MAIN EQUATIONS OF THE SIMPLIFIED ECE THEORY AND
INTERPRETATION OF THE SPIN CONNECTION

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ABSTRACT

The main equations of the ECE engineering model are given, and simplified to give a workable theory for engineers. It is shown that the spin connection of the vacuum plane wave in the simplified theory is the photon energy / momentum within $\mathcal{K}$. This shows that the ECE theory is a theory of general relativity that is able to account for both the plane wave and the photon from pure geometry.

Keywords: ECE theory, simplified engineering model, spin connection as photon energy momentum.

UFT 256
1. INTRODUCTION

In the immediately preceding paper of this series of two hundred and fifty six papers and several monographs to date \(\{1 - 10\}\) it has been shown that the Cartan identity of differential geometry \(\{11\}\) reduces to a simple vector equation in the absence of a magnetic monopole. In Section 2 a simplified ECE theory is developed in which a vector equation is used to derive the spin connection four vector of a vacuum plane wave and to show that it is the energy / momentum four vector of the photon within \(\hbar\). The ECE theory succeeds elegantly in deriving the photon from the plane wave, whereas the standard physics looks at the wave and photon as being the results of two different theories entirely. The connection in general relativity describes the way in which a frame of reference moves. The frame becomes a dynamic entity. In the obsolete Maxwell Heaviside (MH) field theory of electromagnetism of the nineteenth century the field was distinct from the static frame of reference. In ECE theory, electromagnetism becomes part of a perfectly consistent unified field theory based directly on pure geometry, a geometry taught in all good universities and therefore universally accepted, Cartan geometry. As in all theories of general relativity, the field becomes the moving frame, described by a geometrical connection. The result \(\{1 -10\}\) is that both the electromagnetic and gravitational fields are described by field equations which have the same basic structure. In Section 2 these are written out in vector notation by translating the differential form and tensor notations of the Cartan and Evans identities. Both the magnetic and electric charge current densities develop a well defined geometrical structure which ultimately governs the internal structure of the electron and the magnetic monopole, if the latter is observed experimentally in a reproducible and repeatable manner.
2. FIELD EQUATIONS AND SIMPLIFIED VERSION.

The field equations of ECE unified field theory are based on the Cartan identity:

\[ D \wedge T = d \wedge T + \omega \wedge T : = R \wedge \sigma_v - (1) \]

and the Evans identity:

\[ D \wedge \tilde{T} = d \wedge \tilde{T} + \omega \wedge \tilde{T} : = \tilde{R} \wedge \sigma_v - (2) \]

of differential geometry. In this concise notation \( T \) denotes torsion, \( R \) curvature, and \( D \wedge \) the covariant wedge product. The tilde denotes Hodge duality \{1 - 11\}, and \( \sigma \) denotes the Cartan tetrad. In the four dimensions of spacetime, \( T \) and \( R \) are antisymmetric tensors, whose Hodge duals are also antisymmetric tensors. So in four dimensions the Evans identity is an example of the Cartan identity. Written out in more detail, eqs. (1) and (2) become:

\[ d \wedge T^a + \omega^a_b \wedge T^b : = R^a_b \wedge \sigma_v^b - (3) \]

and

\[ d \wedge \tilde{T}^a + \omega^a_b \wedge \tilde{T}^b : = \tilde{R}^a_b \wedge \sigma_v^b - (4) \]

respectively. The \( a \) and \( b \) indices were originally devised by Cartan to denote a Minkowski spacetime tangential at point \( P \) to a base manifold \{1\}, denoted by \( \mu \) and \( \nu \). In applications to electromagnetism within ECE unified field theory, \( a \) and \( b \) are indices of the complex circular basis \((1), (2), (3)\) of three dimensional space \{1 - 10\}. It will be shown later in this section that they can be removed entirely without loss of rigour, and the theory greatly simplified for engineering applications. In Eqs. (3) and (4) \( \omega_{\mu b}^a \) denotes the spin connection, which defines the way in which the frame denoted \((1), (2), (3)\) rotates.
and translates. As shown in previous papers Eqs. 3 and 4 can be written as:

\[ \partial_{\mu} T^{a \mu} + \omega^{a}_{\mu b} T^{b \mu} := R^{a \mu} - (5) \]

and

\[ \partial_{\mu} T^{a \mu} + \omega^{a}_{\mu b} T^{b \mu} := R^{a \mu} - (6) \]

respectively in tensor notation. Eq. (5) gives the homogeneous field equations, and Eq. (6) gives the inhomogeneous field equations both of electromagnetism and gravitation.

