

THE GEOMETRICAL THEORY OF CHARGE CURRENT DENSITY : SPIN
CONNECTION RESONANCE, LENR AND BELTRAMI STRUCTURES.

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ABSTRACT

The ECE unified field theory produces a geometrical structure for magnetic and electric charge current four densities. The geometrical structure of the electric charge density is used to show that the vacuum potential investigated by Eckardt and Lindstrom gives spin connection resonance that can give rise to electric power from spacetime and give a plausible theory of low energy nuclear reactors. It is shown that the Beltrami structure can give rise to several of the fundamental equations of physics, including the free particle Schroedinger equation, and can also occur in the vacuum itself through the orbital and spin curvature. Low energy nuclear reactors (LENR) can be understood in outline by considerations of spin connection resonance produced by the vacuum potential.

Keywords: ECE theory, geometrical theory of charge current density, spin connection resonance, Beltrami equation, low energy nuclear reactors.

UFT 259

1. INTRODUCTION

In immediately preceding papers of this series of two hundred and fifty nine papers and monographs to date {1 - 10} it has been shown that Cartan geometry has a Beltrami structure, so in ECE unified field theory this finding works itself through into electrodynamics and gravitational theory. Notably, the space part of the Cartan identity is a Beltrami equation in the absence of a magnetic monopole. The geometrical structure of magnetic and electric charge current density leads to numerous insights. In Section 2 the new method is applied to electrostatics and it is shown that spin connection resonance (SCR) originates in this geometrical structure provided the Eckardt Lindstrom vacuum potential is present. It is shown that the vector potential obeys a Beltrami equation which inter relates several of the fundamental equations of geometry and physics, notably the Cartan identity, the Euler Bernoulli equation, the Coulomb law, the Helmholtz equation, the Schroedinger equation and fermion equation (the chiral Dirac equation). For example the Schroedinger equation can be derived from a Beltrami equation for momentum. In Section 3 it is shown that the orbital and spin curvature vectors of ECE theory are also Beltrami equations. These insights open up several new subject areas because it is known that Beltrami equations are important in electrodynamics, hydrodynamics, aerodynamics and cosmology.

2. DEVELOPMENT OF THE GEOMETRICAL NATURE OF CHARGE DENSITY

As usual this section is a synopsis of the main results contained in the background notes accompanying UFT259 on www.aias.us. The charge density in ECE theory has been shown in recent papers to be defined by Cartan geometry. So the structure of elementary particles such as the electron and proton is also defined by Cartan geometry. This insight leads to a new theory of elementary particles. The electric charge density is defined {1 - 10} by:

$$\underline{\rho}^a = \epsilon_0 \left(\underline{\omega}^a_b \cdot \underline{E}^b - c \underline{A}^b \cdot \underline{R}^a_b(\text{orb}) \right) \quad - (1)$$

where ϵ_0 is the vacuum permittivity, $\underline{\omega}^a_b$ is the vector spin connection, \underline{E}^b is the electric field strength, c is the universal constant known as the vacuum speed of light, and $\underline{R}^a_b(\text{orb})$ is the orbital part of the curvature vector. For ease of reference some quantities and S. I.

Units are given as follows:

$$\begin{aligned} E &= \text{Vm}^{-1} = \text{Jc}^{-1}\text{m}^{-1} \\ A &= \text{Js}^{-1}\text{c}^{-1}\text{m}^{-1} \\ \epsilon_0 &= \text{J}^{-1}\text{c}^2\text{m}^{-1} \\ B &= \text{Js}^{-1}\text{c}^{-1}\text{m}^{-2} = \text{tesla} \\ \rho &= \text{Cm}^{-3} \\ J &= \text{Cm}^{-2}\text{s}^{-1} \\ \omega &= \text{m}^{-1} \\ R &= \text{m}^{-2} \end{aligned} \quad - (2)$$

The first Cartan structure equation {1 - 10} defines the electric field strength as:

$$\underline{E}^a = -c \underline{\nabla} A^a - \frac{\partial \underline{A}^a}{\partial t} - c \omega^a_{ob} \underline{A}^b + c A^b \cdot \underline{\omega}^a_b \quad - (3)$$

where the four potential of ECE electrodynamics is defined by:

$$\underline{A}^a_\mu = (A^a_0, -\underline{A}^a) = \left(\frac{\phi^a}{c}, -\underline{A}^a \right) \quad - (4)$$

Here ϕ^a is the scalar potential. If it is assumed that the subject of electrostatics is defined by:

$$\underline{B}^a = \underline{0}, \underline{A}^a = \underline{0}, \underline{J}^a = \underline{0} \quad - (5)$$

the Coulomb law in ECE theory is given by:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a_b \cdot \underline{E}^b \quad - (6)$$

The electric current density in ECE theory is defined by:

$$\underline{J}^a = \epsilon_0 c \left(\omega^a_b \underline{E}^b - c A^b_0 \underline{R}^a_b(\alpha b) + c \underline{\omega}^a_b \times \underline{B}^b - c \underline{A}^b \times \underline{R}^a_b(\text{spin}) \right) \quad - (7)$$

where \underline{R}^a_b is the spin part of the curvature vector and where \underline{B} is the magnetic flux density.

