quad -(11)$$

which in standard differential form notation {1 - 11} is:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b. \quad -(12)$$

On both sides of this equation there are antisymmetrized tensor products of a one form and a two form {11}. So in tensor notation the identity is the cyclic sum:

$$\partial_\mu T^a{}_{\nu\rho} + \partial_\rho T^a{}_{\mu\nu} + \partial_\nu T^a{}_{\rho\mu} + \omega^a{}_{\mu b} T^b{}_{\nu\rho} + \omega^a{}_{\rho b} T^b{}_{\mu\nu} + \omega^a{}_{\nu b} T^b{}_{\rho\mu} := q^b{}_\mu R^a{}_{b\nu\rho} + q^b{}_\rho R^a{}_{b\mu\nu} + q^b{}_\nu R^a{}_{b\rho\mu} \quad -(13)$$

The Evans identity is in shorthand notation:

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(14)$$

which gives the antisymmetrized cyclic sum:

$$d \wedge \tilde{T}^a + \omega^a{}_b \wedge \tilde{T}^b := \tilde{R}^a{}_b \wedge q^b \quad -(15)$$

where the tilde denotes Hodge dual {1 - 11}. The Evans identity follows from the fact that the Hodge dual of a two form in four dimensions is also a two form.

Consider the wedge product:

$$A_\mu \wedge D_{\nu\rho} = A_\mu D_{\nu\rho} + A_\rho D_{\mu\nu} + A_\nu D_{\rho\mu} \quad -(16)$$

where:

$$D_{\nu\rho} = B_\nu C_\rho - B_\rho C_\nu \quad -(17)$$
$$D_{\mu\nu} = B_\mu C_\nu - B_\nu C_\mu \quad -(18)$$
$$D_{\rho\mu} = B_\rho C_\mu - B_\mu C_\rho \quad -(19)$$

For space like indices there are terms such as :

$$A_1 \wedge D_{23} = A_1 (B_2 C_3 - B_3 C_2) = A_1 D^1 \quad -(20)$$

so for spacelike indices the product (16) translates as follows into vector notation:

$$A \wedge D \rightarrow \underline{A} \cdot \underline{B} \times \underline{C} . \quad -(21)$$

The spacelike part of the Cartan identity is the spin part, or in electromagnetism the magnetic part. It is shown as follows that the spin Cartan identity reduces to a simple vector identity.

For spin torsion the Cartan identity is:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{q}^b \cdot \left(\underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right)$$

i.e.:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a - \underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b + \underline{\omega}^a{}_b \cdot \left(\underline{\nabla} \times \underline{q}^b - \underline{\omega}^a{}_c \times \underline{q}^c \right)$$
$$= \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(21)$$

Now use:

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{\omega}^a{}_b \cdot \underline{\omega}^a{}_c \times \underline{q}^c \quad -(22)$$

and:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q}^a = 0 \quad -(23)$$

to find that:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{q}^c = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(24)$$

which is the vector identity:

$$\underline{\nabla} \cdot \underline{F} \times \underline{G} = \underline{G} \cdot \underline{\nabla} \times \underline{F} - \underline{F} \cdot \underline{\nabla} \times \underline{G} \quad -(25)$$

Q. E. D.

The structure of the homogeneous field equation in ECE theory is {1 - 10}:

$$\underline{\nabla} \cdot \left(\underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c \right) = 0 \quad -(26)$$

It follows immediately that:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{q}^c = 0. \quad -(27)$$

From Eqs. (21) and (26):

$$\underline{\omega}^a{}_b \cdot \left(\underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c \right) = \underline{q}^b \cdot \left(\underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right) \quad -(28)$$

i. e. :

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b = \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(29)$$

and Eq. (27) follows again, Q. E. D..

The magnetic flux density in the ECE engineering model is defined by {1 - 10}:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(30)$$

and Eq. (27) in ECE electromagnetism becomes:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{A}^c = 0 \quad -(31)$$

so it follows that:

$$\underline{\nabla} \cdot \underline{B}^a = 0. \quad -(32)$$

There is no magnetic monopole, and self consistently this was the assumption that led to Eq. (26). This is another successful test of the self consistency of ECE theory.

The electric field strength in the ECE engineering model is defined by:

$$\underline{E}^a = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} - c\omega^a{}_{0b}\underline{A}^b + \phi^b \underline{\omega}^a{}_b \quad -(33)$$

i.e. by the orbital torsion. The electromagnetic potential is the tetrad:

$$A^a_\mu = \left(\frac{\phi^a}{c}, -\underline{A}^a \right) \quad -(34)$$

where ϕ^a is the scalar potential and A the vector connection. The spin connection is defined by:

$$\omega^a_{\mu b} = \left(\omega^a{}_{0b}, -\underline{\omega}^a{}_b \right). \quad -(35)$$