These are summarized in comprehensive detail in the ECE engineering model {1-10).

The ECE hypothesis is:

\[ A^{a}_{\mu} = A^{(0)}_{\mu} + A^{a}_{\mu} - (7) \]

and transforms the pure geometry directly into field equations of physics. In Eq. (7) \( A^{a}_{\mu} \) is the electromagnetic four potential in which \( A^{(0)}_{\mu} \) is a scalar magnitude. Therefore:

\[ A^{a}_{\mu} = (A^{a}_{\mu}, -A^{a}) = \left( \frac{\phi^{a}}{c}, -A^{a} \right) - (8) \]

in covariant notation and:

\[ A^{a}_{\mu} = (A^{a}_{\mu}, A^{a}) = \left( \frac{\phi^{a}}{c}, A^{a} \right) - (9) \]

in contravariant notation, where \( \phi^{a} \) is the scalar potential and \( A^{a} \) is the vector potential. As shown in the preceding paper the ECE hypothesis means that the electric field strength in volts per metre is defined as being proportional to the orbital part of torsion:

\[ E^{a} = c A^{(0)}_{\mu} \Omega^{a} (\text{orb}) - (10) \]

and that the magnetic flux density in tesla is defined as being proportional to the spin part of
torsion:

\[
\mathcal{B}^a = A^{(0)} T^a (\text{spin}) \quad - (11)
\]

These definitions come from the first Cartan Maurer structure equation of differential geometry \{1 - 11\}. As in note 256(3) the tensorial format of the homogeneous field equation is:

\[
D \mu F_{\mu \nu} = d \mu F_{\mu \nu} + \omega_{\mu \nu} = A^{(0)} R_{\mu} a \ w^a - (12)
\]

where the covariant four derivative is:

\[
\mathcal{D}_{\mu} = \left( \frac{1}{c} \frac{d}{dt}, \nabla \right) - (13)
\]

and where the covariant spin connection four vector is:

\[
\omega^a_{\mu \nu} = \left( \omega^a_{\nu \mu}, -\omega^a_{\mu \nu} \right) - (14)
\]

Note carefully that there is a sign change between Eqs. \(13\) and \(14\) from basic definitions in relativity theory. The Hodge dual of the electromagnetic field tensor is defined for each index \(a\) by:

\[
F^a = \begin{bmatrix}
0 & -cB_x & -cB_y & -cB_z \\
cB_x & 0 & E_z & -E_y \\
cB_y & -E_z & 0 & E_x \\
cB_z & E_y & -E_x & 0
\end{bmatrix} - (15)
\]

As described in complete detail in note 256(3) the tensorial format \(12\) translates into two vectorial field equations. In deriving these equations the Hodge dual of the curvature tensor is defined for each \(a\) and \(b\) by spin and orbital components of curvature defined in a precisely analogous manner to the field tensor \(15\). So the antisymmetric curvature matrix can be defined as follows for each \(a\) and \(b\):
The right hand sides define respectively the magnetic charge (or monopole) density and the magnetic current density. The experimental evidence for these is still not definitive, but pure geometry and general relativity allow them to exist. They are entirely different in concept from both the Dirac and 't Hooft Polyakov theorems of magnetic charge current density. In the assumed absence of magnetic charge current density:

\[ \nabla \cdot B^a = \frac{\rho^m}{\varepsilon_0 c} = \omega^{a b} \cdot B^b - A^b \cdot R^a_{\, b} (\text{spin}) \]  

and:

\[ \frac{dB^a}{dt} + \nabla \times E^a = \frac{J^m}{\varepsilon_0} - c \left( A^b \times R^a_{\, b} (\text{orb}) - A^b R^a_{\, b} (\text{spin}) \right) \]