From Eqs. (5) and (7):

$$\underline{J}^a = \underline{0} = \epsilon_0 c \left(\omega^a_b \underline{E}^b - c A^b_0 \underline{R}^a_b(\alpha b) \right) \quad - (8)$$

so in ECE electrostatics:

$$\omega^a_b \underline{E}^b = c A^b_0 \underline{R}^a_b(\alpha b) \quad - (9)$$

and:

$$\underline{E}^a = -c \underline{\nabla} A^a_0 + c A^b_0 \underline{\omega}^a_b \quad - (10)$$

with

$$\underline{\nabla} \times \underline{E}^a = \underline{0} \quad - (11)$$

From Eqs. (10) and (11):

$$\underline{\nabla} \times \underline{E}^a = c \underline{\nabla} \times (A^b_0 \underline{\omega}^a_b) \quad - (12)$$

so we obtain the constraint:

$$\underline{\nabla} \times (A^b_0 \underline{\omega}^a_b) = \underline{0} \quad - (13)$$

The magnetic charge density in ECE theory is given by:

$$\rho_{mag}^a = \epsilon_0 c \left(\underline{\omega}^a_b \cdot \underline{B}^b - \underline{A}^b \cdot \underline{R}^a_b (spin) \right) - (14)$$

and the magnetic current density by:

$$\underline{J}_{mag}^a = \epsilon_0 \left(\underline{\omega}^a_b \times \underline{E}^b - c \underline{\omega}^a_b \underline{B}^a - c \left(\underline{A}^b \times \underline{R}^a_b (orb) - \underline{A}^b \cdot \underline{R}^a_b (spin) \right) \right) - (15)$$

These are thought to vanish experimentally in the electromagnetism so:

$$\underline{\omega}^a_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a_b (spin) - (16)$$

and:

$$\underline{\omega}^a_b \times \underline{E}^b - c \underline{\omega}^a_b \underline{B}^a - c \underline{A}^b \times \underline{R}^a_b (orb) + c \underline{A}^b \cdot \underline{R}^a_b (spin) = \underline{0} - (17)$$

In ECE electrostatics Eq. (16) is true automatically because:

$$\underline{B}^b = \underline{0}, \underline{A}^b = \underline{0} - (18)$$

and Eq. (15) becomes:

$$\underline{\omega}^a_b \times \underline{E}^b + c \underline{A}^b \cdot \underline{R}^a_b (spin) = \underline{0} - (19)$$

So in summary the equations of electrostatics are:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a_b \cdot \underline{E}^b - (20)$$

$$\underline{\omega}^a_b \underline{E}^b = \phi^b \underline{R}^a_b (orb) - (21)$$

$$\underline{\omega}^a_b \times \underline{E}^b + \phi^b \underline{R}^a_b (spin) = \underline{0} - (22)$$

$$\underline{E}^a = -\underline{\nabla} \phi^a + \phi^b \underline{\omega}^a_b - (23)$$

It is shown in Section 3 that these equations lead to a solution in terms of Bessel functions, but not to Euler Bernoulli resonance.

In order to obtain spin connection resonance Eq. (20) must be extended to:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a_b \cdot \underline{E}^b - c \underline{A}^b(\text{vac}) \cdot \underline{R}^a_b(\text{orb}) - (24)$$

where $\underline{A}^b(\text{vac})$ is the Eckardt Lindstrom vacuum potential. The static electric field is defined by:

$$\underline{E}^a = -\underline{\nabla} \phi^a + \phi^b \underline{\omega}^a_b - (25)$$

so from Eqs. (24) and (25):

$$\begin{aligned} \nabla^2 \phi^a + (\underline{\omega}^a_b \cdot \underline{\omega}^b_c) \phi^c & - (26) \\ = \underline{\nabla} \cdot (\phi^b \underline{\omega}^a_b) + \underline{\omega}^a_b \cdot \underline{\nabla} \phi^b + c \underline{A}^b(\text{vac}) \cdot \underline{R}^a_b(\text{orb}) \end{aligned}$$

By the ECE antisymmetry law {1 - 10}:

$$-\underline{\nabla} \phi^a = \phi^b \underline{\omega}^a_b - (27)$$

leading to the Euler Bernoulli resonance equation:

$$\nabla^2 \phi^a + (\underline{\omega}^a_b \cdot \underline{\omega}^b_c) \phi^c = \frac{1}{2} c \underline{A}^b(\text{vac}) \cdot \underline{R}^a_b(\text{orb}) - (28)$$

and spin connection resonance. The left hand side contains the Hooke law term and the right hand side the driving term originating in the vacuum potential. Denote:

$$\rho^a(\text{vac}) = \frac{F_0 c}{2} \underline{A}^b(\text{vac}) \cdot \underline{R}^a_b(\text{orb}) - (29)$$

then the equation becomes:

$$\nabla^2 \phi^a + (\underline{\omega}^a_b \cdot \underline{\omega}^b_c) \phi^c = \frac{\rho^a(\text{vac})}{\epsilon_0} - (30)$$

The left hand side of Eq. (30) is a field property and the right hand side a property of the ECE vacuum. In the simplest case:

$$\nabla^2 \phi + \omega_0^2 \phi = \frac{\rho(\text{vac})}{\epsilon_0} - (31)$$

and produces undamped resonance if

$$\nabla^2 \phi + \omega_0^2 \phi = \frac{\rho(\text{vac})}{\epsilon_0} = A \cos \omega t \quad - (32)$$

where A is a constant. The particular integral of Eq. (32) is:

$$\phi = \frac{A \cos \omega t}{\omega_0^2 - \omega^2} \quad - (33)$$

and spin connection resonance occurs at:

$$\omega_0 = \omega \quad - (34)$$

when

$$\phi \rightarrow \infty \quad - (35)$$

and there is a resonance peak of electric field strength from the vacuum.

In Section 3 solutions of Eq. (28) are given in terms of a combination of Bessel functions, and an analysis using the Eckardt Lindstrom vacuum potential as a driving term.

The Beltrami structure of ECE electrodynamics can be developed as in Note 259(3). Consider the magnetic flux density in ECE electrodynamics:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad - (36)$$

and consider the complex circular basis for the indices a and b {1 - 10}:

$$a, b, c = (1), (2), (3). \quad - (37)$$

There is summation over repeated indices b in Eq. (36) so:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \omega^a{}_{(1)} \times \underline{A}^{(1)} - \omega^a{}_{(2)} \times \underline{A}^{(2)} - \omega^a{}_{(3)} \times \underline{A}^{(3)} \quad - (38)$$

Now assume that:

$$\underline{\omega}^a{}_b = \epsilon^a{}_{bc} \underline{\omega}^c \quad - (39)$$

where $\epsilon^a{}_{bc}$ is the totally antisymmetric unit tensor in the three dimensions of a, b and c.