It is convenient to express the electric field strength in terms of the Hodge dual in order to develop the Cartan identity in vector format for orbital torsion as well as spin torsion. The background for this development is given in notes 254(5) and 254(6). The Hodge dual of the curl of the vector potential is defined by:

$$\partial^0 A^1 - \partial^1 A^0 = \epsilon^{0123}\left(\partial_2 A_3 - \partial_3 A_2\right) \quad -(36)$$

$$\partial^0 A^2 - \partial^2 A^0 = \epsilon^{0213}\left(\partial_1 A_3 - \partial_3 A_1\right) \quad -(37)$$

$$\partial^0 A^3 - \partial^3 A^0 = \epsilon^{0312}\left(\partial_1 A_2 - \partial_2 A_1\right) \quad -(38)$$

in which:

$$\epsilon^{0123} = -\epsilon^{0213} = \epsilon^{0312} = 1 \quad -(39)$$

so the Hodge dual of the vector curl is:

$$(\underline{\nabla} \times \underline{A})_{HD} = (\partial^0 A^1 - \partial^1 A^0)\underline{i} + (\partial^0 A^2 - \partial^2 A^0)\underline{j} + (\partial^0 A^3 - \partial^3 A^0)\underline{k} \quad -(40)$$

if the original vector curl is:

$$\underline{\nabla} \times \underline{A} = (\partial_2 A_3 - \partial_3 A_2)\underline{i} - (\partial_1 A_3 - \partial_3 A_1)\underline{j} + (\partial_1 A_2 - \partial_2 A_1)\underline{k} \quad -(41)$$

On the U(1) level (Maxwell Heaviside theory) the electric field strength is therefore defined

by the Hodge dual curl multiplied by the speed of light c in S. I. Units:

$$\underline{E} = -c(\underline{\nabla} \times \underline{A})_{HD} \quad -(42)$$

and the magnetic flux density by the original curl:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(43)$$

In ECE theory the electric field strength is defined by the Hodge dual format:

$$\underline{E}^a = -c(\underline{\nabla} \times \underline{A})^a_{HD} - (\underline{\omega}^a{}_b \times \underline{A}^b)_{HD} \quad -(44)$$

in which the definition (33) is recovered by:

$$-c(\underline{\nabla} \times \underline{A})^a_{HD} = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} \quad -(45)$$

and

$$-(\underline{\omega}^a{}_b \times \underline{A}^b)_{HD} = -c\omega^a{}_{0b}\underline{A}^b + \phi^b \underline{\omega}^a{}_b \quad -(46)$$

The Hodge dual format is especially useful for the free electromagnetic field. On the

U(1) level the four equations of the free field are well known:

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad -(47)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \qquad -(48)$$

$$\qquad -(49)$$

$$\underline{\nabla} \cdot \underline{E} = 0$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0}. \qquad -(50)$$

It is also well known {1 - 10} that they remain the same under the duality transform:

$$\underline{E} \rightarrow ic\underline{B}\ , \ \underline{B} \rightarrow \frac{1}{ic}\underline{E}. \qquad -(51)$$

Therefore under the duality transform:

$$\underline{E} \rightarrow ic\underline{B} = ic\underline{\nabla} \times \underline{A} = c\underline{\nabla} \times (i\underline{A}) \qquad -(52)$$

$$\underline{B} \rightarrow \frac{1}{ic}\underline{E} = \frac{1}{ic}\left(-\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}\right). \qquad -(53)$$

From Eqs. (42) and (52) the electric field strength of the free field is:

$$\underline{E} = -c\left(\underline{\nabla} \times \underline{A}\right)_{HD} = c\underline{\nabla} \times (i\underline{A}) \qquad -(54)$$

showing that the Hodge dual of the curl in this case is defined by the original curl with $\underline{A}$ replaced by $i\underline{A}$.

In ECE theory the duality transform of the free field produces:

$$\underline{E}^a = c\left(\underline{\nabla} \times (i\underline{A}^a) - \underline{\omega}^a{}_b \times (i\underline{A}^b)\right) \qquad -(55)$$

It follows that for the underlying orbital torsion of the free field is defined by:

$$\underline{T}^a_{orbital} = \underline{\nabla} \times (i\underline{q}^a) - \underline{\omega}^a{}_b \times (i\underline{q}^b) \qquad -(56)$$

and is generated from the spin torsion by replacing the tetrad vector $\underline{q}^a$ by $i\underline{q}^a$. It follows that Eqs. (24) and (27) also hold for the electric component of the free field. Finally, when there is interaction of the field with matter the duality transform must be augmented to:

$$\mathbf{E} \rightarrow ic\mathbf{B}, \quad \rho \rightarrow 0, \quad \mathbf{J} \rightarrow \mathbf{0} \qquad (57)$$

$$\mathbf{B} \rightarrow \frac{1}{ic}\mathbf{E}, \quad 0 \rightarrow \rho, \quad \mathbf{0} \rightarrow \mathbf{J} \qquad (58)$$

3. OTHER ASPECTS OF THE VECTOR ANALYSIS OF CARTAN GEOMETRY

Section by Horst Eckardt and Doug Lindstrom.

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