The right hand sides define respectively the magnetic charge (or monopole) density and the magnetic current density. The experimental evidence for these is still not definitive, but pure geometry and general relativity allow them to exist. They are entirely different in concept from both the Dirac and 't Hooft Polyakov theorems of magnetic charge current density. In the assumed absence of magnetic charge current density:

\[ \omega^{a b} \cdot B^b = A^b \cdot R^a_{\, b} (\text{spin}) \]

and

\[ \omega^{a b} \times E^b = c \left( A^b \times R^a_{\, b} (\text{orb}) - A^b R^a_{\, b} (\text{spin}) \right) \]

which are two field equations. They imply the familiar Gauss law of magnetism:

\[ \nabla \cdot B^a = 0 \]

and the Faraday law of induction:
for each a index.

As in note 256(4) the tensorial format of the inhomogeneous field equation is:

$$D_{\mu} F^{\alpha \mu} = \partial_{\mu} F^{\alpha \mu} + \omega_{\mu b} F^{b \mu} = A^{(0)} R^{\alpha \mu} - (23)$$

in which the field tensor for each a is defined as:

$$F^{\mu \nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix} - (24)$$

and in which the curvature tensor for each a and b is defined as:

$$R^{\mu \nu} = \begin{bmatrix} 0 & -R_x(\alpha \beta) & -R_y(\alpha \beta) & -R_z(\alpha \beta) \\ R_x(\alpha \beta) & 0 & -R_z(\gamma \pi) & R_y(\gamma \pi) \\ R_y(\alpha \beta) & R_z(\gamma \pi) & 0 & -R_x(\gamma \pi) \\ R_z(\alpha \beta) & -R_y(\gamma \pi) & R_x(\gamma \pi) & 0 \end{bmatrix} - (25)$$

As explained in all detail in note 256(4), the tensorial equation (23) gives two vector equations:

$$\nabla \cdot E^a = \rho^a = \omega_{a b} \cdot E^b - c A^b \cdot R^{a b} (\gamma \pi) - (26)$$

and

$$\nabla \times B^a - \frac{i}{c^2} \frac{dE^a}{dt} = \mu_0 J^a = \omega_{a b} b \times B^b + \omega_{b c} E^b - A^b \times R^{a b} (\gamma \pi) - (27)$$

Eq. (26) defines the electric charge density:

$$\rho^a = \rho_0 (\omega_{a b} b \cdot E^b - c A^b \cdot R^{a b} (\gamma \pi)) - (28)$$
and Eq. \((28)\) defines the electric current density:

\[
\mathbf{J}^a = \frac{1}{\mu_0} \left( \mathbf{A}^a \times \mathbf{E}^b + c \frac{\partial \mathbf{A}^a}{\partial t} \mathbf{E}^b - \left( \mathbf{A}^b \times \mathbf{R}^a_{\ b} (\text{spin}) + A^b R^a_{\ b} (\text{curvature}) \right) \right) - (29)
\]

With these definitions these equations become for each index \(a\) the Coulomb law:

\[
\nabla \cdot \mathbf{E}^a = \rho^a / \epsilon_0 \quad - (30)
\]

and the Ampere Maxwell law:

\[
\nabla \times \mathbf{B}^a - \frac{1}{c^2} \frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}^a \quad - (31)
\]

However, the presence of the spin connection, curvature and torsion means that these familiar looking laws are now equations of general relativity, and not the Maxwell Heaviside (MH) equations of special relativity. In MH theory there is no spin connection, and the nineteenth century electromagnetic field was an entity distinct from, and superimposed on, a static frame of reference. There was no concept of torsion or curvature in MH theory. In ECE theory the field is the frame itself within \(A\), in other words the field is pure geometry within a factor \(A\). The dynamic frame of reference simultaneously translates and rotates, and this motion is defined by the spin connection in the following proof.