Eq. (39) assumes that the spin connection vector $\underline{\omega}^c$ that is dual to $\underline{\omega}^a{}_b$. Thus:

$$\underline{\omega}^{(1)}{}_{(2)} = \epsilon^{(1)}{}_{(2)(3)} \underline{\omega}^{(3)} = \underline{\omega}^{(3)} \quad - (40)$$

$$\underline{\omega}^{(1)}{}_{(3)} = \epsilon^{(1)}{}_{(3)(2)} \underline{\omega}^{(2)} = -\underline{\omega}^{(2)} \quad - (41)$$

and

$$\underline{B}^{(1)} = \underline{\nabla} \times \underline{A}^{(1)} - \underline{\omega}^{(3)} \times \underline{A}^{(2)} + \underline{\omega}^{(2)} \times \underline{A}^{(3)} \quad - (42)$$

et cyclicum

In the absence of a magnetic monopole the Cartan identity is {1-10}:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad - (43)$$

which implies:

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b = \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad - (44)$$

These results have been derived in immediately preceding papers of this series. Eq. (39) is a possible solution of Eq. (44) and this gives a rigorous geometrical justification for O(3) electrodynamics.

The Cartan identity (43) is itself a Beltrami equation:

$$\underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) = \kappa \underline{\omega}^a{}_b \times \underline{A}^b \quad - (45)$$

From Eqs. (39) and (45):

$$\underline{\nabla} \times (\underline{A}^c \times \underline{A}^b) = \kappa \underline{A}^c \times \underline{A}^b \quad - (46)$$

In the complex circular basis:

$$\begin{aligned} \underline{A}^{(1)} \times \underline{A}^{(2)} &= i A^{(0)} \underline{A}^{(3)*} \\ \underline{A}^{(2)} \times \underline{A}^{(3)} &= i A^{(0)} \underline{A}^{(1)*} \\ \underline{A}^{(3)} \times \underline{A}^{(1)} &= i A^{(0)} \underline{A}^{(2)*} \end{aligned} \quad - (47)$$

so from Eqs. (46) to (47):

$$\begin{aligned} \underline{\nabla} \times \underline{A}^{(1)} &= \kappa \underline{A}^{(1)} & - (48a) \\ \underline{\nabla} \times \underline{A}^{(2)} &= \kappa \underline{A}^{(2)} & - (48b) \\ \underline{\nabla} \times \underline{A}^{(3)} &= \kappa \underline{A}^{(3)} & - (48c) \end{aligned}$$

so the vector potentials obey Beltrami equations.

This result can also be derived self consistently by using the Gauss law:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad - (49)$$

which implies the Beltrami equation:

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a \quad - (50)$$

From Eqs. (36) and (50):

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a = \kappa \left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \right) \quad - (51)$$

so:

$$\begin{aligned} \underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) - \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{A}^b) & \\ = \kappa (\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) & \quad - (52) \end{aligned}$$

Using Eq. (45) gives:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \underline{\nabla} \times \underline{A}^a \quad - (53)$$

which implies Eqs. (48a) to (48c) QED. As shown in the immediately preceding papers

of this series the Beltrami structure also governs the spin connection vector:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa \underline{\omega}^a{}_b. \quad - (54)$$

It follows that the equations:

$$\underline{\omega}^{(3)} = \frac{1}{2} \frac{\kappa}{A^{(0)}} \underline{A}^{(3)} \quad - (55)$$

and

$$\underline{\omega}^{(2)} = \frac{1}{2} \frac{\kappa}{A^{(0)}} \underline{A}^{(2)} \quad - (56)$$

produce O(3) electrodynamics {1 - 10}

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} - \frac{i\kappa}{A^{(0)}} \underline{A}^{(2)} \times \underline{A}^{(3)} \quad - (57)$$

et cyclicum

As shown in Note 259(3) there are many inter related equations of O(3) electrodynamics which all originate in geometry.

In Section 3 it is shown that a consequence of these conclusions is that the spin and orbital curvature vectors also obey a Beltrami structure.

The fact that ECE is a unified field theory also allows the development and interrelation of several basic equations. In ECE electrodynamics the magnetic field is:

$$\underline{B}^{(3)} = \underline{\nabla} \times \underline{A}^{(3)} - \frac{i\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (58)$$

in general, in which:

$$p = \hbar \kappa = e A^{(0)}. \quad - (59)$$

The potentials are related by:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i A^{(0)} \underline{A}^{(3)*} \quad - (60)$$

et cyclicum

and are Beltrami functions:

$$\underline{\nabla} \times \underline{A}^{(1)} = \kappa \underline{A}^{(1)} \quad - (61)$$

$$\underline{\nabla} \times \underline{A}^{(2)} = \kappa \underline{A}^{(2)} \quad - (62)$$

with:

$$\underline{A}^{(1)} = \underline{A}^{(2)*} \quad - (63)$$

These structures follow from Cartan geometry and are true in general. For plane waves:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) \exp(i(\omega t - \kappa z)) \quad - (64)$$

$$\underline{A}^{(3)} = A^{(0)} \underline{k} \quad - (65)$$

so:

$$\underline{B}^{(3)*} = \underline{B}^{(3)} = -\frac{i\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (66)$$

which is the fundamental B field of propagating electromagnetic radiation.