In the absence of a magnetic monopole it was shown in the preceding paper, UFT255, that the spacelike component of the Cartan identity reduces to a simple vector equation:

\[
\nabla \cdot \mathbf{\omega}^a_{\ b} \times \mathbf{\omega}^b = 0 \quad - (32)
\]

from which it follows that:

\[
\nabla \cdot \nabla \times \mathbf{\omega}^a = \nabla \cdot \mathbf{\omega}^a_{\ b} \times \mathbf{\omega}^b = 0 \quad - (33)
\]
In the complex circular basis:

\[ \mathbf{a} \times \mathbf{b} = -i \omega \cdot \mathbf{a} \times \mathbf{b} \]  

For the vacuum plane wave it has been shown {1 - 10} that the tetrad vectors are:

\[ \mathbf{a}^{(1)} = \frac{1}{\sqrt{2}} (i - i^*) \exp(i \phi) \]  
\[ \mathbf{a}^{(2)} = \frac{1}{\sqrt{2}} (i + i^*) \exp(-i \phi) \]  
\[ \mathbf{a}^{(3)} = \mathbf{k} \]  

where the electromagnetic phase is:

\[ \phi = \omega t - \mathbf{k} \cdot \mathbf{Z} \]  

Here \( \omega \) is the angular frequency at point \( t \) and \( \mathbf{k} \) is the wave vector at point \( Z \). The phase causes the frame itself to rotate and translate, so this process must be defined in general relativity by a geometrical connection, in this case the spin connection. With these definitions the complex circular basis is defined by the O(3) symmetry equations:

\[ \mathbf{a}^{(1)} \times \mathbf{a}^{(2)} = i \mathbf{a}^{(3)} \]  
\[ \mathbf{a}^{(3)} \times \mathbf{a}^{(1)} = i \mathbf{a}^{(2)} \]  
\[ \mathbf{a}^{(2)} \times \mathbf{a}^{(3)} = i \mathbf{a}^{(1)} \]  

where * denotes complex conjugate. It also follows that:

\[ \nabla \times \mathbf{a}^{(1)} = \mathbf{k} \cdot \mathbf{a}^{(1)} \]  
\[ \nabla \times \mathbf{a}^{(2)} = \mathbf{k} \cdot \mathbf{a}^{(2)} \]  
\[ \nabla \times \mathbf{a}^{(3)} = 0 \]  

These equations can be simplified greatly by noting that:

\[ \mathbf{a}^{(1)} = e^{i \phi} = \mathbf{a}^{(1)} = \frac{1}{\sqrt{2}} (i - i^*) \exp(i \phi) \]  

where \( q \) is a complex vector in the Cartesian basis. The index (1) has been eliminated without loss of rigour or generality. In the Cartesian basis the \( O(3) \) symmetry cyclic equations (39) to (41) become:

\[
\begin{align*}
\vec{k} \times \vec{q} &= i \vec{q} \quad \text{(44)} \\
\vec{q}^* \times \vec{k} &= i \vec{q}^* \quad \text{(45)} \\
\vec{q} \times \vec{q}^* &= i \vec{k} \quad \text{(46)}
\end{align*}
\]

and are the same equations.

It follows that Eq. (34) reduces to:

\[
\vec{\nabla} \times \vec{q} = \vec{k} \vec{q} = -i \omega \vec{q} \quad \text{(47)}
\]

in which:

\[
\vec{q} = \vec{q}^a = \vec{q}^b \quad \text{(48)}
\]

The solution of Eq. (47) is:

\[
\omega = k \vec{k} \quad \text{(49)}
\]

and the details are given in note 256(6). The spin connection is the momentum of the photon within \( \hbar \):

\[
\hat{p} = \hat{p} \omega = \hat{p} k \quad \text{(50)}
\]

Therefore the theory has derived the existence of the photon propagating along \( Z \) from the plane wave, which has theoretically infinite lateral extent along \( X \) and \( Y \). The old MH theory could not achieve this because it had no spin connection. In the obsolete standard physics the MH and photon theories were completely different. In ECE theory they are unified elegantly.

The ECE hypothesis is simplified to:

\[
\begin{align*}
\vec{E} &= c \vec{A}^{(0)} \mathcal{T} (\text{orb}) \quad \text{(51)} \\
\vec{B} &= \vec{A}^{(0)} \mathcal{T} (\text{spin}) \quad \text{(52)}
\end{align*}
\]
so the first Cartan structure equation gives:
\[
\begin{align*}
E &= -\nabla \phi - \frac{\partial A}{\partial t} - \omega_0 A + \phi \alpha \\
B &= \nabla \times A - \omega \times A.
\end{align*}
\]  

In the absence of magnetic charge current density the Cartan and Evans identities give:
\[
\begin{align*}
\nabla \cdot B &= 0, \\
\frac{\partial B}{\partial t} + \nabla \times E &= 0, \\
\nabla \cdot E &= \rho / \varepsilon_0, \\
\nabla \times B - \frac{1}{c^2} \frac{\partial E}{\partial t} &= \mu_0 J.
\end{align*}
\]

The spacelike part of the Cartan identity gives:
\[
\nabla \cdot \omega \times A = 0. - (55)
\]

The absence of magnetic charge density gives:
\[
\omega \cdot B = A \cdot R(\text{spin}). - (56)
\]

The absence of magnetic current density gives:
\[
\omega \times E - c \omega_0 B = c \left( \frac{\mu}{\varepsilon} R(\text{spin}) + A \times R(\text{orb}) \right). - (57)
\]

The electric charge density is:
\[
\rho = \varepsilon_0 \left( \omega \cdot E - c A \cdot R(\text{orb}) \right). - (58)
\]

and the electric current density is:
\[
\jmath = \frac{1}{\mu_0} \left( \frac{\omega \times B + \omega_0 E}{c} - (A \times R(\text{spin}) + A_0 R(\text{orb})) \right). - (59)
\]

The orbital curvature simplifies as follows:
\[
R(\omega_b) = -\nabla \omega_o - \frac{1}{c} \frac{d\omega_o}{dt} = \omega_o \dot{\omega} + \omega_o \omega
\]

and the spin curvature simplifies to:

\[
R(s_{\text{spin}}) = \nabla \times \omega = 0.
\]

Using Eq. (59) it follows that for vacuum plane waves:

\[
R(\omega_b) = R(s_{\text{spin}}) = 0.
\]

and vacuum plane waves have no curvature. They are made up of pure spacetime torsion. In view of Eq. (49) the charge density of a vacuum plane wave vanishes self consistently:

\[
\rho = \epsilon_0 \omega \cdot E = 0.
\]

Using \{1 - 10\}:

\[
E = \frac{E(0)}{\sqrt{2}} (i \dot{i} - j \dot{j}) e^{i \phi} \quad (64)
\]

\[
B = \frac{B(0)}{\sqrt{2}} (i \dot{i} + j \dot{j}) e^{i \phi} \quad (65)
\]

\[
E(0) = \frac{c B(0)}{\sqrt{2}} \quad (66)
\]

the current density of a vacuum plane wave also vanishes self consistently:

\[
\hat{J} = \frac{1}{\mu_0} \left( \omega \times B + \omega_o \frac{E}{c} \right) = 0. \quad (67)
\]

if:

\[
\omega_o = \kappa = \frac{\omega}{c}. \quad (68)
\]

Therefore the spin connection four vector is the wave four vector:

\[
\omega^\mu = \kappa^\mu = \left( \frac{\omega}{c}, \kappa \right) \quad (69)
\]
for a vacuum plane wave. Within this the spin connection four vector is the energy momentum four vector of the photon:
\[ \rho^\mu = \left( \frac{E}{c}, \mathbf{P} \right) = \frac{\hbar}{c} \omega^\mu = \frac{\hbar}{c} k^\mu, \quad -(76) \]

Therefore the Planck / Einstein and de Broglie postulates have been derived from geometry within a unified field theory. They are no longer postulates, they are a manifestation of pure geometry - Cartan geometry.

The free electromagnetic field / photon is described by the four equations:
\[ \begin{align*}
\mathbf{E} \cdot \mathbf{B} &= 0 \\
\mathbf{E} \cdot \mathbf{E} &= 0 \\
\omega^2 \mathbf{B} - \frac{1}{c^2} \omega \times \mathbf{E} &= 0 \\
\omega \mathbf{E} + c \omega \times \mathbf{B} &= 0
\end{align*} \quad -(71) \]

and simultaneously by the familiar looking equations:
\[ \begin{align*}
\nabla \cdot \mathbf{B} &= \nabla \cdot \mathbf{E} = 0 \\
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} &= \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = 0
\end{align*} \quad -(72) \]

These may look familiar, but they are equations of general relativity, no longer of special relativity. The free gravitational field has an identical structure to Eqs. (71) and (72).