It can be written as:

$$\underline{B} = -i \frac{e}{\hbar} \underline{A} \times \underline{A}^* = B^{(0)} \underline{k} = B_z \underline{k} \quad - (67)$$

and in this format can be used as the definition of a static magnetic field. This is important for the subject of magnetostatics and the development of the fermion equation with:

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r} \quad - (68)$$

Eq. (67) also gives the transition from classical to quantum mechanics. In ECE

electrodynamics A must always be a Beltrami field. As shown in recent work this is the

direct result of the Cartan identity. So it is necessary to solve:

$$\underline{B} = -i \frac{e}{\hbar} \underline{A} \times \underline{A}^* \quad - (69)$$

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r} \quad - (70)$$

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad - (71)$$

This can be done by using the principles of general relativity, so that the electromagnetic field is a rotating and translating frame of reference. The position vector is therefore:

$$\underline{r} = \underline{r}^* = \frac{r^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad - (72)$$

where

$$\underline{r} = \underline{r}^{(1)}, \quad \underline{r}^* = \underline{r}^{(2)}, \quad \phi = \omega t - \kappa z \quad - (73)$$

so

$$\underline{r}^{(1)} \times \underline{r}^{(2)} = i r^{(0)} \underline{r}^{(3)*} \quad - (74)$$

et cyclicum

It follows that:

$$\underline{\nabla} \times \underline{r}^{(1)} = \kappa \underline{r}^{(1)} \quad - (75)$$

$$\underline{\nabla} \times \underline{r}^{(2)} = \kappa \underline{r}^{(2)} \quad - (76)$$

$$\underline{\nabla} \times \underline{r}^{(3)} = 0 \underline{r}^{(3)} \quad - (77)$$

The results (72) to (77) for plane waves can be generalized to any Beltrami solutions. It follows that spacetime itself has a Beltrami structure.

From Eqs. (70) and (72):

$$\underline{A} = \underline{A}^{(1)} = \frac{B^{(0)} r^{(0)}}{2\sqrt{2}} (\underline{i}\underline{i} + \underline{j}) e^{i\phi} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i}\underline{i} + \underline{j}) e^{i\phi} \quad - (78)$$

where:

$$A^{(0)} = \frac{1}{2} B^{(0)} r^{(0)} \quad - (79)$$

From Eq. (78):

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad - (80)$$

QED. Therefore it is always possible to write the vector potential in the form of Eq. (70) provided that spacetime itself has a Beltrami structure. This conclusion ties together several branches of physics because Eq. (70) is used to produce the Landé factor, ESR, NMR and so on from the Dirac equation, which becomes the fermion equation in ECE physics.

In ECE physics the tetrad postulate of Cartan geometry gives:

$$(\square + \kappa_0^2) \underline{A} = \underline{0} \quad - (81)$$

under all conditions. Eq. (81) as always in ECE physics is the result of geometry. The fermion or chiral Dirac equation is a factorization of Eq. (81). The wave number κ_0 is the result of Cartan geometry as shown in detail on Note 259(4) and is given by:

$$\kappa_0^2 = \tilde{e}_a^\nu \partial^\mu (\omega_{\mu\nu}^a - \Gamma_{\mu\nu}^a) \quad - (82)$$

where \tilde{e}_a^ν is the inverse tetrad, $\omega_{\mu\nu}^a$ is the mixed index spin connection and $\Gamma_{\mu\nu}^a$ the mixed index gamma connection. Now use the ECE hypothesis:

$$A_\mu^a = A^{(0)} \tilde{e}_\mu^a \quad - (83)$$

to find that:

$$(\square + \kappa_0^2) A_\mu^a = 0. \quad - (84)$$

Finally use:

$$A_\mu^a = (A_0^a, -\underline{A}^a) \quad - (85)$$

so that for each a:

$$(\square + \kappa_0^2) \underline{A}_0 = 0 \quad - (86)$$

$$(\square + \kappa_0^2) \underline{A} = \underline{0} \quad - (87)$$

which gives Eq. (81), QED.

The d'Alembertian is defined by:

$$\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \quad - (88)$$

From Eq. (80):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \kappa \underline{\nabla} \times \underline{A} = \kappa^2 \underline{A} \quad - (89)$$

and

$$\underline{\nabla} \cdot \underline{A} = 0 \quad - (90)$$

because

$$\underline{A} = \frac{1}{\kappa} \underline{\nabla} \times \underline{A} \quad - (91)$$

and

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{A} = 0. \quad - (92)$$

From vector analysis:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla} (\underline{\nabla} \cdot \underline{A}) - \nabla^2 \underline{A} \quad - (93)$$

so from Eqs. (89) to (93):

$$(\nabla^2 + \kappa^2) \underline{A} = \underline{0} \quad - (94)$$

which is the Helmholtz wave equation. In ECE electrodynamics this is true for each \hat{a} :

$$(\nabla^2 + \kappa^2) \underline{A}^a = \underline{0}. \quad - (95)$$

The Helmholtz wave equation is the result of the Beltrami equation.

From Eq. (87):

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \kappa_0^2 \right) \underline{A} = 0 \quad - (96)$$

so:

$$\frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} + (\kappa_0^2 + \kappa^2) \underline{A} = 0 \quad - (97)$$

This is the equation for the time dependence of \underline{A} . The Helmholtz and Beltrami equations are for the space dependence of \underline{A} . Eq. (97) is satisfied by:

$$\underline{A} = \underline{A}_0 \exp(i\omega t) \quad - (98)$$

where:

$$\frac{\omega^2}{c^2} = \kappa^2 + \kappa_0^2 \quad - (99)$$

Eq. (99) is a generalization of the Einstein energy equation for a free particle:

$$E^2 = c^2 p^2 + m^2 c^4 \quad - (100)$$

where:

$$E = \hbar \omega, \quad \underline{p} = \hbar \underline{\kappa} \quad - (101)$$

using:

$$\kappa_0^2 = \left(\frac{mc}{\hbar} \right)^2 = \eta^{\nu\alpha} \eta^{\mu\beta} (\omega_{\mu\nu}^a - \pi_{\mu\nu}^a) \quad - (102)$$

So mass in ECE theory is defined by geometry.