Eq. (58) in the presence of matter resembles Ohm’s law, and Eq. (59) resembles the Lorentz force equation:
\[ \mathbf{F} = e \left( \mathbf{E} + \gamma \times \mathbf{B} \right). \quad -(73) \]

The cross product in Eq. (46) is known as the conjugate product \( \{1 - 10\} \):
\[ \mathbf{q} \times \mathbf{q}^* = \mathbf{q}^{(1)} \times \mathbf{q}^{(2)} \quad -(74) \]

and defines the B(3) field \( \{1 - 10\} \) as follows
The spin connection vector is defined as:

\[
\omega = -i \omega_0 \mathbf{\epsilon}_1 \times \mathbf{\epsilon}_2 = -i \omega_0 \mathbf{\epsilon}_1 \times \mathbf{\epsilon}_2^* = \omega_0 \mathbf{k}
\]  

so it follows that:

\[
B^{(3)} = B^{(0)} \frac{\omega}{\omega_0}
\]  

The spin connection vector is defined as:

\[
B^{(3)} = B^{(0)} \mathbf{k} = -i B^{(0)} \mathbf{\epsilon}_1 \times \mathbf{\epsilon}_2 = -i B^{(0)} \mathbf{\epsilon}_1 \times \mathbf{\epsilon}_2^* = \omega_0 \mathbf{k}
\]  

The B(3) field is defined by the spin connection vector. This means that general relativity is needed to define the B(3) field, which is observed in the inverse Faraday effect as the conjugate product of circularly or elliptically polarized electromagnetic radiation of any frequency. The old MH theory did not define B(3) because it did not define the spin connection, being a theory of special relativity. The old MH theory did not allow B(3) because it allowed only transverse states of polarization in the vacuum. This absurdity followed from the incorrect assumption of zero photon mass. The U(1) gauge invariance of the old MH theory relied on these absurdities. The U(1) x SU(2) electroweak theory also relied on the same absurdities, and in UFT225 of this series was shown to be wildly incorrect algebraically at the very place in which the Higgs mechanism was introduced. The old theory had no real explanation for the inverse Faraday effect because it did not allow longitudinal magnetic flux density in the vacuum, despite the fact that this longitudinal vacuum magnetic flux density manifests itself as longitudinal magnetization in the inverse Faraday effect in any material matter. All these absurdities, and many others, disappear in ECE theory. The latter is perfectly self consistent because its underlying Cartan geometry is flawless.
Main equations of the simplified ECE theory and interpretation of the spin connection

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3 Some implications of the spin connection in engineering

In the preceding section it has been shown that electric charge density and current can be described by pure geometric quantities as torsion and curvature, more precisely by the electric and magnetic field, electric and magnetic potential, and spin connections and spin/orbital curvature (Eq.(29)). For one polarization direction this equation reads

\[ J = \frac{1}{\mu_0} \left( \frac{\omega_0}{c} E + \omega \times B - A \times R(\text{spin}) + \frac{\Phi}{c} R(\text{orb}) \right). \]  \hspace{1cm} (78)

In case of the free vacuum field, the curvature terms vanish. This gives us an argument to neglect these terms in Eq.(78) and write the current density as

\[ J = J_\sigma + J_{\omega} \]  \hspace{1cm} (79)

with a term

\[ J_\sigma = \frac{\omega_0}{\mu_0 c} E = \sigma E \]  \hspace{1cm} (80)

which represents Ohm’s law with conductivity \( \sigma \) and

\[ J_{\omega} = \frac{1}{\mu_0} \omega \times B \]  \hspace{1cm} (81)

which looks similar to a Lorentz force law. Ohm’s law is derived phenomenologically in MH theory. Here it follows from Cartan geometry directly. The term \( J_{\omega} \) should have a similar meaning in electromagnetic engineering. This will be investigated further as follows.