The general solution of Eq. (84) is therefore:

$$A_{\mu}^a = A_{\mu}^a(0) \exp(i(\omega t - \kappa Z)) \quad - (103)$$

where:

$$\omega^2 = c^2 (\kappa^2 + \kappa_0^2). \quad - (104)$$

It follows that there exist:

$$(\square + \kappa_0^2) \phi^a = 0 \quad - (105)$$

and

$$(\nabla^2 + \kappa^2) \phi^a = 0 \quad - (106)$$

where ϕ^a is the scalar potential in ECE physics. For each a:

$$(\nabla^2 + \kappa^2) \phi = 0. \quad - (107)$$

Now write:

$$\kappa_0 = \frac{mc}{\hbar} \quad - (108)$$

where m is a mass. The relativistic wave equation for each a is:

$$(\square + \kappa_0^2) \phi = 0 \quad - (109)$$

which is the quantized form of

$$E^2 = c^2 p^2 + m^2 c^4 = c^2 p^2 + \hbar^2 \kappa_0^2 c^2. \quad - (110)$$

Eq. (110) is:

$$E = \gamma mc^2 \quad - (111)$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad - (112)$$

and where the relativistic momentum is:

$$\underline{p} = \gamma m \underline{v}. \quad - (113)$$

Define the relativistic kinetic energy as:

$$T = E - mc^2 \quad - (114)$$

and it follows that:

$$T = \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) mc^2 \sim \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) mc^2 = \frac{1}{2} mv^2 \quad - (115)$$

which is the non-relativistic limit of the kinetic energy, i.e. :

$$T = \frac{p^2}{2m} \quad - (116)$$

Using

$$T = i\hbar \frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar \underline{\nabla} \quad - (117)$$

Eq. (116) quantizes to the free particle Schroedinger equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \phi = T \phi \quad - (118)$$

which is the Helmholtz equation:

$$\left(\nabla^2 + \frac{2mT}{\hbar^2} \right) \phi = 0 \quad - (119)$$

It follows that the free particle Schroedinger equation is a Helmholtz equation but with the vector potential \underline{A} replaced by a scalar potential ϕ . The scalar potential plays the role of the wave function. It also follows in the non relativistic limit that:

$$\left(\nabla^2 + \frac{2mT}{\hbar^2} \right) \underline{A} = \underline{0} \quad - (120)$$

so:

$$k^2 = \frac{2mT}{\hbar^2} \quad - (121)$$

The Helmholtz equation (119) can be written as:

$$(\nabla^2 + \kappa^2)\phi = 0 \quad - (122)$$

which is an Euler Bernoulli equation without a driving term on the right hand side. In the presence of potential energy V , Eq. (118) becomes:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\phi = E\phi \quad - (123)$$

where H is the hamiltonian and E is the total energy:

$$E = T + V. \quad - (124)$$

Eq. (123) is:

$$(\nabla^2 + \kappa^2)\phi = \left(\frac{2mV}{\hbar^2}\right)\phi \quad - (125)$$

which is similar to an Euler Bernoulli equation with a driving term on the right hand side.

However Eq. (125) is an Eigen equation rather than an Euler Bernoulli equation as conventionally defined {1 - 10}, but Eq. (125) has very well known resonance solutions in quantum mechanics. Eq. (125) may be written as:

$$(\nabla^2 + \kappa_1^2)\phi = 0 \quad - (126)$$

where:

$$\kappa_1^2 = \frac{2m}{\hbar^2}(E - V) \quad - (127)$$

and in UFT226 ff. on www.aias.us was used in the theory of low energy nuclear reactors (LENR). Eq. (126) is well known to be a linear oscillator equation which can be used to define the structure of the atom and nucleus. It can be transformed into an Euler Bernoulli equation as follows:

$$(\nabla^2 + \kappa_1^2)\phi = A \cos(\kappa_2 z) \quad - (128)$$

where the right hand side represents a vacuum potential. $\underline{I}t$ is precisely the kind of structure obtained from the ECE Coulomb law.

The free particle Schroedinger equation can be obtained from the Beltrami equation, for momentum:

$$\underline{\nabla} \times \underline{p} = \kappa \underline{p} \quad - (129)$$

an equation which implies:

$$(\nabla^2 + \kappa^2) \underline{p} = \underline{0} \quad - (130)$$

and:

$$\underline{\nabla} \cdot \underline{p} = 0. \quad - (131)$$

If \underline{p} is a linear momentum in the classical straight line then:

$$\kappa = 0. \quad - (132)$$

In general however \underline{p} has intricate Beltrami solutions animated in UFT258 on www.ajias.us.

Now quantize Eq. (130):

$$\underline{p} \psi = -i\hbar \underline{\nabla} \psi \quad - (133)$$

so:

$$(\nabla^2 + \kappa^2) \underline{\nabla} \psi = \underline{0}. \quad - (134)$$

Use:

$$\nabla^2 \underline{\nabla} \psi = \underline{\nabla} \nabla^2 \psi \quad - (135)$$

and:

$$\underline{\nabla} (\kappa^2 \psi) = \kappa^2 \underline{\nabla} \psi \quad - (136)$$

assuming that:

$$\underline{\nabla} \kappa^2 = \underline{0}. \quad - (137)$$

Eqs. (133) to (137) give:

$$\nabla \left(\nabla^2 \psi + \kappa^2 \psi \right) = 0 \quad - (138)$$

A possible solution is:

$$\left(\nabla^2 + \kappa^2 \right) \psi = 0 \quad - (139)$$

which is the Helmholtz equation for the scalar ψ , the wave function of quantum mechanics. The Schroedinger equation for a free particle is obtained by applying Eq. (133)

to:

$$E = \frac{p^2}{2m} \quad - (140)$$

so:

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi \quad - (141)$$

and

$$\left(\nabla^2 + \frac{2Em}{\hbar^2} \right) \psi = 0 \quad - (142)$$

Eqs. (139) and (142) are the same if:

$$\kappa^2 = \frac{2Em}{\hbar^2} \quad - (143)$$

QED. Using:

$$p = \hbar \kappa \quad - (144)$$

then:

$$p^2 = 2Em \quad - (145)$$

which is Eq. (140), QED.

Therefore the free particle Schroedinger equation is:

$$\nabla \times p = \left(\frac{2Em}{\hbar^2} \right)^{1/2} p \quad - (146)$$

with:

$$p \psi = -i \hbar \nabla \psi \quad - (147)$$

The origin of the Schroedinger equation is a Beltrami equation for p .

The geometrical theory of charge current density: spin connection resonance, LENR and Beltrami structures

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3 Beltrami structure of curvature, Bessel solutions and graphics

3.1 Resonance solutions in cartesian coordinates

The equations to be solved are Eqs.(20-23). In single polarization and with a "driving term" charge density ρ_0 they can be written:

$$\nabla \cdot \mathbf{E} = \boldsymbol{\omega} \cdot \mathbf{E} + \frac{\rho_0}{\epsilon_0}, \quad (148)$$

$$\omega_0 \mathbf{E} = \phi \mathbf{R}(\text{orb}), \quad (149)$$

$$\boldsymbol{\omega} \times \mathbf{E} + \phi \mathbf{R}(\text{spin}) = 0, \quad (150)$$

$$\phi \nabla \times \boldsymbol{\omega} + \nabla \phi \times \boldsymbol{\omega} = 0, \quad (151)$$

$$\mathbf{E} = -\nabla \phi + \phi \boldsymbol{\omega}. \quad (152)$$

ρ_0 is a vacuum charge density which may induce resonance effects. These are 13 equations with 15 variables so there is no unique solution. The situation can be made manageable by omitting the equations containing curvature. Both the orbital and spin curvature appear only in one equation so these are definitions for those variables and can be omitted. The scalar spin connection occurs only in one of the curvature equations and therefore can be omitted also. Only seven equations with seven unknowns remain. Inserting the definition of \mathbf{E} , Eq.(152), into Eqs.(148) and (151) then gives four remaining equations

$$\Delta \phi - \boldsymbol{\omega} \cdot \nabla \phi + \omega^2 \phi = -\frac{\rho_0}{\epsilon_0}, \quad (153)$$

$$\phi \nabla \times \boldsymbol{\omega} + \nabla \phi \times \boldsymbol{\omega} = 0. \quad (154)$$

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The first of both equations is an Euler-Bernoulli resonance with damping term for a negative ω . For further simplification we assume only a dependence of all variables on the X coordinate. These equations then reduce to

$$-\frac{d^2}{dX^2}\phi + \omega_X \left(\frac{d}{dX} \phi \right) + \left(\frac{d}{dX} \omega_X \right) \phi = \frac{\rho}{\epsilon_0} - \omega_X \frac{d}{dX} \phi + (\omega_X^2 + \omega_Y^2 + \omega_Z^2) \phi, \quad (155)$$

$$\begin{bmatrix} 0 \\ \omega_Z \left(\frac{d}{dX} \phi \right) - \left(\frac{d}{dX} \omega_Z \right) \phi \\ \left(\frac{d}{dX} \omega_Y \right) \phi - \omega_Y \left(\frac{d}{dX} \phi \right) \end{bmatrix} = 0. \quad (156)$$

Both equations in (156) are symmetric in ϕ or ω_Y and ω_Y , respectively. For a given ϕ , the equations have the solutions

$$\omega_Y = C_1 \phi, \quad (157)$$

$$\omega_Z = C_2 \phi \quad (158)$$

with constants C_1, C_2 . Inserting these solutions into Eq.(155) leads to the strongly non-linear equation

$$\frac{d^2}{dX^2}\phi - 2\omega_X \frac{d}{dX}\phi - \left(\frac{d}{dX} \omega_X \right) \phi + \omega_X^2 \phi = - (C_1^2 + C_2^2) \phi^3 - \frac{\rho}{\epsilon_0}. \quad (159)$$

This equation can be linearized by setting the constants C_1 and C_2 to zero and assuming a constant or slowly varying ω_X :

$$\frac{d^2}{dX^2}\phi - 2\omega_X \frac{d}{dX}\phi + \omega_X^2 \phi = - \frac{\rho}{\epsilon_0}. \quad (160)$$

Thus an Euler-Bernoulli resonance equation in one dimension similar to Eq.(155) is obtained. The well-known type of solution (with neglected damping) has been discussed in Eqs.(31-35).

3.2 Resonance solutions in cylindrical coordinates

Now we used cylindrical coordinates r, θ, Z to represent the vectors \mathbf{E} and ω . Assuming only radial dependencies of these variables, Eq.(151) takes the form

$$\begin{bmatrix} 0 \\ \omega_Z \left(\frac{d}{dr} \phi \right) - \left(\frac{d}{dr} \omega_Z \right) \phi \\ \phi \left(\frac{d}{dr} \omega_\theta \right) + \frac{\omega_\theta}{r} - \omega_\theta \frac{d}{dr} \phi \end{bmatrix} = 0. \quad (161)$$

Solutions are

$$\omega_\theta = C_1 \phi, \quad (162)$$

$$\omega_Z = C_2 \frac{\phi}{r}. \quad (163)$$

The divergence of \mathbf{E} is

$$\nabla \cdot \mathbf{E} = - \frac{d^2}{dr^2} \phi + \omega_r \frac{d}{dr} \phi + \left(\frac{d}{dr} \omega_r \right) \phi - \frac{\frac{d}{dr} \phi - \omega_r \phi}{r}. \quad (164)$$

Inserting this and Eqs.(162) and (163) into (151) results in

$$\frac{d}{dr} \left(\omega_r \phi - \frac{d}{dr} \phi \right) - \frac{\frac{d}{dr} \phi + \omega_r \phi}{r} = \frac{\rho}{\epsilon_0} + \frac{C_2 \omega_\theta \phi}{r} + \omega_r \left(\omega_r \phi - \frac{d}{dr} \phi \right) + C_1 \omega_Z \phi. \quad (165)$$

This equation can only be hadled if we make the assumptions

$$\omega_r = 0, \quad (166)$$

$$C_1 = C_2 = 0. \quad (167)$$

Computer algebra then gives

$$\left(\frac{d^2}{dr^2} \phi \right) r^2 + \left(\frac{d}{dr} \phi \right) r = -\frac{r^2 \rho}{\epsilon_0} \quad (168)$$

with the solution

$$\phi = \frac{1}{\epsilon_0} \left(\int r \rho(r) \log(r) dr - \log(r) \int r \rho(r) dr + k_1 \log(r) + k_2 \right). \quad (169)$$