As electric conductivity is present in conductive materials, it is assumed that magnetic effects invoke also a current in a conductive material according

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to Eq.(81). Magnetic fields are usually induced by source currents as described by the Ampère-Maxwell law Eq.(31). This is depicted in Fig. 1. The curl operator leads to closed B field lines around the conductor, and the B field is described by a curl of the vector potential A:

\[ B = \nabla \times A. \] (82)

In case of Eq.(81) the situation is reversed, the existing magnetic field effects an electric current in a conductor. The existence of this current depends on the vector spin connection \( \omega \). This can be interpreted as a spatial rotation axis of the magnetic field. Therefore this effect requires a curved magnetic field. For the free field we have from Eq.(71)

\[ \omega \cdot B = 0, \] (83)

e.g. \( \omega \) is perpendicular to \( B \). We can assume this to be the case outside of materials in the same way as electric and magnetic fields are perpendicular to each other. According to Eq.(81), the induced current is perpendicular to \( \omega \) and \( B \) so that the situation of Fig. 2 appears: \( J_\omega \) points into the direction of the curvature radius of \( B \).

We want to give an example how this situation may look like in engineering applications. We consider two bar magnets positioned in an acute angle to each other. Then the magnetic field around the nearest points is significantly curved. We have performed a two-dimensional finite element calculation of this situation. The resulting induction field \( B \) is shown in Fig. 3. The equations being solved are

\[ \nabla \times A = B \] (84)
\[ H = \frac{B - M}{\mu_0 \mu_r} \] (85)
\[ \nabla \times H = 0 \] (86)

where \( M \) is the magnetization within the bar magnets. We used \( M = 100 \text{A/m} \) and \( \mu_r = 1000 \). The vector potential is the primary variable to be solved. The question now is how to represent spin connection. The above calculation is MH theory and does not contain this variable, therefore we try to determine it a posteriori like in perturbation theory. From the ECE antisymmetry conditions we have

\[ \omega_0 A = \omega \Phi \] (87)

which means that the vector spin connection is proportional to the vector potential if \( \omega_0 \) and the scalar potential \( \Phi \) are constant. From Eq.(71) it is required for the free field that the vector spin connection is perpendicular to the magnetic field which is by the above equation:

\[ \omega || A. \] (88)

Then we have

\[ J_\omega \propto A \times B. \] (89)

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Since the FEM calculation was in two dimensions only, the magnetic vector potential has only one component perpendicular to the $B$ plane. The result is shown in Fig. 4 as a contour plot. Its magnitude is highest where the bar magnets are nearest to each other and internally at the sides facing each other. Correspondingly, the spin connection current (89) is highest in these regions as can be seen from the plot in Fig. 5. If this approach is realistic, a voltage should arise across the magnets which should be detectable if the magnets are made of conducting material. The spin connection current density is also high in the region where both magnets are near together. In such a vacuum region no current can arise but we could place a non-magnetic wire here and measure whether a current is induced by connecting it with a test circuit. Both effect, if existing, would be an effect completely unknown to the MH theory.

Another method of constructing a spin connection vector is to compute the rotation axis of the $B$ field by a curl operation. This would lead to

$$J_{\omega} \propto (\nabla \times B) \times B.$$  (90)

This current density is graphed in Fig. 6. Because of the parallel behaviour of $B$ vectors within the magnets there is only a significant contribution in the curved region where the magnetic field bends from one magnet to the other, see Fig. 6. In this region the current density behaves similar as for the approach of Eq. (89). In total such a current would be induced by the spin connection in a magnetostatic environment. This is new physics compared to the MH theory.

Figure 1: Induction principle for a magnetic field generated by a current.
Figure 2: Spin connection current induced by a curved B field.

Figure 3: Magnetic flux density in and around two bar magnets.
Figure 4: Z component of the magnetic vector potential.

Figure 5: Spin connection current from $\mathbf{A} \times \mathbf{B}$. 
Figure 6: Spin connection current from $(\nabla \times B) \times B$. 
3. SOME IMPLICATIONS OF THE SPIN CONNECTION IN ENGINEERING

Section by Dr. Horst Eckardt.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS, particularly Dr. Douglas Lindstrom, for many interesting discussions. Dave Burleigh is thanked for posting and Alex Hill is thanked for translation. Robert Cheshire and Alex Hill are thanked for broadcasting.

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