This is a logarithmic solution with a particular integral. Somewhat more familiar results are obtained by assuming that the spin connection has the simplified form

$$\boldsymbol{\omega} = \begin{bmatrix} 0 \\ \frac{C_1}{r} \\ C_2 \end{bmatrix}. \quad (170)$$

We then arrive at the equation

$$\left(\frac{d^2}{dr^2} \phi \right) r^2 + \left(\frac{d}{dr} \phi \right) r = -\phi r^2 C_2^2 - \phi C_1^2 - \frac{r^2 \rho(r)}{\epsilon_0}. \quad (171)$$

By substituting the variable r with $C_2 r$ (or κr) this is Bessel's differential equation with an inhomogenous term at the right hand side:

$$\left(\frac{d^2}{dr^2} \phi(\kappa r) \right) r^2 + \left(\frac{d}{dr} \phi(\kappa r) \right) r + (C_1^2 + r^2) \phi(\kappa r) = -\frac{r^2 \rho(\kappa r)}{\epsilon_0}. \quad (172)$$

This equation can be solved analytically, giving in this case the complex Bessel functions $J_\alpha(\kappa r)$ (with complex value α) plus two complicated integral terms stemming from the inhomogeneity. With the imaginary unit i the solution is:

$$\begin{aligned} \phi(\kappa r) &= k_1 J_{iC_1}(\kappa r) + k_2 J_{-iC_1}(\kappa r) \\ &\quad - 2 J_{-iC_1}(\kappa r) \int \frac{\rho(\kappa r) J_{iC_1}(\kappa r)}{\epsilon_0 N} dr \\ &\quad + 2 J_{iC_1}(\kappa r) \int \frac{\rho(\kappa r) J_{-iC_1}(\kappa r)}{\epsilon_0 N} dr \end{aligned} \quad (173)$$

with denominator function

$$\begin{aligned} N &= J_{-iC_1}(\kappa r) J_{iC_1+1}(\kappa r) - J_{-iC_1}(\kappa r) J_{iC_1-1}(\kappa r) \\ &\quad - J_{iC_1}(\kappa r) J_{1-iC_1+1}(\kappa r) + J_{iC_1}(\kappa r) J_{-iC_1-1}(\kappa r). \end{aligned} \quad (174)$$

For purpose of demonstration we show the real and imaginary part of the complex Bessel functions J_i and J_{i+1} in Fig. 1. The real part of the denominator function N of Eq.(174) is plotted in Fig. 2. Because the Bessel functions differ widely in amplitude, the denominator looks quite irregular. This could also be a numerical artifact to a certain extent. We did not further investigate this point since we would have to provide then an own routine for calculation of Bessel functions. Zero crossings of the denominator indicate resonance-like enhancements of the potential.

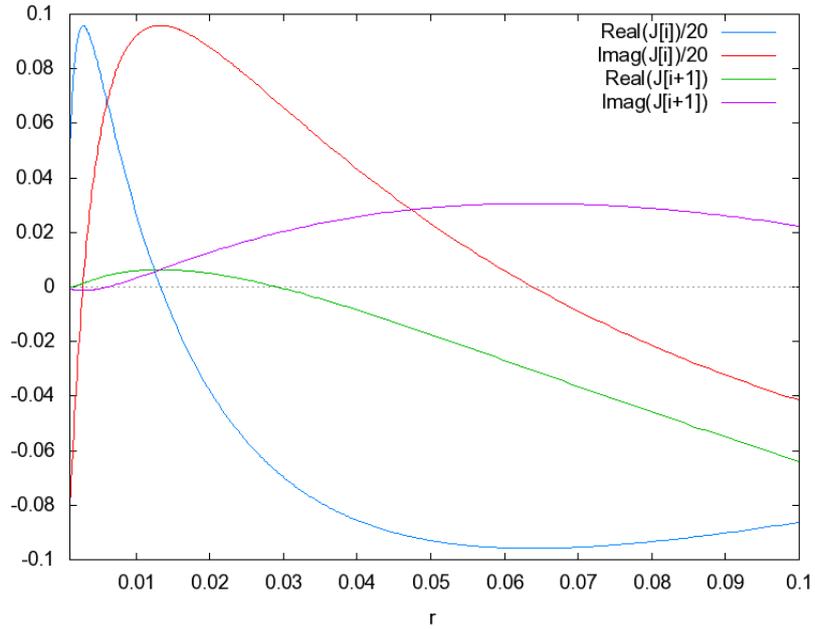


Figure 1: Real and imaginary part of the complex Bessel functions J_i (rescaled) and J_{i+1} .

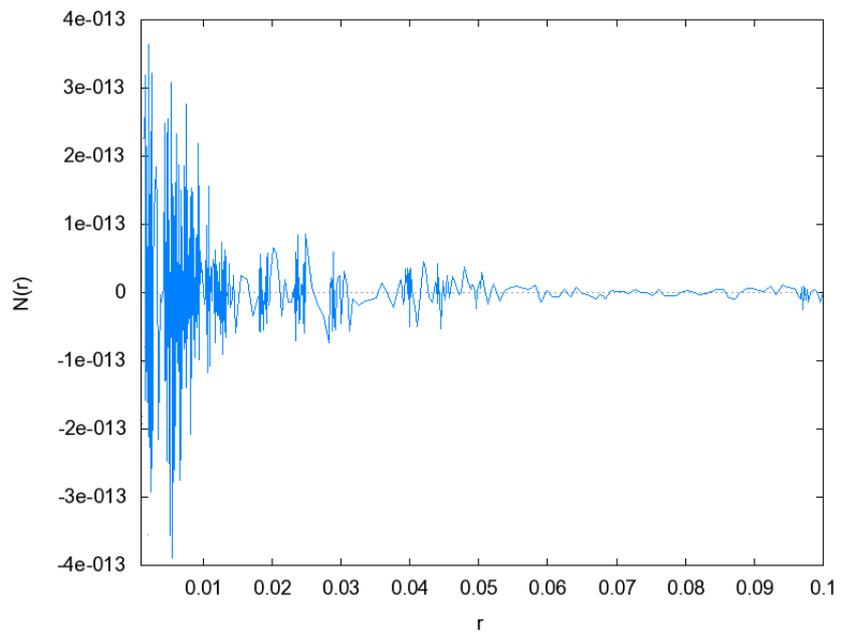


Figure 2: Real part of the denominator function $N(r)$.

Part 3.3 Curvature in the Single Polarization ECE Vacuum

To be a Beltrami field, the vector \mathbf{u} must satisfy the following [11]

$$\mathbf{u} \times \underline{\nabla} \times \mathbf{u} = \mathbf{0} \quad (175)$$

This is equivalent to [12]

$$\underline{\nabla} \times \mathbf{u} = k \mathbf{u} \quad (176)$$

A Beltrami field of the first kind [12] is a potential field where

$$\mathbf{u} = \underline{\nabla} \psi \quad (177)$$

with

$$k = 0 . \quad (178)$$

A Beltrami field of the second kind [12] occurs when

$$k = \text{constant} \quad (179)$$

Meaning that

$$\underline{\nabla} \cdot \mathbf{u} = 0 . \quad (180)$$

A Beltrami field of the third kind [12] occurs when k a function of the coordinate variables and equations (175) or (176) apply.

The ECE vacuum for a single state of polarization is given by

$$\mathbf{B} = \underline{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} = \mathbf{0} \quad (181)$$

$$\frac{E}{2} = -\underline{\nabla} \phi + \boldsymbol{\omega} \phi = -\omega_0 \mathbf{A} - \frac{\partial \mathbf{A}}{\partial t} = 0 \quad (182)$$

For the sake of simplicity, the vacuum state label has been omitted from the variables.

For any scalar potential ϕ

$$\underline{\nabla} \times \underline{\nabla} \phi = 0 . \quad (183)$$

Therefore the field given by $\underline{\nabla} \phi$ is a Beltrami field of the first kind.

From (182) therefore, since $\boldsymbol{\omega} \phi = \underline{\nabla} \phi$, it is a Beltrami field of the first kind, i.e.

$$\underline{\nabla} \times \boldsymbol{\omega} \phi = 0 \quad (184)$$

Using the standard vector expansion, equation (184) can be written

$$\underline{\nabla} \times \boldsymbol{\omega} \phi = \phi \underline{\nabla} \times \boldsymbol{\omega} + \underline{\nabla} \phi \times \boldsymbol{\omega} = \mathbf{0} \quad (185)$$

But from equation (182)

$$\underline{\nabla}\phi \times \boldsymbol{\omega} = \underline{\nabla}\phi \times \frac{\underline{\nabla}\phi}{\phi} = 0$$

Thus equation (185) becomes

$$\underline{\nabla} \times \boldsymbol{\omega}\phi = \phi \underline{\nabla} \times \boldsymbol{\omega} = \mathbf{0} \quad (186)$$

making $\boldsymbol{\omega}$ a Beltrami field of the first kind for the single polarization vacuum, as shown earlier.

Since, the single polarization curvature is given by

$$\mathbf{R}_{spin} = \underline{\nabla} \times \boldsymbol{\omega} \quad (187)$$

We note that the single polarization vacuum requires from equation (186)

$$\mathbf{R}_{spin} = 0 . \quad (188)$$

Also for a single polarization, from equations (8) and (15) we have that

$$\mathbf{R}_{orb} \times \mathbf{A} = \mathbf{0} \quad (189)$$

$$\mathbf{R}_{orb} \cdot \mathbf{A} = 0 \quad (190)$$

This leads to the conclusion that

$$\mathbf{R}_{orb} = 0 \quad (191)$$

From equation (182)

$$-\omega_0 \mathbf{A} = \frac{\partial \mathbf{A}}{\partial t} \quad (192)$$

so that

$$\omega_0 = -\frac{1}{A_i} \frac{\partial A_i}{\partial t} = -\frac{\partial}{\partial t} \text{Log}(A_i) . \quad (193)$$

Using

$$\mathbf{R}_{orb} = -\underline{\nabla}\omega_0 - \frac{\partial \boldsymbol{\omega}}{\partial t} = 0 \quad (194)$$

We see that

$$\boldsymbol{\omega} = \underline{\nabla} \int \frac{\partial}{\partial t} \text{Log}(A_i) dt = \underline{\nabla} \text{Log}(A_i) . \quad (195)$$

Therefore

$$\boldsymbol{\omega} \times \mathbf{A} = \underline{\nabla} \text{Log}(A_i) \times \mathbf{A} \quad (196)$$

A typical term for this cross product can be written

$$\frac{1}{A_i} \left(A_j \frac{\partial A_i}{\partial k_k} - A_k \frac{\partial A_i}{\partial k_j} \right)$$

which in [13] was shown to be zero for the single polarization vacuum.

Therefore

$$\boldsymbol{\omega} \times \mathbf{A} = \underline{\nabla} \times \mathbf{A} = \mathbf{0} \quad (197)$$

Thus \mathbf{A} and by equation (192), $\omega_0 \mathbf{A}$ are Beltrami fields of the first kind.

In summary, we have for the single polarization vacuum, that \mathbf{R}_{spin} and \mathbf{R}_{orb} are zero and that \mathbf{A} , $\boldsymbol{\omega}$, $\underline{\nabla} \phi$, $\omega_0 \mathbf{A}$, and $\boldsymbol{\omega} \phi$ are Beltrami fields of the first kind.

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3 BELTRAMI STRUCTURE OF CURVATURE, BESSEL SOLUTIONS AND GRAPHICS.

(Section by Horst Eckardt and Doug Lindstrom)

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